A NUMERICAL METHOD FOR THE RAY-TRACING OF POLARIZED BEAMS

François Méot
Groupe Théorie
Laboratoire National Saturne
CE SACLAY, 91191 Gif-sur-Yvette Cedex, France

Abstract

A numerical method for solving the Thomas-BMT differential equation is presented. It has been implemented in the ray-tracing code Zgoubi. This is illustrated by ray-tracing in the spectrometer SPES 2 and multiturn tracking of resonant depolarization in the synchrotron Saturne.

1 INTRODUCTION

A numerical method for solving the Thomas-BMT [1] differential equation of spin motion is presented. It is based on the numerical formalism used for the tracking of charged particles in the ray-tracing code Zgoubi [2], in which it has been implemented recently [3]. This is illustrated by ray-tracing in the spectrometer SPES 2 [4], and by a detailed study of resonant depolarization by multiturn tracking in the synchrotron Saturne [5].

2 NUMERICAL SPIN TRACKING

The ray-tracing code Zgoubi provides a numerical solving of the Thomas-BMT differential equation of motion of the spin $\vec{S}$ of a particle travelling in a magnetic field $\vec{B}$ [1].

$$d\vec{S}/dt = (q/\gamma m)\vec{S} \times \vec{\Omega} \qquad (1)$$

where $\vec{\Omega} = (1+\gamma G)\vec{B} + G(1-\gamma)\vec{B}_{\|}$ (2)
q, m, γ and G are the charge, rest mass, Lorentz factor and gyromagnetic anomaly ; $\vec{B}_{\|}$ is the component of $\vec{B}$ parallel to the particle velocity $\vec{v}$. The code handles more practical notations : let $v = \| \vec{v} \|$, $ds = vdt$ (differential path), $\vec{S}' = d\vec{S}/ds = (1/v)\ d\vec{S}/dt$, B= $\| \vec{B} \|$ and $B\rho = \gamma mv/q$ (rigidity), $\vec{b} = \vec{B}/B\rho$ and $\vec{b}_{\|} = \vec{B}_{\|}/B\rho$; (2) becomes

$$\vec{\omega} = \vec{\Omega}/B\rho = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{\|} \qquad (3)$$

and (1) can then be written

$$\vec{S}' = \vec{S} \times \vec{\omega} \qquad (4)$$

This equation is solved by Taylor expansion of $\vec{S}$

$$\vec{S}(M_1) = \vec{S}(M_0) + \vec{S}'(M_0)ds + \vec{S}''(M_0)ds^2/2 + \ldots \qquad (5)$$

which gives the spin $\vec{S}$ at point M_1 from its value at point M_0, after a step ds. The derivatives $\vec{S}^{(n)}$ involved in (5) are obtained by differentiating (4) :

$$\vec{S}'' = \vec{S}' \times \vec{\omega} + \vec{S} \times \vec{\omega}',$$

$$\vec{S}''' = \vec{S}'' \times \omega + 2\vec{S}' \times \vec{\omega}' + \vec{S} \times \vec{\omega}'', etc.. \qquad (6)$$

where (3) provides the $\omega^{(n)}$; the normalized field $\vec{b}$ and its derivatives $\vec{b}^{(n)}$ are intrinsically provided by the code (they define the magnetic element of concern); noting $\vec{u} = \frac{\|\vec{v}\|}{v}$, $\vec{b}_{\|}$ and $\vec{b}_{\|}^{(n)}$ are given by

$$\vec{b}_{\|} = (\vec{b}\cdot\vec{u})\vec{u},\ \vec{b}_{\|}' = (\vec{b}'\cdot\vec{u} + \vec{b}\cdot\vec{u}')\vec{u} + (\vec{b}\cdot\vec{u})\vec{u}',\ \text{etc...} \qquad (7)$$

3 THE RAY-TRACING CODE ZGOUBI

Zgoubi [2] has been used since long for ray-tracing in beam lines and spectrometers. The numerical method described above is drawn from that used in Zgoubi for solving the Lorentz equation (normalized)

$$\vec{u}' = \vec{u} \times \vec{b} \qquad (8)$$

by Taylor expension of the vector position $\vec{R}(M_1)$

$$\vec{R}(M_1) = \vec{R}(M_0) + \vec{u}(M_0)ds + \vec{u}'(M_0)ds^2/2 + \ldots \qquad (9)$$

This resulted in a straightforward implementation of the spin tracking [3]. Fig. 1 gives an illustration of ray-tracing in the measured 2-D field maps of the QDD spectrometer SPES 2 [4]. Upon spin tracking, such results as mean polarization matrices can be obtained (Table 1).

