Treatment of Nonlinearities in Achromatic Trajectory Corrections for Future Linacs

C. Fischer and G. Guignard
CERN CH-1211 Geneva 23

Abstract

The control of transverse emittance blow-up is quite a challenging issue for the next generation of linear colliders. To deal with this problem, a 'dispersion-free' algorithm has been proposed at SLAC in order to optimize the beam trajectory in a suitable momentum range. The method requires a good knowledge of the machine model at different momenta and the question is then to determine the coefficients of transfer matrices which are strongly nonlinear in $\Delta p/p_0$. It is shown that the function to be minimized can be expressed as a series of n + 1 terms; the first one describes the contribution of the trajectory excursions at nominal momentum, the n consecutive terms being the influence of dispersive effects to increasing orders in momentum. The application of the method is also discussed and an example to second order is given in the case of the CLIC main linac.

1. INTRODUCTION

In future linear colliders, transverse wake fields and dispersive errors will dilute the transverse emittances. These perturbations are due to trajectory offsets in both the magnets and the accelerating structures that are typically misaligned. In this paper, the study of the dispersive errors and their compensation is emphasized. When the beam travels off-axis in a quadrupole, particles with different energies are deflected differently. The dispersion-free algorithm, proposed at SLAC [1] and shown to be effective in the case of the NLC design, deals with this question. Here this method is investigated on a general basis and applied to CLIC for different sets of parameters.

The presence of trajectory deviations and trajectory differences in the function to minimize [1] gives basically two handles to the correction. To keep the generality and to take into account the nonlinearities with momentum of the transfer-matrix coefficients [2], the second contribution that includes dispersive (or chromatic) effects can be split into terms to increasing order in momentum. The relative importance of the trajectory and of the dispersive effects as well as the relative impact of the different chromatic terms can then be studied, and the dependence of this relative importance (and unavoidable trajectory–dispersion correlation) on the hypothesis made on parameters and misalignments can be analysed.

2. ACHROMATIC CORRECTION TO HIGHER ORDERS

The quantities of interest are the measured trajectory distortions x_j and trajectory differences Δx_j taken for different energy excursions $\delta = \Delta p/p_0$. If t_j is the actual trajectory, the quantities x_j and Δx_j become

$$x_j = t_j + \xi_j - b_j$$
$$\Delta x_j = x_j(\delta) - x_j = t_j(\delta) - t_j + \xi_j(\delta) - \xi_j \tag{1}$$

in presence of random position-monitor precision errors ξ_j (of r.m.s. value σ_ξ) and random position-monitor misalignments b_j (of r.m.s. value σ_b). An 'achromatic' correction should by definition reduce both x_j and Δx_j, i.e. the trajectory and the dispersion effects. This is typically achieved by applying dipole kicks ϑ_i along the linac or by moving the quadrupoles of the lattice transversely (which is equivalent). Therefore, the next important quantities are the calculated trajectory distortions X_j and trajectory differences ΔX_j due to kicks ϑ_i (downstream projection, i.e. j > i),

$$X_j = \sum_{i<j} R_{12}(i,j,0)\vartheta_i \tag{2a}$$

$$\Delta X_j = X_j(\delta) - X_j$$
$$= \sum_{i<j}\left[R_{12}(i,j,\delta)\frac{1}{1+\delta} - R_{12}(i,j,0)\right]\vartheta_i \tag{2b}$$

where R_{12} are the transfer matrix elements transforming a deflection ϑ_i into an excursion X_j (at j > i), either on-momentum ($\delta = 0$) or off-momentum ($\delta \neq 0$). After correction, the sum of the quantities (1) and (2) must be minimum, i.e. $x_j + X_j$ and $\Delta x_j + \Delta X_j$. It is important to note at this point that nonlinearities with momentum develop in $R_{12}(i, j, \delta)$ as the separation (j – i) increases [2]. These nonlinear variations are mainly quadratic with δ, when $j - i \lesssim 70$ and δ remains between $\pm$ 4% about, but even wilder outside. The trajectory variations Δx_j with δ are also not necessarily always linear. Hence, in order to include higher orders in the model for study purposes, the quantities Δx_j and ΔX_j have to be developed in δ. A first step gives:

$$\frac{1}{1+\delta} = 1 - \delta + \delta^2 - \ldots + (-1)^n\delta^n$$

$$R_{12}(i,j,\delta) - R_{12}(i,j,0) = \sum_m c_m^{ij}\delta^m \tag{3a}$$

with $$R_{12}(\delta)\frac{1}{1+\delta} - R_{12}(0)$$
$$= \frac{1}{1+\delta}\left\{\left[R_{12}(\delta) - R_{12}(0)\right] - \delta R_{12}(0)\right\} \tag{3b}$$

