The Beam Steering in Beamlines and Rings Using the Methods of Sequential Filtering

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1. INTRODUCTION

The methods of sequential filtering were passed over in the field of accelerator monitoring and control, despite the fact of these methods' wide application in different fields of science and technic (dynamic system control, radio location, radio technic, communication etc.).

Here we try to stimulate the application of these methods in wide field of accelerator identification and control.

That's why the identification theory and sequential filtering survey, together with algorithms based on the methods of sequential filtering for the beam-kick errors and beam-focus errors estimation, for the trajectory and closed orbit correction in the beam-lines and rings are presented.

The general conclusions are illustrated by the simulation results.

2. THE MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 The Choice of Model and Optimization Criterion

The object and observation on the K beam position monitor (BPM) are described by the equation

$$Z_k = h_k X + \nu_k \qquad k=0,1,\dots,N \qquad (1)$$

where Z_k is the measurement vector on the K BPM (X,Y), X is the vector of unknown parameters, ν_k is additive white noise with 0 mean value, h_k is the influence matrix of X parameter vector on the Z_k measurement. The matrix R is the intensity or covariance matrix of ν white noise.

The main problem of the estimation theory is finding the estimation of the system parameter vector X using the measurement vector Z. In our case the system parameter X can be errors in the dipole or quadrupole strength and in BPM, beam transverse displacements and $\Delta p/p$, correctors strength or corrector calibration coefficient, etc. The Z measurement is the sum of beam transverse displacement and the noise of the BPM.

Under the optimization criterion of estimation for the identification problem one understands the losses connected with not achieving absolutely precise identification.

The choice of criterion depends on apriori knowledge about the stochastic features of X and noise ν [1].

I. Least Square Criterion (LS).

The LS criterion has a form

$$J = 0.5\sum_{0}^{N}[Z_k - h_k X]^T S_k [Z_k - h_k X] \qquad (2)$$

where S_k is nonnegativly defined symmetric weight matrix. LS estimate $\hat{X}_{LS} = \min_X J$ is found from condition

$$\left[\frac{\partial J}{\partial X}\right]_{X=\hat{X}_{LS}} = 0.$$

For the LS estimation no apriori information about the stochastic features of X and ν is required.

II. Maximum Likelihood Criterion (ML).

Let the function $P(Z|X)$ be probability density of Z measurement for fixed X. ML estimate $\hat{X}_{ML} = \max_X P(Z|X)$ is found from condition

$$\left[\frac{\partial P(Z|X)}{\partial X}\right]_{X=\hat{X}_{ML}} = 0,$$

which under certain conditions is equivalent to results of functional

$$J = 0.5\sum_{0}^{N}[Z_k - h_k X]^T R_k^{-1}[Z_k - h_k X] \qquad (3)$$

minimization.

For ML estimation we need only information about the R_k error covariance matrix on K measurement.

III. Maximum Aposteriori Criterion (MAP).

Obviously, as the estimate of parameter vector X it's natural to take such X which maximize the probability density $P(X|Z)$ for fixed measurement Z $\hat{X}_{MAP} = \max_X P(Z|X)$. The necessary condition for MAP criterion is

$$\left[\frac{\partial P(X|Z)}{\partial X}\right]_{X=\hat{X}_{MAP}} = 0,$$

Here we need information about matrix R and error covariance matrix of estimation $P_o = E[\Delta X_o, \Delta X_o]$, where $\Delta X_o = E[X] - \hat{X}$ and $E[\cdot]$ is mean value. It can be shown, that under certain conditions the maximization of $P(X|Z)$ is equivalent to minimization of functional

$$J = 0.5\Delta X_o^T P_o^{-1}\Delta X_o + 0.5\sum_{0}^{N}[Z_k - h_k X]^T R_k^{-1}[Z_k - h_k X] \quad (3)$$

The mathematical form derived for MAP criterion includes those derived for ML and LS criterions. So, if the matrix R_k and P_o are known one can use MAP estimator. For ML estimation equation (4) mathematically reduces to (3) if $P_o^{-1} \to 0$, which corresponds to no apriori information $P_o \to \infty$. If we have no a priori information about noise, too, equation (4) reduces to (2) $P_o \to \infty$ and S_k weight matrix can be a unit matrix as well as any nonnenegativly defined matrix.