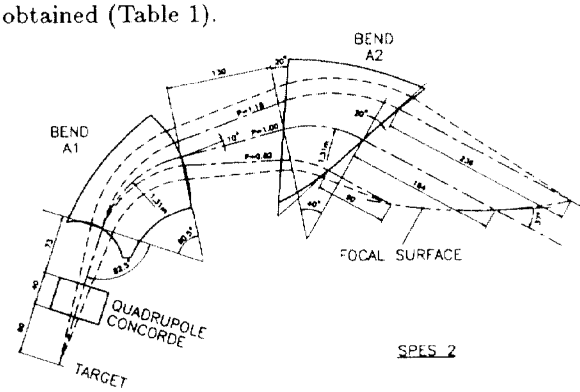

Fig. 1. The QDD spectrometer SPES 2 of Laboratoire National SATURNE. Particles with different

initial coordinates are ray-traced with Zgoubi. Those travelling in non homogeneous field regions experience slight depolarization (Table 1).

700 MeV/c			826 MeV/c		
$\gamma G\alpha = 234.82°$			$\gamma G\alpha = 250.93°$		
0.94	−0.01	0.00	0.94	0.00	−0.02
0.00	0.99	0.01	0.01	0.99	0.00
0.00	−0.01	0.94	−0.02	−0.01	0.95

Table 1. Average polarization matrices [S] of SPES 2 (such that $(< Sx >, < Sy >, < Sz >)_{focal\ plane} = [S]\ (Sx, Sy, Sz)_{target}$), for two momenta: 700 (central momentum) and 700+18% MeV/c. The means < > of the polarization components at the focal plane are calculated from a sampling of 200 particles leaving the target at angles randomly distributed within the acceptance of SPES 2 (± 50 mrd horizontally and vertically). $\gamma G\alpha$ is the spin rotation of a particle undergoing an horizontal deflection α.

4. MULTITURN TRACKING IN SATURNE

Zgoubi has been improved in order to provide options for multiturn tracking [3], thus allowing the analyzes, and correction study of resonant depolarizations. This is illustrated by the study of the resonance $\gamma G = 7 - \nu_z$, in the synchrotron Saturne [5] (Fig.2). Details of these investigations are given in Figs 3-5 and their captions. We show the stability and accuracy of multiturn particle tracking in terms of emittance preservation. We then study the static case at the vicinity of the resonance, and its related formulations [6] and show the agreement of depolarization crossing with the Froissard-Stora formula [7] and its alteration by momentum dispersion. Finally, spin tracking is performed through two strong neighbouring resonances (Fig. 6).