forgetting in Eq. (3b) the indices i, j for simplicity. All the coefficients c_m^{ij} can be computed by simulation. Assuming that Δx_j development is deduced from measurements and introducing Eqs. (3) into Eq. (2b), one gets

$$\Delta x_j = \sum_n a_n^j \delta^n$$

$$\Delta X_j = \sum_{i<j}\vartheta_i \sum_n\left[\left(\sum_{m=1}^{n}(-1)^{n+m}c_m^{ij}\right) + (-1)^n R_{12}(i,j,0)\right]\delta^n$$

$$\stackrel{\text{def}}{=} \sum_{i<j}\vartheta_i\sum_n C_n^{ij}\delta^n = \sum_n\delta^n\sum_i\vartheta_i C_n^{ij} = \sum_n A_n^j\delta^n. \tag{4}$$

Equations (4) show that both quantities Δx_j and ΔX_j can be developed to increasing orders in δ and that, for each order, the sum of the corresponding terms can be retained for a minimization. One can then imagine a correction that minimizes, besides the nominal trajectory, the successive dispersive terms, i.e. the linear dispersion, the quadratic chromatic trajectory dependence, and so on to higher orders. This consists in minimizing an error function Φ that contains a quadratic sum for the trajectory and each dispersive term

$$\Phi = \sum_{j=1}^{N} \left\{ w_0 \left(x_j + X_j \right)^2 + \sum_{n \geq 1} w_n \delta^{2n} \left(a_n^j + A_n^j \right)^2 \right\} \quad (5)$$

where w_0 and w_n are weights and N the total number of measurement points. It is natural to choose weights inversely proportional to the measurement errors [1]. In this case, w_0^{-1} is equal to the quadratic sum of the r.m.s. precision errors and misalignment errors, while w_n^{-1} is given by twice the precision errors (in a trajectory difference, misalignment contributions cancel). But appropriate weights can also be introduced to better investigate the relative importance of each term. Taking for δ the particular value δ_0 used in the trajectory difference measurements, the function Φ becomes

$$\Phi = \sum_{j=1}^{N} \left[\frac{\left(x_j + \sum_i R_{12}^{ij} \vartheta_i \right)^2}{\sigma_\xi^2 + \sigma_b^2} + \sum_{n \geq 1} \frac{\left(a_n^j + \sum_i C_n^{ij} \vartheta_i \right)^2}{2\sigma_\xi^2 / \delta_0^{2n}} \right]. \quad (6)$$

The minimum of Φ is obtained by looking for the kicks ϑ_i that bring to zero the partial derivatives of Φ with respect to ϑ_i. This is equivalent to saying that the following linear system must be satisfied (index i varying from 1 to N):

$$\begin{aligned} 0 = & \sum_{j=i+1}^{N} \left(\frac{x_j R_{12}^{ij}}{\sigma_\xi^2 + \sigma_b^2} + \sum_{n \geq 1} \frac{a_n^j C_n^{ij}}{2\sigma_\xi^2 / \delta_0^{2n}} \right) \\ & + \sum_{k=0}^{N-1} \vartheta_k \left[\sum_{j=j_m}^{N} \left(\frac{R_{12}^{ij} R_{12}^{kj}}{\sigma_\xi^2 + \sigma_b^2} + \sum_{n \geq 1} \frac{C_n^{ij} C_n^{kj}}{2\sigma_\xi^2 / \delta_0^{2n}} \right) \right] \end{aligned} \quad (7)$$

where j_m is the maximum of i + 1 and k + 1. Equations (7) have to be solved for the ϑ_k. In the present studies, they have been coded up to second order in δ in Eq. (4) ($n \leq 2$), since the contributions to the dispersion are expected to decrease with this order.

3. Application of the Method

To order n in δ, n coefficients a_n^j and A_n^j come in the development (4) of Δx_j and ΔX_j; these coefficients are determined by the measurement of the trajectory and the calculation of the transfer matrix coefficients R_{12} (i, j) at the nominal momentum p_0 and for n different values of the energy excursion δ. Application to second order therefore requires one to know the trajectory and transfer coefficients for two different values of δ, which can be chosen symmetrically around p_0. The exercise was carried out in the case of the CLIC main linac structure. The method for extracting the necessary parameters using the same tracking program with different initial conditions is described elsewhere [3]. Only the vertical plane is considered here; this is the most critical one as far as transverse blow-up is concerned owing to the emittance ratio (> 10) expected in CLIC. Therefore, only QDs of the 90° FODO lattice are selected for kick applications to correct the vertical trajectory and for pick-up locations to measure it.

