Study of Injection Transient in Coupled Cell Linac Cavities

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1 INTRODUCTION

In high intensity proton beam linear accelerators of the Meson Factory type with final energy \( \approx 1000 \text{ MeV} \) and average current \( n \times 100\mu A \) there are stringent requirements for maximum losses of particles \( \sim 10^{-3} \div 10^{-4} \) and momentum spread of beam at exit \( \langle \frac{\Delta p}{p} \rangle_\sigma \sim 1 \div 2 \times 10^{-3} \) [1]. The momentum spread minimization in linear accelerator injectors for high energy, high luminosity colliders defines largely all the following stages of accelerating complex.

These requirements arise stringent tolerances for stabilization of the RF accelerating field amplitude and phase in time and in space especially in Coupled Cell Linac with stepped-phase-velocity [2,3].

In case of the stationary deviations of RF parameters from design value (mistuning of cavity) the special procedure can be used with mutual compensation of RF and geometrical parameters errors [4,5,6].

To stabilize the average level of RF amplitude and phase in cavity (fundamental mode) during the injection transient the special feedback with high coefficient and synchronization of beam with compensating pulse of generator are used. However if the generator inputs power in cavity locally in some point and excites all modes while the beam radiates along cavity only the waves with phase velocity smaller than beam velocity in accordance of Cherenkov's law. So reaction of the cavity on the beam and the generator is different. In result it leads to time and space distortions of field along cavity. So a perfect compensation can not be reached.

A careful consideration of "beam loading" effect in multimode approach shows that in some accelerating structure real momentum spread is in excess of waited design value calculated in one mode approach. Especially it is very important for linac where the beam pulse length comparable with the transient time. The permanent distortion of RF field depends on characteristics of accelerating structure so as the couple coefficient, the quality factor and the length of cavity (number of cells).

In this paper the interaction of beam with the cavity is considered in multimode approach. The final expressions are presented in simplifying assumption through usual characteristics of accelerating structure. Some practical recommendations for decreasing the RF field distortion are suggested as well.

2 THE MULTIMODE APPROACH TO THE CAVITY DESCRIPTION

The most distributed structure for proton beam accelerating is the standing wave structure consisting of resonators-cells sequence. The behavior of standing wave accelerator cavity made up of chains of resonant cells is conveniently studied by considering the properties of a chain of coupled resonators with general couplings, frequencies and quality factors [8]. This coupled resonator model is useful so long as it is possible to describe the behavior using separated modes of a single cavity. Numerical calculations show that we lose insignificant part of information in this model and it may be used to analyze the radiotechnical properties of structure.

However the study of the transient is rather by the eigenfunction method. Why is it so? It is known that the problem of forced oscillation of string can be solved by the expansion string eigenfunction series (Bernoulli's method) and by analysis of running and reflected waves (d'Alembert's method). It is obviously that both methods lead to the same result. Moreover the transition from each to other is not difficult and is required sometimes for interpretation. Really at first moment of transient there is only direct and back waves running from the point of power input:

\[
E = \sum_s \left( C_s E_s + C_{-s} E_{-s} \right),
\]

where \( C_s \) and \( C_{-s} \) - functions of longitudinal coordinate \( z \) and

\[
E_s = E^0_s(r, z) e^{i h_{rs}},
\]

\[
E_{-s} = E^0_s(r, z) e^{i h_{rs}},
\]

where \( E^0_s(r, z) \) is distribution function of field in resonant cell, \( h_r \), is waveguide number. In the waveguides with dissipation of energy \( h_r = \mu / d + j \alpha, 0 \leq \mu \leq \pi \), where \( \mu \) - advanced phase per period \( d \), \( \alpha \) is the damping coefficient of wave and equals to \( 1/l_4 \), where \( l_4 \) is the length on which the wave damps in \( e \) times. The transient in cavity can be described by summing up direct and reflected waves. As example, let's write the following expression for any \( s - th \) mode through the sum of direct and \( k \) times reflected waves:

\[
E_s = E^0_s e^{j \varphi_s} e^{-j \alpha_s} e^{j \mu_s / d} \left( 1 + (1 + \ldots + (1 + e^{2j \mu_s / d} e^{-2j \alpha_s} \hat{G}_1 \hat{G}_2) e^{2j \mu_s / d} e^{-2j \alpha_s} \hat{G}_1 \hat{G}_2) \ldots e^{2j \mu_s / d} e^{-2j \alpha_s} \hat{G}_1 \hat{G}_2) \right)
\]
where \( L_c \) is the length of cavity and \( G_1, G_2 \) are the complex coefficients of reflection at ends of cavity. If for \( k \)th mode the resonance condition \( G_1 * G_2 = 1 \) and \( \mu L_c / d = \pi m, (m = 1, 2, 3,...) \) is fulfilled itself then

\[
E_r = \frac{1}{1 - e^{-2\omega L_c/\omega_{group}}} E_0 \exp(-\alpha x) e^{i\mu z / d},
\]

(5)

The propagation time of wave front along cavity is \( \tau_{space} = L_c / \omega_{group} \). Taking into account that \( \tau_{space} \) \( k = t \), and \( 1/(\alpha * \omega_{group}) = 2 * Q / \omega \) we can get \( E_r = (1 - e^{-t/\tau}) \), where \( \tau_{time} = 2Q / \omega \) is the constant of damping of standing wave in time. The relation between \( \tau_{time}, \tau_{space} \) and the beam pulse length \( \tau_{beam} \) defines four types of accelerating structure:

1. \( \tau_{beam} \gg \tau_{time} \gg \tau_{space} \) is the INR variant.
2. \( \tau_{beam} \gg \tau_{space} \gg \tau_{time} \) is the waveguide variant.
3. \( \tau_{beam} \gg \tau_{time} \gg \tau_{space} \) is the LAMPF variant.
4. \( \tau_{beam} \sim \tau_{space} \sim \tau_{time} \) is the SSC variant.

As it will be shown below this relation determinates the parameters of beam during the transient. During \( \tau_{space} \) the cavity can be considered as waveguide where the spectrum of modes with \( \mu \) variation is continuous. Approximately after one-two runs of the waves the modes with discrete \( \mu \) predominate. The general solution is found as the expansion in a series of eigen functions for cavity in the following form not forgetting that during first \( \tau_{space} \) we have to integrate over all \( \mu \) range:

\[
E = \sum A_s(t) E_s(\tau) e^{-j\omega t},
\]

(6)

\[
H = \sum B_s(t) H_s(\tau) e^{-j\omega t},
\]

(7)

Here \( E_s \) is the sum of the direct and reflected waves:

\[
E_s = E_s^0 + E_s^\prime \cos(\mu z / d),
\]

(8)

besides \( \mu \) has additional requirement \( \mu N = \pi s \).

\[
E_s = E_s^0 \cos(\pi s N / N),
\]

(9)

where \( n \) is number of cell. The same is for eigenfunction of magnetic field \( H_s \). The eigen functions \( E_s \) and \( H_s \) are connected with each other by the Maxwell homogeneous equations:

\[
\text{rot} E_s = j \mu_0 w_0 H_s,
\]

(10)

\[
\text{rot} H_s = -j \omega_0 w_0 E_s.
\]

(11)

Now substituting (6,7,10,11) in Maxwell unhomogeneous equations we get

\[
\frac{\partial A_s}{\partial t} - j w A_s + j w B_s = - \frac{1}{N_s} \int j^m E_s^m d\nu
\]

(12)

\[
\frac{\partial B_s}{\partial t} - j w B_s + j w A_s = - \frac{1}{N_s} \int j^m H_s^m d\nu
\]

(13)

\[
N_s = \epsilon \int E_s E_s^* d\nu
\]

(14)

where \( j^m \) and \( j^e \) are magnetic and electric currents.

