Self-field-driven rms emittances of a field-photoemitted intense short relativistic electron beam

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Abstract

The photoinjector is presently considered as the best source for those low-emittance, short relativistic electron beams that various applications require. For high-current beams, the emittance growth is dominated by the self-field effects, and essentially located near the photocathode. In this region the electrons are submitted to a very strong acceleration so that the self-field effects cannot be considered as space-charge ones: relativistic acceleration and retardation phenomena have to be taken into account [1]. By extending preliminary results on the r.m.s. transverse emittances [2], this work presents a systematic study of transverse, longitudinal and 3D r.m.s. emittances, as a function of beam parameters and RF field intensity.

1. INTRODUCTION

As in several other applications of electron or ion beams, a low emittance is required in free electron lasers (FEL); the smaller the aimed laser wavelength, the lower the emittance. For RF-FEL, the RF photoinjector seems presently to be the best low-emittance source. Illuminated by laser pulses in the ps-duration range, the cathode emits short photoelectron pulses which are very rapidly accelerated by the RF-field to relativistic energies. Powerful FEL require intense pulses. For \( J > \) greater than some tens of A/cm\(^2\), self-field effects are dominant in the evolution of the various rms emittances, among which we shall distinguish transverse, longitudinal and 3D ones.

It is near the cathode, just after the beam pulse has been emitted, that transverse rms emittance growth - and longitudinal rms emittance decrease - is the most rapid. It is to this region the present work is devoted. For a typical RF-field intensity on the cathode such as 30 MV/m, electron-positron pairs are emitted by the cathode, from \( t = 0 \) to \( t = \tau \), with a constant and uniform current density \( J \), in the RF field \( B_0(r,\theta,\phi) \). Typical values will be \( \pi R^2 = 1 \, \text{cm}^2 \), \( \tau = 10-100 \, \text{ps} \), \( J = 100 \, \text{A/cm}^2 \), \( B_0(0,0,0) = 15.45 \, \text{MV/m} \).

As said before, it is the early emittance growth we are interested in, the one experienced by the beam pulse during its extraction from the cathode: \( 0 \leq t \leq \tau \) (where \( \tau \) is the pulse duration), or during a few \( \tau \). The accelerating RF electric field \( E_0 \) may be considered as constant and uniform as long as, on the one hand: \( \tau \ll 1/\nu \) (where \( \nu \) is the RF frequency), and on the other hand the beam radius \( R \) is small compared to the cavity radius. For the low-frequency photoinjector of the "ELSA" LEL [3] (CEA, Bruyères-le-Châtel) where \( \nu = 144 \, \text{MHz} \), the first condition means \( \tau \approx 7 \, \text{ns} \).

Self-field effects being dominant in intense beams, the latter will be treated as laminar.

2. THEORETICAL MODEL

2.1. Assumptions. Definitions

The beam pulse is assumed to be axisymmetric (radius \( R \)), emitted by the cathode, from \( t = 0 \) to \( t = \tau \), with a constant and uniform current density \( J \), in the RF field \( E_0(r,\theta,\phi) \). Typical values will be \( \pi R^2 = 1 \, \text{cm}^2 \), \( \tau = 10-100 \, \text{ps} \), \( J = 100 \, \text{A/cm}^2 \), \( E_0(0,0,0) = 15.45 \, \text{MV/m} \).

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2.2. Transverse rms emittance

In cylindrical coordinates \((r,\theta,z)\), the radial rms transverse emittance \( \varepsilon_r \) of an axisymmetric beam is defined by (with a coefficient 2, in order to allow the identification with the phase emittance for a KV beam):

\[
\varepsilon_r = 2 \left[ \frac{1}{\tau^2} \left( \frac{1}{mc^2} \right)^2 \right] = \left( \frac{1}{mc} \right)^2,
\]

where \( p \) is the mechanical momentum, and \( \langle \cdot \rangle \) the global phase average over the whole beam, at a given time \( t \).
\[ \langle G \rangle = \int G(x,p) f(x,p) \, dx \, dp, \]

where \( f(x,p) \) is the beam normalized distribution function.

For the considered cold beam: \( f(x,p) = \left[ n(x)/N \right] \delta(p-p(x)) \), where \( n \) is the number density, \( N \) the total number of particles in the beam pulse.

### 2.3. Longitudinal emittance

Although identification with some ideal KV-like beam is no longer possible, we shall keep the factor 4 one finds in the cartesian transverse emittances \( \epsilon_x, \epsilon_y \), and adopt for the longitudinal rms emittance \( \langle \Delta x \rangle \langle \Delta p_x \rangle = \langle \Delta x \rangle \langle \Delta p_x \rangle \): \( \epsilon_z = 4 \left[ \langle \Delta z^2 \rangle \left( \frac{\Delta p_x}{mc} \right)^2 \right]^{1/2} \).

### 2.4. 3D rms emittance

As long as the beam distribution function may be considered as satisfying a 6D Liouville equation (Vlasov equation), it is possible [4] to build, from its successive moments, linear invariants, i.e. quantities which would be invariant during the transport if all the electromagnetic fields (external fields as well as self-fields) experienced by the electrons were linear with respect to \( x \langle x \rangle \).