The behaviour of the R_{12} (i, j) coefficients versus δ in the case of the CLIC structure has already been presented [2]. For a given reference quadrupole i their parabolic shape holds for roughly 26 cells (52 quadrupoles) as long as δ is limited to within ±3.5%; these values have been kept and the process applied on bins of 60 quadrupoles with an overlap of 10 quadrupoles between consecutive bins. In principle, five iterations are applied on each bin for a better convergence. CLIC (option 1) with a final energy of 250 GeV, a linac length of 3500 m, and 350 quadrupoles is used.

The algorithm efficiency was considered for various sets of parameters: initial emittance, alignment errors of lattice elements (quadrupoles, pick-ups), the main appreciation criterion being the relative emittance blow-up at the linac end.

Another key parameter is the relative contribution (weight) of the two terms in Eq. (5), describing the basic trajectory and the dispersive effects, as well as the relative impact of the linear and quadratic terms in the latter.

4. Examples

Quadrupoles are supposed to be misaligned within an r.m.s. error of 5 µm. This value is realistic for CLIC and maintains the beam trajectory before correction, and hence wake fields, at a reasonable level for the purpose of this study. To reduce the tracking time, RF cavities are neither cut in sections nor misaligned. R.m.s. alignment errors σ_b up to 10 µm have been considered for pick-ups; above this value CLIC wake-field effects dominate, masking the relative influence of trajectory and dispersion minimization. Resolution errors σ_ξ are supposed to amount to a few tenths of a micron and are not considered when measuring trajectories.

Up to σ_b = 3 µm the most efficient correction, considering both convergence speed and asymptotic value, is got when the trajectory is weighted more than the dispersion (up to a ratio of 10). In this case when the trajectory is corrected the dispersion follows and both contributions are simultaneously reduced.

Figures 1, 2 and Figs. 3–5 deal with σ_b = 5 µm and σ_b = 10 µm respectively. In the first case, the blow-up curves given after the same number of iterations concern different weighting configurations: curve (a) corresponds to the case where both the trajectory and the dispersion have the same weight, whereas in curve (b) the weight of the trajectory is stressed by a factor 10; curve (c) is obtained when starting with an emphasis on the trajectory to converge rapidly and then giving more power to the dispersion to improve the final result. The blow-up between cases (b) and (c) drops by a factor of 2 (Fig. 1). The observed improvement corresponds to a better correction of the dispersion (Fig. 2).

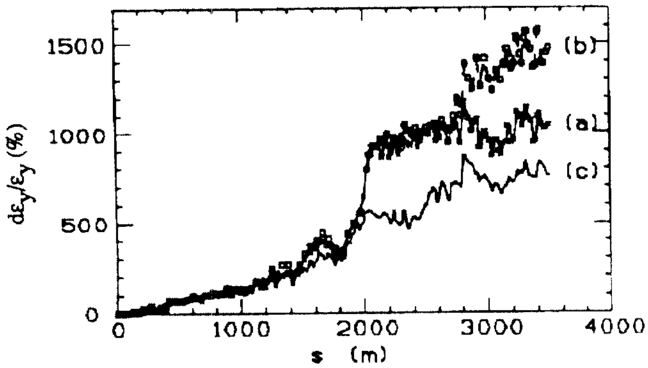

Figure 1. V EMITTANCE GROWTH RATE (σ_b = 5 μm)
(a) trajectory and dispersion have the same weight
(b) trajectory weighted 10 times more than dispersion
(c) weighting more trajectory first then dispersion in last iterations

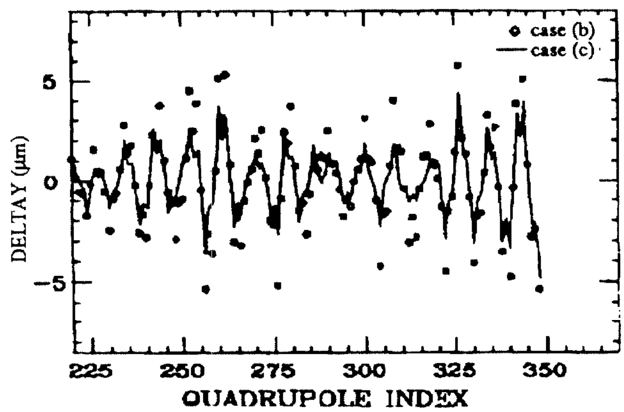

Figure 2. V TRAJECTORY OFFSET FOR δ = 3.5% (σ_b = 5 μm)
Dispersion after correction in cases (b) and (c)

In Figs. 3–5, when σ_b = 10 μm, the same observations hold (cases (a) and (b)). In addition here, when in the last iterations the dispersion is reinforced 10 times, its quadratic term is also weighted 10 times more than the linear one (case (c)). A significant improvement can be further observed – comparison between (b) and (c). Both the trajectory and the dispersion are reduced in case (c) – Figs. 4 and 5.