Eliminating \( A_s \) and \( B_s \) and taking into account that \( \omega_s = \omega_0 N_s / (1 - j/Q) \) we can receive the following equations:

\[
\frac{\partial A_s}{\partial t} + \frac{i(w_0^2 - w_s^2) + w_s^2 / Q_s}{2w} A_s = \frac{1}{2N_s} \int j^m E_s^m d\nu - \frac{w_s^2}{2N_s} \int j^m H_s^m d\nu
\]

(15)

Solving this differential equation we have:

\[
A_s = \frac{Q_s}{(N_s w_s)} \left[ j s j^m E_s d\nu + j s j^m H_s d\nu \right] \int \frac{1 + j2\Delta_s \omega_s Q_s / w_s}{1 - e^{-2\Delta_s \omega_s e^{-(W_0 + t)/2Q_s}}}
\]

(16)

where \( \Delta_s = w - w_s \).

The equation with \( B_s \) is similar. For simplification the norm \( N_s \) is determined as the stored energy in cavity at the nominal level of accelerating field. Then integral \( \int j^m E_s d\nu \) is a power of Cherenkov's beam radiation of \( s \)th mode and other integral \( \int j^m H_s d\nu \) is the generator power of \( s \)th mode.

It is obviously that for proton linac the beam has to be injected only after stabilizing of electromagnetic field at nominal level (\( A_s = 1 \) for fundamental mode). Before beam injection full power from generator is spend on support electromagnetic field and goes to losses (\( P_{loss} \)). In moment of beam switching the generator has to increase a power on value approximately equals to the beam power.

Here we would like to emphasize two features.

1. First moment. Generator inputs power in some point, but beam radiates power distributively almost uniformly along cavity. So if amplitudes \( j^e \) and \( E_s \) don't depend on the longitudinal coordinate then integral \( \int j^m E_s d\nu \) equals to zero for all modes except fundamental mode of beam. And we can suppose that for fundamental mode \( P_{generator} = P_{beam} \). In the same time the generator excites all modes proportionally to ratio \( RP = P_{beam} / (Q_s P_{loss}) \) This value is usually about 0.3 (LAMPH,INR,SSC).

2. Second moment. The velocity of beam propagation along cavity equals to phase velocity (synchronization condition). But the velocity of the wave front of generator equals to the group velocity \( \omega_{group} - K_c \pi \text{phase} / 4 \). \( K_c \) is the couple coefficient of structure ( for SSC and LAMPF \( K_c = 0.05 \) for INR \( K_c = 0.5 \)). Besides the coefficient of space damping for a generator wave (\( K_c \)) times as much as the coefficient of space damping for Cherenkov's wave excited by beam.

3 THE STEADY REGIME

In general case the excitation of any mode can be estimated by following expression:

\[
A_s = \frac{RP(1 - j2\Delta_s \omega_s Q_s / w_s)}{1 + 4(\Delta_s \omega_s Q_s / w_s)^2 \left[1 - e^{-2\Delta_s \omega_s e^{-(W_0 + t)/2Q_s}}\right]}
\]

(17)
In the steady regime when \( t \gg \tau_{\text{time}} \) and \( 2\Delta_\omega Q/u_s \approx 1 \), then

\[
A_s \simeq \frac{RP}{2\Delta_\omega Q/u_s}.
\]  

(19)

Let's calculate the sum of two neighbor modes nearest to the fundamental mode \( S = N/2 \pm 1 \) which are the slope of field along a cavity with the dispersion relation \( \omega_s = \omega_{N/2}(1 + K/c_2 \cos(\pi s/N)) \)

\[
A_{-+,+} \simeq \frac{2RP}{(2\Delta_\omega Q/u_s)^2}.
\]  

(20)

The last demonstrates the main advantage of structure with \( \frac{\pi}{2} \) fundamental mode.