For the parameter estimation we'll use algorithms based on sequential Kalman filter [2]. Generally Kalman filter needs information about mean value and dispersion of unknown parameter and noise. but we use less information. Naturally, the estimation will be suboptimal, but the uniform structure of the filter is preserved and it's easy to get the optimum estimate if more information is available.

2.2 Sequential Filtering Algorithms

I. Parameter Estimation

If our object and observation are described by equation (1), the sequential estimation algorithm of parameter vector X has a form [2]

$$\hat{X}^{k+1}=\hat{X}^{k} + P_{k+1}h_{k+1}^{T}R_{k}^{-1}[Z_{k+1}-h_{k+1}\hat{X}^{k}] \qquad (5)$$

$$P_{k+1}=P_{k} - P_{k+1}h_{k+1}^{T}[h_{k+1}P_{k}h_{k+1}^{T} + R_{k}]^{-1}h_{k+1}P_{k}$$

where $\hat{X}^{k}$ is the parameter vector estimation after the measurement Z_k is used for estimation. Matrices h_k, R_k, P_k are already described in equation (2-4).

The initial values of $\hat{X}^{0}$ and P_0 will be changed in dependence on our knowledge about the stochastic features of the unknown parameter vector X as it was described above. If E[X] and P_0 are known $\hat{X}^{0}$=E[X] and algorithm (5) will produce MAP estimate, if E[X] is unknown then $P_0 \to \infty$ (in the numerical realization P_0 can be a diagonal matrix, if we have no any information about the correlation between the elements of vector X, and the diagonal elements are chosen to be $10^{2}-10^{4}$ order of magnitude of expected value X) and $\hat{X}^{0}$=0. In this case (5) produces ML estimate. If R_k matrix is unknown, any weight matrix can be taken and LS estimation will be obtained from (5). The weight matrix can be chosen in dependence on our assumptions about the measurement features of BPM.

Up to now we treat with the parameters, which have linear influence on the measurement. For nonlinear parameter estimation Kalman nonlinear filter is assumed. Particularly for the quadrupole strength estimation this algorithm can be applied.

Here the observation will be described by the equation

$$dZ_k=h_k(X_Q,dC)+\nu_k \qquad (6)$$

where X_Q is the unknown strength of the quadrupole, dC is the known kick applied on the corrector situated upstream of the unknown quadrupole, $h_k(X_Q,dC)$ is the influence matrix of X_Q and dC on K BPM, dZ_k is the difference in the trajectory measurement on K BPM before and after dC kick is applied.

For the X_Q estimation the nonlinear sequential filter has a form [2]

$$\hat{X}_Q^{k+1}=\hat{X}_Q^{k} + P_{k+1}H_{k+1}^{T}R_{k}^{-1}[Z_{k+1}-h_{k+1}(\hat{X}_Q^{k},dC)] \quad (7)$$

$$P_{k+1}=P_{k} - P_{k}H_{k+1}^{T}[H_{k+1}P_{k}H_{k+1}^{T}+ R_{k}]^{-1}H_{k+1}P_{k}$$

where

$$H_{k+1}= \left[\frac{\partial h_{k+1}}{\partial X_Q}\right]\hat{X}_Q^{k}$$

and all matrices and vectors have the same meaning as in (5).

II.The Estimation of Control

For the estimation of control kicks of the correctors sequential algorithm is used, which is based on the Kalman filter [3,4].

The trajectory and closed orbit control problem is the minimization of measurement Z_k with respect to the central trajectory. It means the minimization of functional

$$J=0.5\sum_{N1}^{N2}[Z_k+h_kX]^{T}R_k^{-1}[Z_k+h_kX] \qquad (8)$$

where X is the vector of used correctors, N1 and N2 are the numbers of first and last BPM of the section in the beamline. For the minimization of functional algorithm (5) is assumed, where only the first equation is changed into

$$\hat{X}^{k+1}=\hat{X}^{k} + P_{k+1}h_{k+1}^{T}R_{k}[-Z_{k+1}-h_{k+1}\hat{X}^{k}] \quad (8')$$

The structure of algorithm is the same for the coupled case.