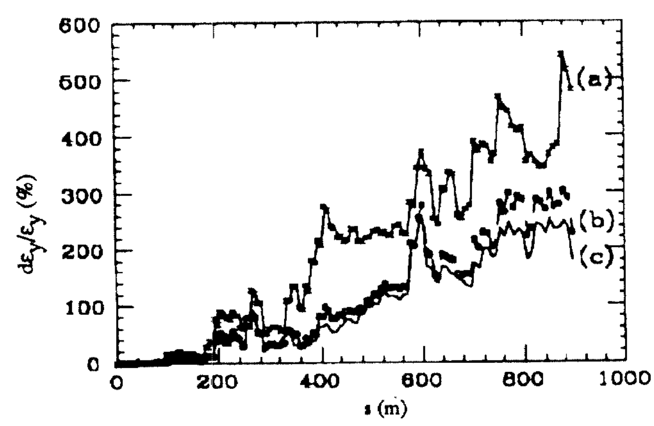

Figure 3. V EMITTANCE GROWTH RATE (σ_b = 10 μm)
(a) trajectory weighted 10 times more than dispersion
(b) *idem* (a) with dispersion weighted 10 times more in last iterations
(c) *idem* (b) with quadratic term reinforced 10 times in last iterations

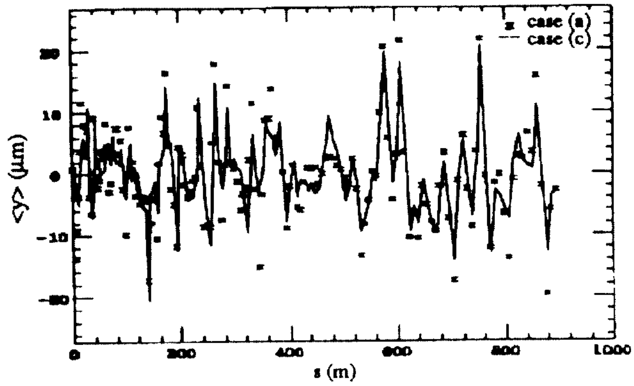

Figure 4. V TRAJECTORY AT NOMINAL MOMENTUM (σ_b = 10 μm)
Comparison between cases (a) and (c)

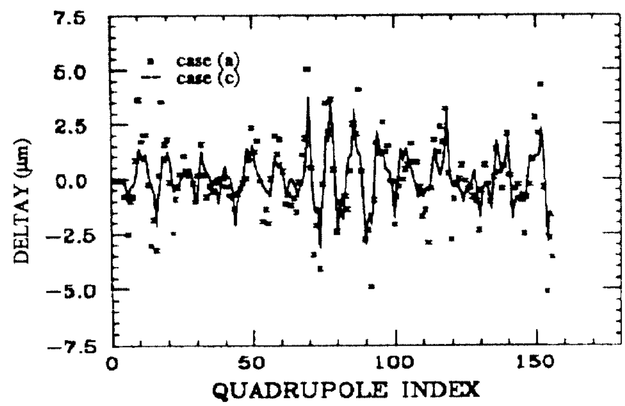

Figure 5. V TRAJECTORY OFFSET FOR δ = 3.5% (σ_b = 10 μm)
Dispersion after correction in cases (a) and (c)

5. Conclusion

This study shows that a closer look at an achromatic correction algorithm is of interest for an optimum control of the blow-up. The contribution of the trajectory preponderates in most cases at the beginning of the correction. An emphasis of the dispersive term in the next iterations can still improve the result depending on the conditions (initial emittance, wake fields, alignment errors) as dispersion can be further reduced. It may even be beneficial to cope with non-linearities versus energy deviation affecting the optics.

In practice this can be carried out by an appropriate and dynamical weighting process of the various terms that come in the algorithm.

6. References

[1] T.O. Raubenheimer and R. Ruth, "A New Trajectory Correction Technique for Linacs", in Proc. 2nd European Particle Accelerator Conf., Nice, June 1990.

[2] G. Guignard, C. Fischer, A. Millich, "Investigations on Beam Damping Simulations and the Associated Model of CLIC", in Proc. Particle Accelerator Conf., San Francisco, May 1991.

[3] C. Fischer, "A Process to Correct Trajectory and Dispersion in CLIC", to be published.