Substituting \( \Delta_\omega Q/u_s = \frac{\pi K}{N} \) in (20), which is satisfied for nearest modes, get:

\[
A_{-+,+} \simeq RPP(\frac{\sqrt{2N}}{\pi K Q_s})^2.
\]  

(21)

The slope amplitude equals \( 2 + A_{-+,+} \) as long as the expression between the brackets much less unit the structure with fundamental \( \tau/2 \) mode saves its advantages and has enough small distortion along cavity. For instance, this slope of field for INR equals \( 2 \times 10^{-4} \), for LAMPF \( 1 \times 10^{-2} \) and SSC \( 6 \times 10^{-2} \).

4 THE TRANSIENT

During the transient \( t \sim \tau_{\text{time}} \) (for INR, LAMPF, SSC \( \tau \sim 3 \div 5 \mu s/\text{sec} \) we have to take in account all expression for \( A_s \).

Eliminating the real part of \( A_{-+,+} \) in (18) let's write the expression for two symmetrical modes:

\[
Re A_{-,+} = \frac{RP}{\Delta_\omega Q/u_s} e^{-\omega_s t/2Q_s} = \frac{2N}{\pi K Q_s} RPP\sin(\Delta_\omega wt)e^{-\omega_s t/2Q_s}.
\]  

(22)

Now calculate the distortion of field along cavity for all modes taking in account its distribution (8)

\[
\frac{\delta E}{E}(z,t) = RPe^{-t/\tau} \sum_{s=0}^{N/2} \sin(\Delta_\omega wt) e^{-\omega_s t/2Q_s} \cos\frac{2\pi z}{T}.
\]  

(23)

where \( T = 2L, \) and \( \tau = 2Q_s/u_s \) is taking approximately the same for all modes.

Now we can do the convolution of this series using \( t = z/v_{\text{group}} \) and the linearized dispersion relation:

\[
\frac{\delta E}{E}(z,t) = e^{-t/\tau} \frac{N}{K/Q} RPP(z - v_{\text{group}}t)
\]  

(24)

where \( K = 1 \), if \( z \leq v_{\text{group}}t \) and 0 if \( z \geq v_{\text{group}}t \). During \( \tau_{\text{space}} \) we solve this problem through waveguide eingen function \( E = \int A_s(z) E_x e^{2\pi i z/d} dz \) and just overwrite the final result:

\[
\frac{\delta E}{E}(z) = e^{-t/\tau} u L \cos \delta/(2Q v_{\text{group}})
\]  

(25)

5 CONCLUSION

It is obviously that if we will excite cavity in middle of cavity then \( \delta E/E \) will be approximated by series with the less number of modes, but the distortion will remain almost the same. After injection of beam and the simultaneous input of power in cavity three types of wave run alone cavity: wave of generator fundamental mode, Cherenkov's wave of beam and additional wave of nonfundamental modes with amplitude proportional to \( \tau_{\text{space}}/\tau_{\text{time}} \). The nonfundamental wave being damped proportionally to \( e^{-t/\tau_{\text{time}}} \) propagates with group velocity. After reflection at end of cavity the wave changes sign every times. Since the beam velocity is much more than the group velocity of the nonfundamental waves wave the beam "sees" significant distortion of field during transient. This effect will give additional momentum spread of beam largely to side of high energy. One may suppose that the beam's excitation of the waves with phase velocity only smaller than the beam velocity will lead to changing of the advanced phase per cell and hence to changing of the structure phase velocity. To decrease this effect it should use the beam pulse with \( \tau_{\text{front}} \approx 1/\Delta_\omega \). However the beam intensity modulation can lead to the intensity modulation of current in the synchrotron if the injection time is comparable with the beam pulse length in linac. Other way is to use regime with more longer pulse of beam and less loading. Nevertheless in any case the structure with high coupling coefficient improves quality of beam.

6 REFERENCES