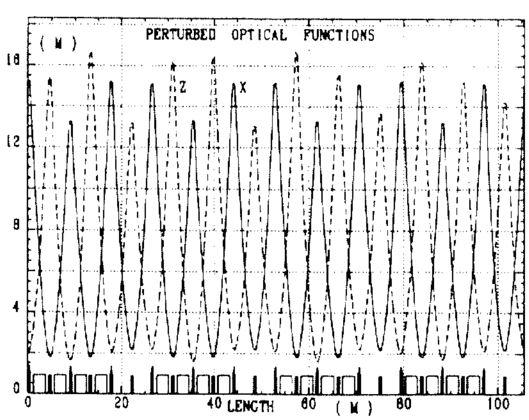

Fig. 2. Optical functions of Saturne Synchrotron, as obtained from preliminary matrix calculations: their values at $s = 0$ are $\beta_x = 15.18$ m, $\gamma_x = 0.581\ m^{-1}$ (horizontal) and $\beta_z = 2.063$ m, $\gamma_z = 0.572\ m^{-1}$ (vertical), including a gradient perturbation of 1% in the second quadrupole. The tunings are $\nu_x = 3.6375$, $\nu_z = 3.6089$. The $\gamma G = 7 - \nu_z$ resonance strength, as calculated analytically [6] for $\varepsilon_z/\pi = 12.2\ 10^{-6}$ m.rd is $|\ \varepsilon\ | = 2.98\ 10^{-4}$, and the Froissart-Stora formula [7] gives $S_z/S_0 = 2\ \exp(-\pi\ m\ |\ \varepsilon\ |^2\ /\ 2G\rho R\dot{B})$ = 0.443 (for $\dot{B} = 2.1$ T/s, ρ = magnetic radius = 6.3381 m, R =geometric radius = 16.8 m).

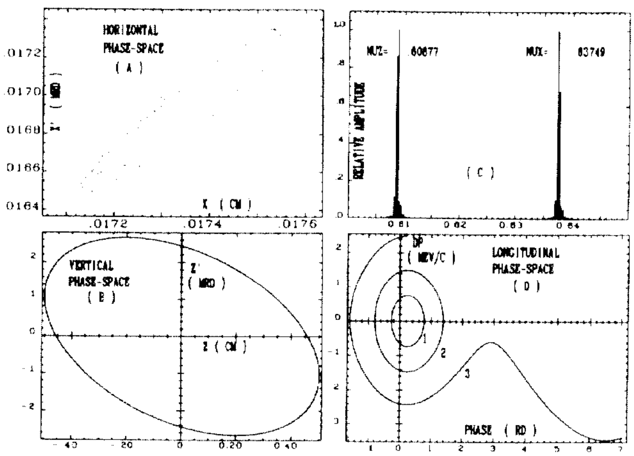

Fig. 3. Tracking of a particle over 3000 turns, with the code Zgoubi. These preliminary calculations show how precisely the first order parameters and motions are reproduced by the multiturn ray-tracing. (A) Horizontal phase-space. The particle starts near-by the betatron closed orbit and with $z_0 = z'_0 = 0$. Due to the numerical imprecision the 3000 points undergo spreading, but with negligible extent ($\varepsilon_x/\pi \simeq 0$). (B) Vertical phase-space. The particle starts with $z_0 = 4.58\ 10^{-3}$ m, $z'_0 = 0$. A least-square fit by the ellipse $\gamma_z z^2 + 2\alpha_z zz' + \beta_z z'^2 = \varepsilon_z/\pi$ gives $\beta_z = 2.05$ m, $\gamma_z = 0.582\ m^{-1}$, $\varepsilon_z/\pi = 12.2\ 10^{-6}$ m.rd in accordance with the preliminary matrix calculations (Fig. 2). (C) Tune numbers obtained by Fourier analysis of the phase-space ellipses for $\varepsilon_x/\pi = \varepsilon_z/\pi \simeq 12\ 10^{-6}$ m.rd: $\nu_x = 3.6375$, $\nu_z = 3.6088$; again these values agree with those of Fig. 2. (D) Longitudinal phase-space. The particles are accelerated at 1405 eV/turn ($\dot{B} = 2.1$ T/s) with a momentum dispersion of 5 10^{-4} (1), 10^{-3} (2), 1.65 10^{-3} (3) (out of acceptance); note that analytical calculations give a momentum acceptance of 1.65 10^{-3}. These four figures do prove the stability of the numerical ray-tracing, in terms of preservation of the emittances.

