Self-field-driven rms emittances of a field-photoemitted intense short relativistic electron beam

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Abstract

The photoinjector is presently considered as the best source for those low-emittance, short relativistic electron beams that various applications require. For high-current beams, the emittance growth is dominated by the self-field effects, and essentially located near the photocathode. In this region the electrons are submitted to a very strong acceleration so that the self-field effects cannot be considered as space-charge ones : relativistic acceleration and retardation phenomena have to be taken into account [1]. By extending preliminary results on the r.m.s. transverse emittances [2], this work presents a systematic study of transverse, longitudinal and 3D r.m.s. emittances, as a function of beam parameters and RF field intensity.

1. INTRODUCTION

As in several other applications of electron or ion beams, a low emittance is required in free electron lasers (FEL) ; the smaller the aimed laser wavelength, the lower the emittance. For RF-FEL, the RF photoinjector seems presently to be the best low-emittance source. Illuminated by laser pulses in the ps-duration range, the cathode emits short photoelectron pulses which are very rapidly accelerated by the RF-field to relativistic energies. Powerful FEL require intense pulses. For J greater than some tens of A/cm^2, self-field effects are dominant in the evolution of the various rms emittances, among which we shall distinguish transverse, longitudinal and 3D-ones.

It is near the cathode, just after the beam pulse has been emitted, that transverse rms emittance growth -and longitudinal rms emittance decrease- is the most rapid. It is to this region the present work is devoted. For a typical RF-field intensity on the cathode such as 30 MV/m, photoelectrons are accelerated from a thermal velocity of the order of $5\times10^{-4}c$ to $c/4$ in 0.5 mm. Of course, it is impossible in these conditions to deduce the rms emittances from a static electric field map calculated in any particular beam proper Lorentz frame. Moreover, due to the extreme acceleration experienced by the photoelectrons, electrodynamic effects of acceleration-radiation field, or retardation, have to be taken into account.

In 2, we describe the theoretical model used to calculate the emittances, and precise definitions and assumptions.

In 3, some sample results concerning the transverse emittance ε_r are given for $t=\tau$, the beam pulse duration, i.e. when the pulse is wholly emitted but still in contact with the cathode. They complete -and sometimes correct- preliminary results [2].

In 4, the longitudinal emittance ε_z, at $t=\tau$ also, is shown as a function of beam parameters, RF field intensity, and distance to the cathode.

In 5, the early ($\tau<t<$a few τ) following evolution of ε_r and ε_z are studied, and a global linear invariant, which may be called the 3D rms emittance, is deduced from ε_r and ε_z.

2. THEORETICAL MODEL

2.1. Assumptions. Definitions

The beam pulse is assumed to be axisymmetric (radius R), emitted by the cathode, from $t=0$ to $t=\tau$, with a constant and uniform current density J, in the RF field $\mathbf{E}_0(r,z,t)$. Typical values will be πR^2=1 cm^2, τ =10-100 ps, J =100 A/cm^2, $E_0(0,0,0)$=15-45 MV/m.

As said before, it is the early emittance growth we are interested in, the one experienced by the beam pulse during its extraction from the cathode : $0\le t\le\tau$ (where τ is the pulse duration), or during a few τ. The accelerating RF electric field $\mathbf{E}_0$ may be considered as constant and uniform as long as, on the one hand : $\tau \ll 1/\nu$ (where ν is the RF frequency) , and on the other hand the beam radius R is small compared to the cavity radius. For the low-frequency photoinjector of the "ELSA" LEL [3] (CEA, Bruyères-le-Châtel) where ν= 144 MHz, the first condition means $\tau \ll 7$ ns.

Self-field effects being dominant in intense beams, the latter will be treated as laminar.

2.2. Transverse rms emittance

In cylindrical coordinates (r,θ,z), the radial rms transverse emittance ε_r of an axisymmetric beam is defined by (with a coefficient 2, in order to allow the identification with the phase emittance for a KV beam) :

$$\varepsilon_r = 2\,[<r^2><(\frac{p_r}{mc})^2> - <r\frac{p_r}{mc}>^2]^{1/2},$$

where $\mathbf{p}$ is the mechanical momentum, and < > the global phase average over the whole beam, at a given time t :

$$<G> = \int G(\mathbf{x},\mathbf{p}|t)\, f(\mathbf{x},\mathbf{p}|t)\; d^3x\; d^3p,$$

where $f(\mathbf{x},\mathbf{p}|t)$ is the beam normalized distribution function.

For the considered cold beam : $f(\mathbf{x},\mathbf{p}|t) = [n(\mathbf{x}|t)/N]\,\delta[\mathbf{p}-\mathbf{p}(\mathbf{x}|t)]$, where n is the number density, N the total number of particles in the beam pulse.