It's more efficient to correct the trajectory in relatively short sections of beamlines using correctors from that sections. It's possible to use bump approach to the trajectory correction in the beamlines i.e. correct the trajectory in a section without disturbing the one outside the section. It can be realized by minimizing the functional (8) with the constrains

$$h_{N2+1}X=0,\ h_{N2+2}X=0. \qquad (9)$$

It's supposed to include these constrains into the functional (8) with big weight matrices.

$$J=0.5\sum_{N1}^{N2+2}[Z_k+h_kX]^{T}R_k^{-1}[Z_k+h_kX] \qquad (10)$$

where $Z_{N2+1}=Z_{N2+2}=0$ and the diagonal elements of the matrices R_{N2+1} and R_{N2+2} must have big values ($\sim$100÷300). Two constrains are changed into four for the coupled case:

$$h_{N2-1}X=0,\ h_{N2}X=0,\ h_{N2+1}X=0,\ h_{N2+2}X=0. \qquad (11)$$

The control kicks estimation with these algorithms is attractive, because the algorithms have a common structure for different requirement to the control problem and don't need calculations of inverse matrix. The elements of R matrix can be chosen in a way to exclude the measurements of doubtful BPM or increase the influence of those where the trajectory minimization is more desirable.

The sequential algorithms can be applied for the closed orbit correction, too. In equation (8) the matrix h_k is the influence matrix of correctors vector X on closed orbit on K BPM. In some cases it is desirable to minimize the closed orbit in the long sections using the correctors from the sections without disturbing the one in the rest part of the ring. We suppose the application of multi corrector bump for this problem. Here two correctors are fixed in the ends of section, which don't take part in minimization, then with remained correctors the minimization procedure is applied for the chosen section. With the consideration of bump condition the i corrector influence on j BPM will be

$$-\sqrt{\beta_i\beta_j}\,\frac{\sin(\varphi_{jo})\sin(\varphi_{ni})}{\sin\varphi_{no}} \qquad j\le i,$$

$$-\sqrt{\beta_i\beta_j}\,\frac{\sin(\varphi_{io})\sin(\varphi_{nj})}{\sin\varphi_{no}} \qquad j>i,$$

where β_i is beta function on i position, φ_{ij} is phase advance between positions i and j, 0 and n are the positions of first and last fixed correctors.

The algorithm (8') is supposed to be used for the minimization. The strengths of two end correctors and defined from bump condition

$$C_0=\frac{1}{\sqrt{\beta_0}\,\sin\varphi_{n0}}\sum_{1}^{n-1}\sqrt{\beta_i}\,\sin(\varphi_{in})C_i$$

$$C_n=\frac{1}{\sqrt{\beta_n}\,\sin\varphi_{n0}}\sum_{1}^{n-1}\sqrt{\beta_i}\,\sin(\varphi_{0i})C_i$$

where C_i $i=1,\ldots,n-1$ are the correctors strengths, which are the result of the algorithm (8´).

One of the main difficulties in the closed orbit correction problem are the BPM and correctors big number. Since the suggested sequential algorithm processes the BPM measurement sequentially and doesn´t need the inverse matrix calculation, the one side of problem is seemed to be overcome, but the dimensions of P matrix can cause calculation problem for the big number of correctors (the dimension is N*N, where N is the number of the correctors).

Here the application of singular analyses is possible for the decrease of the estimation parameters´ number [5].