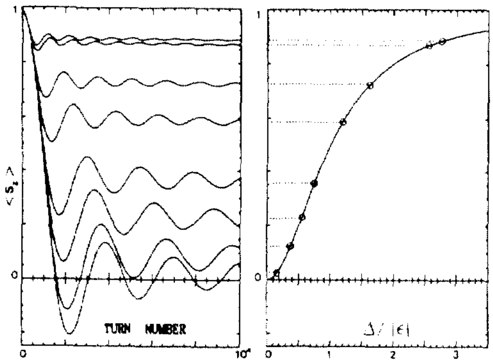

Fig. 4. Depolarization in the vicinity of $\gamma G = 7 - \nu_z$ in the static case. Particles starting with $\varepsilon_x/\pi = 0$, $\varepsilon_z/\pi = 12.2\ 10^{-6}$ m.rd and spin vertical, are tracked over 10^4 turns, for several values of $\Delta = \gamma G - 7 + \nu_z$. (A) S_z oscillates around the local eigenvector with a period $P(turns) = (\Delta^2 + |\varepsilon|^2)^{-1/2}$, and reaches the asymptotic mean $< S_z >= \Delta^2/\ (\Delta^2 + |\ \varepsilon\ |^2)$ [6]. Fitting these two equations with the plots gives $|\ \varepsilon\ |=$ $3.3\ 10^{-4}$ and $\nu_z = 3.608$, which is in good agreement with the results stated in Figs. 2 and 3. (B) Plot of the theoretical curve $< S_z >$ v.s. Δ, together with the "experimental" points derived from Fig. 4A, for showing the good agreement between numerical and analytical calculations.

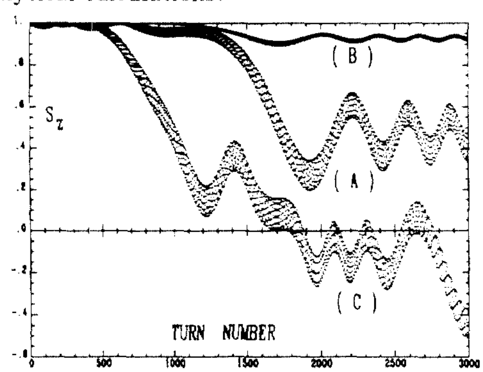

Fig. 5. Crossing of $\gamma G = 7 - \nu_z$, at $\dot{B} = 2.1$ T/s. (A) $\varepsilon_z/\pi = 12.2\ 10^{-6}$ m.rd. The strength of the resonance is $|\ \varepsilon\ |= 3.3\ 10^{-4}$ as derived from the static case (Fig. 4). As expected from the Froissart-Stora formula [7], the asymptotic polarization is about 0.44. (B) The emittance is now $\varepsilon_z/\pi = 1.2\ 10^{-6}$ m.rd; comparison with (A) shows that $|\ \varepsilon\ |$ is proportional to $\sqrt{\varepsilon_z}$, in agreement with the theory [6]. (C) Crossing of this resonance for a particle having a momentum dispersion of 10^{-3}.

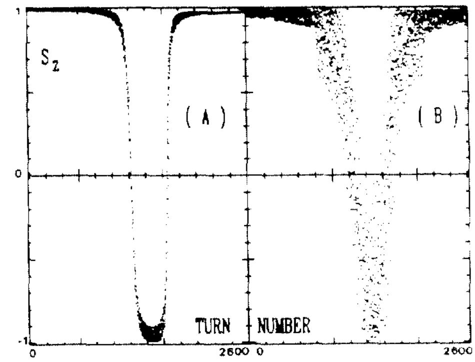

Fig. 6. ν_z is now 3.88. The plot shows the vertical spin component of a particle starting with its spin vertical and $\varepsilon_x/\pi \simeq 0$, $\varepsilon_z/\pi = 25\ 10^{-6}$ m.rd (A) or 200 10^{-6} m.rd (B), when crossing successively the two systematic resonances ν_z, 8-ν_z, at $\dot{B} = 420$ T/s (not realistic but allows faster tracking), with $|\ \varepsilon\ |\simeq$ 0.03 for both. The distance between the two is 8-$2\nu_z = 8\ |\ \varepsilon\ |$ (A), or $2.8|\ \varepsilon\ |$ (B).

5. CONCLUSION

The method of spin tracking described in this note has been implemented recently in the ray-tracing code Zgoubi, and work still remains to be done about checking its efficiency and accuracy. It henceforth appears to be a powerfull and promising tool, worth being utilized in the field of depolarizing phenomena and their compensation or corrections.

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