2.3. Longitudinal emittance

Although identification with some ideal KV-like beam is no longer possible, we shall keep the factor 4 one finds in the cartesian transverse emittances ε_x or ε_y, and adopt for the longitudinal rms emittance ($\Delta\mathbf{x}=\mathbf{x}-<\mathbf{x}>$, $\Delta\mathbf{p}=\mathbf{p}-<\mathbf{p}>$):

$$\varepsilon_z = 4\left[<\Delta z^2><\left(\frac{\Delta p_z}{mc}\right)^2> - <\Delta z\,\frac{\Delta p_z}{mc}>^2\right]^{1/2}.$$

2.4. 3D rms emittance

As long as the beam distribution function may be considered as satisfying a 6D Liouville equation (Vlasov equation), it is possible [4] to build, from its successive moments, linear invariants, i.e. quantities which would be invariant during the transport if all the electromagnetic fields (external fields as well as self-fields) experienced by the electrons were linear with respect to $\mathbf{x}-<\mathbf{x}>$.

Among this infinity of linear invariants there is one, built with the second order moments, which is directly related to the ordinary rms emittances :

$$I = \sum_{ij} <\Delta x_i \Delta x_j><\Delta p_{x_i} \Delta p_{x_j}> - <\Delta x_i\, \Delta p_{x_j}><\Delta x_j\, \Delta p_{x_i}>.$$

For a beam pulse moving along the z axis, $\Delta x=x$, $\Delta y=y$, $\Delta p_x=p_x$, $\Delta p_y=p_y$; as for the couple (Δz, Δp_z), it may be replaced by (Δt, $-\Delta H$), where Δt and ΔH denote the differential transit time and energy, respectively, with respect to the beam pulse centroid.

For an axisymmetric beam :

$$\frac{4\sqrt{I}}{mc} = \left(\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2\right)^{1/2} = \left(2\varepsilon_r^2 + \varepsilon_z^2\right)^{1/2},$$

reduces to $\sqrt{2}\,\varepsilon_r$ for a longitudinally monokinetic beam, and to ε_z for a transversally cold beam. For any beam, we propose for $(4/mc)\sqrt{I}$ the name of *3D rms emittance* ε_{3D}.

2.5. Self-field map and electron trajectories

Due to the extreme acceleration the photoelectrons experience in the RF photoinjector electric field $\mathbf{E}_0$, self-field effects are not space-charge effects one could calculate from a Poisson equation written in any particular beam-pulse Lorentz proper frame. As it has been previously indicated, acceleration-radiation field effects, as well as retardation effects, have to be taken into account. The main outlines of the method followed to calculate the beam pulse self-field map : $(\mathbf{E},\mathbf{B})(r,z,t)$, and the electron longitudinal motion, for $0 \le t \le \tau$ (or a few τ), are described in a companion paper [5]. Two assumptions -rather well verified by complete 3D calculations or numerical simulations- simplify the analysis : the beam is paraxial from the very beginning ; the electron trajectories, for $0 \le t \le \tau$, are straight lines at a good approximation.

Besides the electron longitudinal motion : $[z(t,t_0), p_z(t|t_0)]$, where t_0 is emission time, one needs $p_r(t|r,t_0)$ to calculate the emittances. The latter momentum is deduced, knowing $(\mathbf{E},\mathbf{B})(r,z,t)$ and $z(t,t_0)$, by integration of :

$$\frac{d}{dt}p_r(t|r,t_0) = -e\left\{E_r\,[r,z(t|t_0),t] - \beta\,(t|t_0)cB_\theta\,[r,z(t|t_0),t]\right\},$$

with $p_r(t_0|r,t_0)=0$.

3. TRANSVERSE RMS EMITTANCE ε_r AT $t=\tau$

At $t=\tau$, the beam pulse is wholly emitted, but still in contact with the cathode. A sample of slice- $\varepsilon_r(z)$, as well as global- ε_r, rms transverse emittances had been given in [2] for various J, τ, R, and E_0. Figures 1-2 show some new results about ε_r, at time $t=\tau$ after photoemission beginning, on the one hand as a function of τ, for E_0=15 or 30 MV/m, and on the other hand as a function of E_0 for S=1 or 2 cm^2.

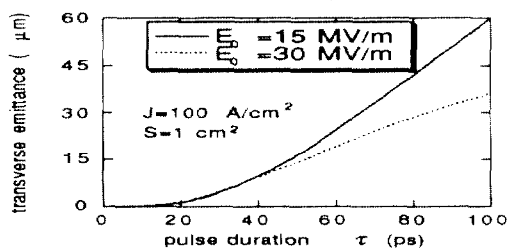

Figure 1

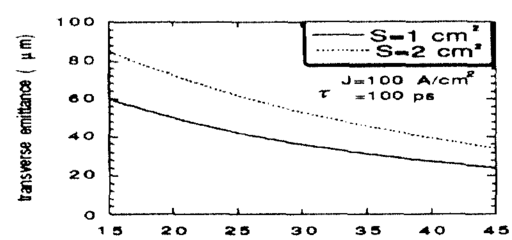

Figure 2

4. LONGITUDINAL EMITTANCE ε_z AT $t=\tau$

At 0-order, ε_z at time $t=\tau$ may be analytically expressed, in a reduced form : $\varepsilon'^{(0)}_z = H\varepsilon^{(0)}_z$, as a function of $\tau' = Hc\tau$, where $H=eE_0/mc^2$ (dimension L^{-1}) :

$$\varepsilon'^{(0)}_z = \frac{2}{3\tau'}\Big\{2(\tau'^2+4)(1+\tau'^2)^{1/2} - 2(3\tau'^2+4) + 6\tau'\,[(1+\tau'^2)^{1/2}-1]\,\sinh^{-1}\tau' - 3(\tau'\,\sinh^{-1}\tau')^2\Big\}^{1/2}$$

Figure 3 shows this $\varepsilon'^{(0)}_z(\tau')$. For given E_0, $\tau' \propto \tau$, $\varepsilon'^{(0)}_z \propto \varepsilon^{(0)}_z$: the longitudinal transverse emittance $\varepsilon^{(0)}_z(\tau)$ increases rapidly with τ, the behaviour being the one of Figure 3.