Among this infinity of linear invariants there is one, built with the second order moments, which is directly related to the ordinary rms emittances:

\[ I = \sum_{i,j} \langle \Delta x_i \Delta x_j \rangle \langle \Delta p_{x,i} \Delta p_{x,j} \rangle - \langle \Delta x_i \Delta p_{x,i} \rangle \langle \Delta x_j \Delta p_{x,j} \rangle. \]

For a beam pulse moving along the \( z \) axis, \( \Delta x = \Delta y = 0 \), \( \Delta p_x = p_x, \Delta p_y = p_y \); as for the couple \( \langle \Delta x, \Delta p_x \rangle \), it may be replaced by \( \langle \Delta x, -\Delta t \rangle \), where \( \Delta t \) denotes the differential transit time and energy, respectively, with respect to the beam pulse centroid.

For an axisymmetric beam:

\[ \frac{4\sqrt{1}}{mc} = \left( \epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 \right)^{1/2} = \left( 2\epsilon_x^2 + \epsilon_z^2 \right)^{1/2}, \]

reduces to \( \sqrt{2} \epsilon_z \) for a longitudinally monokinetic beam, and to \( \epsilon_z \) for a transversally cold beam. For any beam, we propose for \( (4/mc)^{1/4} \) the name of 3D rms emittance \( \epsilon_{3y} \).

### 2.5. Self-field map and electron trajectories

Due to the extreme acceleration the photoelectrons experience in the RF photoinjector electric field \( E_{0y} \), self-field effects are not space-charge effects one could calculate from a Poisson equation written in any particular beam-pulse Lorentz proper frame. As it has been previously indicated, acceleration-radiation field effects, as well as retardation effects, have to be taken into account. The main outlines of the method followed to calculate the beam pulse self-field map: \( (E,B)(r,z,t) \), and the electron longitudinal motion, for \( 0 \leq t \leq T \) (or a few \( \tau \)), are described in a companion paper [5]. Two assumptions -rather well verified by complete 3D calculations or numerical simulations- simplify the analysis: the beam is paraxial from the very beginning; the electron trajectories, for \( 0 \leq t \leq \tau \), are straight lines at a good approximation.

Besides the electron longitudinal motion: \( \langle z(t_0) \rangle, p_z(t_0) \rangle \), where \( t_0 \) is emission time, one needs \( p_z(t_0) \) to calculate the emittances. The latter momentum is deduced, knowing \( (E,B)(r,z,t) \) and \( z(t_0) \), by integration of:

\[ \frac{dp_z}{dt} = -eE_z, \]

with \( p_z(t_0) = 0 \).

### 3. Transverse RMS Emittance \( \epsilon_y \) at \( t=\tau \)

At \( t=\tau \), the beam pulse is wholly emitted, but still in contact with the cathode. A sample of slice \( \epsilon_y(z) \), as well as global \( \epsilon_y \), rms transverse emittances had been given in [2] for various \( J, \tau, R \), and \( E_0 \). Figures 1-2 show some new results about \( \epsilon_y \) at time \( t=\tau \) after photoemission beginning, on the one hand as a function of \( \tau \), for \( E_0=15 \) or 30 MV/m, and on the other hand as a function of \( E_0 \) for \( S=1 \) or 2 cm².

### 4. Longitudinal Emittance \( \epsilon_z \) at \( t=\tau \)

At 0-order, \( \epsilon_z \) at time \( t=\tau \) may be analytically expressed, in a reduced form:\n
\[ \epsilon_z(0) = H \epsilon_z(0), \]

where \( H = eE_0/mc^2 \) (dimension \( L^{-1} \)):

\[ \epsilon_z(0) = \frac{2}{3\tau} \left\{ 2(\tau^2 + 1)(1 + \tau^2)^{1/2} - 2(3\tau^2 + 4) + 6\tau^2 \right\} \sin^{-1} \tau - 3(\tan^{-1} \tau^2)^{1/2} \]

Figure 3 shows this \( \epsilon_z(0)(\tau) \). For given \( E_0, \tau \approx \pi, \epsilon_z^{(0)} \approx \epsilon_z^{(0)} \) the longitudinal transverse emittance \( \epsilon_{z(0)}(\tau) \) increases rapidly with \( \tau \), the behaviour being the one of Figure 3.
For given pulse duration $\tau$, $\varepsilon_z^{(0)}(\tau) = (mc^2/eE_0/E_z^{(0)}(E_0/mc)$ is an increasing function of $E_0$. Taking the longitudinal self-field into account slightly modify $\varepsilon_z(\tau)$ (Figure 4).

\[\varepsilon_z^{(0)}(\tau) = \frac{mc^2}{eE_0}\]

5. EARLY FOLLOWING EVOLUTION OF $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_{3D}$: $\tau < t < A FEW \tau$

$\varepsilon_z(t \geq \tau)$ may also be analytically expressed at order 0; the formula is too long to be reproduced here. $\varepsilon_x^{(0)}(t \geq \tau)$, where $t=t=H\tau t$, is a rapidly decreasing function. For instance, for $\tau = 100$ ps, $J=100$ A/cm$, E_0=15$ MV/m, $\varepsilon_x^{(0)}$ would decrease from about 789 $\mu$m at $t=\tau$, to 15 $\mu$m at $t=5\tau$, i.e. when the pulse centroid is only 10.6 cm away from the cathode. But here, taking the longitudinal self-field into account greatly modifies the result. Figures 5 and 6 show $\varepsilon_z^{(0)}$ and $\varepsilon_z$ as a function of $t/\tau$, for $E_0=15$ MV/m or $30$ MV/m, and for $J=100$ A/cm$, S = 1$ cm$^2$. At the end, Figure 7 shows $\varepsilon_z$, $\varepsilon_y$, and $\varepsilon_{3D}$ as a function of $t/\tau$, for $E_0=15$ MV/m, and $15t/\tau<5$.

6. REFERENCES