Our model (1) can be written in a general form

$$Z=HX+\nu$$

where
$Z=[Z_1,Z_2,\ldots,Z_N]^T$, $H=[H_1,H_2,\ldots,H_N]^T$, $\nu=[\nu_1,\nu_2,\ldots,\nu_N]^T$. If $H=USV^T$ decomposition is realized, where U and V are orthogonal matrices, S is a diagonal matrix and $g=U^TZ$, $X=VP$ the new system will be

$$g=SP+\nu.$$

Since S is a diagonal matrix, the influence of new parameters P on the residuals is uncorrelated. The sum of squares of residuals is

$$\rho^2=||Z-HX||^2=||g-SP||^2$$

and one can choose desirable sum of squares of residuals in dependence on the number of nonzero elements of vector P. So if $P=[P_1,P_2,\ldots,P_k,0,\ldots,0]$, then

$$\rho^2=\sum_{k+1}^{n}g_i^2$$

So number of nonzero elements in vector P can be chosen and then minimization of functional

$$J=[Z-HVP]^TR^{-1}[Z-HVP]$$

with algorithm of sequential filtering is applied.

3. THE RESULTS OF SIMULATION

Set of programs is written for the investigation of suggested algorithms. They can be applied for the identification and model-based control of beamlines and rings. The investigation are carried out on the model of the electron beamline from ´PETRA´ to ´HERA´ (DEZY). The results of beam initial parameter estimation ($X,X´,Z,Z´,\Delta p/p$) are presented in [3,4].

In fig.1 the example of trajectory correction on a section of the beamline is presented. Here the trajectory between sixth and twelfth monitors is corrected by the sequential algorithm. The trajectory measurements are supposed to be done by the screen monitors with accuracy of 0.2mm.

In fig.2 the application of bump control between eighth and fourteenth monitors is shown. In both figures the dashed line is the trajectory before correction and the solid line the one after correction.

In fig.3 an example of quadrupole strength estimation is presented. The real strength of quarupole is $0.8137m^{-2}$. Here the sequential estimation versus the number of measurements is presented.

4. CONCLUSIONS

So, the algorithms based on the sequential filtering methods are seemed to be efficient in accelerator control and identification. They have a common structure and are flexible for the different requirements upon the control problems and different levels of apriori information.

These algorithms don´t need the calculation of inverse matrix are very stable to measurement errors. They don´t need big computer memory due to sequential processing of measurements and self-learning character of algorithms.

5. REFERENCES

[1] A.P. Sage and J.L. Melsa, System Identification, N.-Y. & London: Academic Press, 1971.

[2] A.P. Sage and J.L. Melsa, Estimation Theory with Application to Communication & Control, N.-Y.: McGraw-Hill, 1972.

[3] S.H. Ananian and R.H. Manoukian DESY-M-90-15, November 1990.

[4] S.H. Ananian and R.H. Manoukian "The Program for Automatic Control of Beam Transfer Lines", Proceedings of IEEE Particle Accelerator Conference, San-Francisco, USA, 1991.

[5] Ch.L. Lawson and R.J. Hanson, Solving Least Squares Problems, Englewood Cliffs, New-Jersey: Prentice-Hall,Inc, 1974.

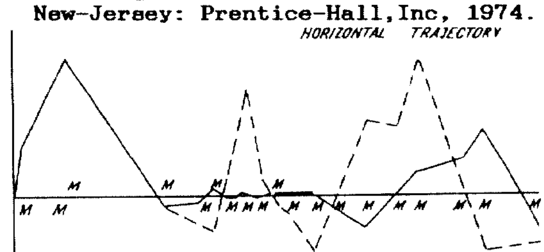

fig.1

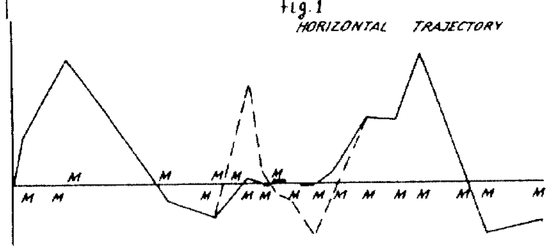

fig.2

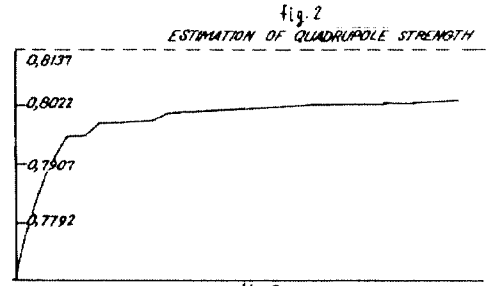

fig.3