For given pulse duration τ, $\varepsilon_z^{(0)}(\tau) = (mc^2/eE_0)\ \varepsilon'^{(0)}_z\ (eE_0\tau/mc)$ is an increasing function of E_0. Taking the longitudinal self-field into account slightly modify $\varepsilon_z(\tau)$ (Figure 4).

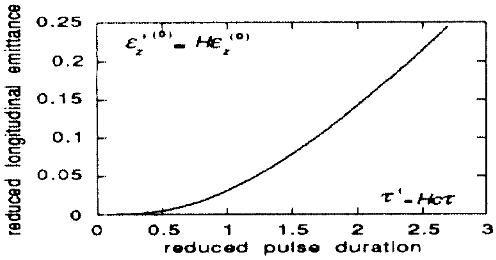

Figure 3

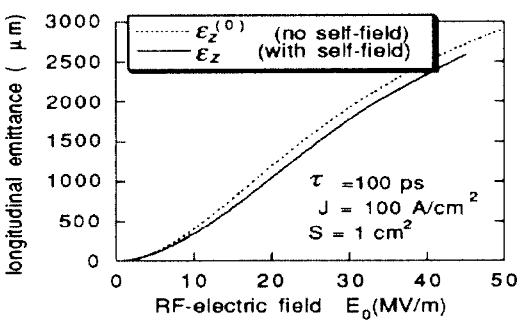

Figure 4

5. Early following evolution of ε_r, ε_z, ε_{3D}: $\tau<t<$a few τ

$\varepsilon_z(t \geq \tau)$ may also be analytically expressed at order 0 ; the formula is too long to be reproduced here. $\varepsilon'^{(0)}_z\ (t' \geq \tau')$, where $t'=Hct$, is a rapidly decreasing function. For instance, for τ =100 ps, J=100 A/cm^2, E_0=15 MV/m, $\varepsilon_z^{(0)}$ would decrease from about 789 µm at $t=\tau$, to 15 µm at $t=5\tau$, i.e. when the pulse centroid is only 10.6 cm away from the cathode. But here, taking the longitudinal self-field into account greatly modifies the result. Figures 5 and 6 show $\varepsilon_z^{(0)}$ and ε_z as a function of t/τ, for E_0=15 MV/m or 30 MV/m, and for J=100 A/cm^2, S =1 cm^2. At the end, Figure 7 shows ε_r, ε_z, and ε_{3D} as a function of t/τ, for E_0=15 MV/m, and $1 \leq t/\tau \leq 5$.

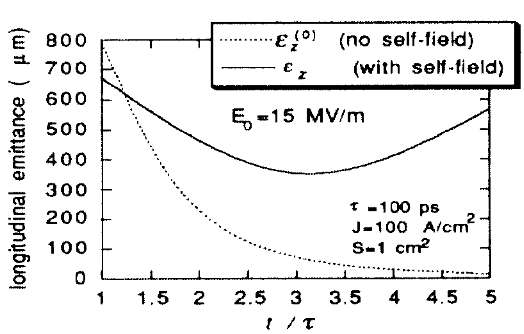

Figure 5

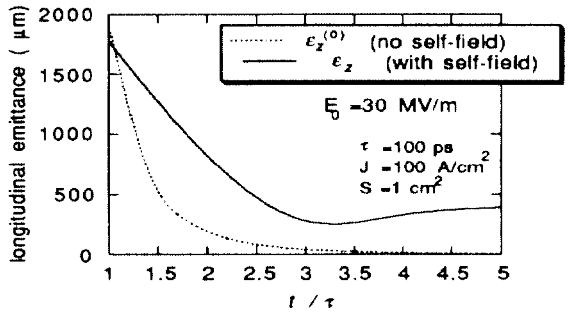

Figure 6

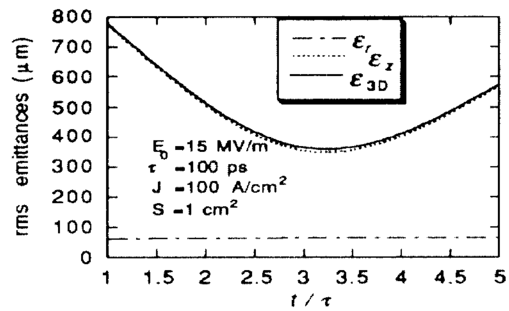

Figure 7

6. References

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