# Effects of the $G\gamma = 2$ Imperfection Resonance on the Spin Motion *

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*Abstract*

Experiments with polarized protons performed at the Indiana University Cyclotron Facility (IUCF) Cooler Ring revealed an apparent 1.8 MeV downward shift[1] in the energy at which the $G\gamma = 2$ imperfection resonance occurs. The shift in the resonance energy results from the presence of a type-3 snake[2] which shifts the spin tune of the orbiting proton. The type-3 snake is comprised of the electron beam confinement magnets in the cooling region. Along with the type-3 snake, the imperfection resonance may also shift the spin tune. The spin tune is found to be shifted away from the resonance energy. The combined effects of the type-3 snake and the imperfection resonance on the spin motion of the proton is included in the analysis of the data in the immediate vicinity of the resonance. The imperfection resonance strength parameters affecting the spin motion are deduced.


## 1 SPIN MOTION IN THE PRESENCE OF ORBIT DISPLACEMENTS IN THE COOLER

The leading order spin precession elements in the cooling section of the IUCF Cooler Ring are VS,CS,VS′, T,MS,T,VS′,CS,VS, where MS is the main solenoid, T are toroids which serve to bend the electron beam trajectory to match that of the proton, CS are compensating solenoids which are set to cancel the coupling of the betatron motion introduced by the main solenoid, and VS and VS′ are vertical steerers which remove vertical orbit perturbations introduced by the radial part of the toroidal field.

Neglecting the contribution from the imperfection resonance, the spin equation of motion in the presence of the electron cooling system may be expressed as follows:

$$e^{-i\pi\nu_s(\vec{n}_s\cdot\vec{\sigma})} = e^{-i\frac{G\gamma}{2}(2\pi-\theta)\sigma_3}\; C\; e^{-i\frac{G\gamma}{2}\sigma_3\theta}, \qquad (1)$$

where $\theta$ is the angle at which the detectors are located relative to the cooling section ($\theta = \pi/3$), $C$ is the matrix defining the spin precession in the cooling region and is given by

$$C = [e^{-i\frac{\phi}{2}\sigma_1} e^{i\frac{\psi_c}{4}\sigma_2} e^{i\frac{\phi}{2}\sigma_1}] e^{-i\frac{\psi}{2}\sigma_2} [e^{-i\frac{\phi}{2}\sigma_1} e^{i\frac{\psi_c}{4}\sigma_2} e^{i\frac{\phi}{2}\sigma_1}],$$

where $\vec{n}_s \equiv (\cos\alpha_x, \cos\alpha_y, \cos\alpha_z)$ is the stable spin direction vector, $\nu_s$ is the proton spin tune, $\sigma_i$ are the Pauli matrices, and $\psi$, $\psi_c$, and $\phi$ are the spin precession angles due to cooling solenoid, the compensating solenoids, and the vertical steerers, respectively; i.e. $\psi = (1+G)\int B_\parallel^{main} dl/B\rho$ for the cooler main solenoid, $\psi_c = (1+G)\int B_\parallel^{comp} dl/B\rho$ for the compensating solenoid, $\phi$ is the effective vertical steerer precession angle.

The spin tune, $\nu_s$, can be obtained from the trace of Eq.(1). Assuming that the injected polarization is purely vertical, the measured radial, longitudinal, and vertical components of the polarization are given by $P_L = P_{inj}\cos\alpha_z\cos\alpha_y$; $P_R = P_{inj}\cos\alpha_z\cos\alpha_x$; $P_V = P_{inj}\cos^2\alpha_z$, where $(\cos\alpha_x, \cos\alpha_y, \cos\alpha_z)$ are obtained from Eq.(1). Using Taylor series expansions in the limit of small $\phi$, we obtain

$$\cos\pi\nu_s \approx \cos\frac{\psi_c-\psi}{2}\cos(\pi G\gamma - \frac{\psi\phi}{2})$$
$$P_R \approx P_{inj}\sin\frac{\psi_c-\psi}{2}\sin(\frac{2G\gamma\pi}{3})\frac{\cos\alpha_z}{\sin\pi\nu_s}$$
$$P_V \approx P_{inj}\cos^2\alpha_z$$
$$\cos\alpha_z \approx \cos(\frac{\psi-\psi_c}{2})\sin(\pi G\gamma - \frac{\psi\phi}{2})\frac{1}{\sin\pi\nu_s},$$

which indicates that the spin tune is shifted from the expected value of $G\gamma$ to $G\gamma - \frac{\psi\phi}{2\pi}$. Therefore the imperfection depolarizing resonance will occur when $G\gamma - \frac{\psi\phi}{2\pi} = n$ with integer $n$. The shift in the spin tune leads to a shift in the resonance energy, $\Delta E = mc^2\frac{\psi\phi}{2\pi G}$.

## 2 EXPERIMENTAL DATA ANALYSIS

Experimental data[1] ranging from 104.2 to 120.0 MeV at the Indiana University Cyclotron Facility (IUCF) Cooler Ring were used in our data analysis. Fig.1 shows data taken at 104.5, 105.9, 106.9, 107.8, and 110.5 MeV along with the results of the analysis. Plotted are the measured values of the vertical and the radial polarizations as a function of the net longitudinal field error, $\int B_\parallel dl$ in Tesla-meters. The net longitudinal field error is calculated given the known currents, $I_{main}$, $I_{tor}$, and $I_{comp}$ in the main cooling solenoid, the toroids, and the compensating solenoids, respectively using the following calibration equation: $\int B_\parallel dl$ [Tm] $= 0.4065 I_{main} - 1.3072 I_{comp} + 0.3279 I_{tor}$, where the currents are in kiloAmperes. Typical operating currents are $I_{main} = 675$A and $I_{tor} = 399$A, where $I_{comp}$ ranges from 260 to 340 Amperes. The dips in the polarization data observed at 104.5 and 107.8 MeV are due to synchrotron depolarization resonances.

The dotted curves indicate the predicted behavior taking into account only the longitudinal field imperfection with zero vertical steerer angle (i.e. with the type-3 snake off). The solid curves are calculated using Eq.(1), which include the effects of the type-3 snake in the electron cooling region assuming zero closed orbit error elsewhere in the ring. The resonance energy lies between 105.9 and 106.9 MeV as can be seen by the change in the sign of the radial polarization. The average precession angle due to the

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vertical steerers was determined, by an optimal fit to the experimental data, to be -30 mrad at all incident beam energies except for data taken during the earlier stages of cooler development.

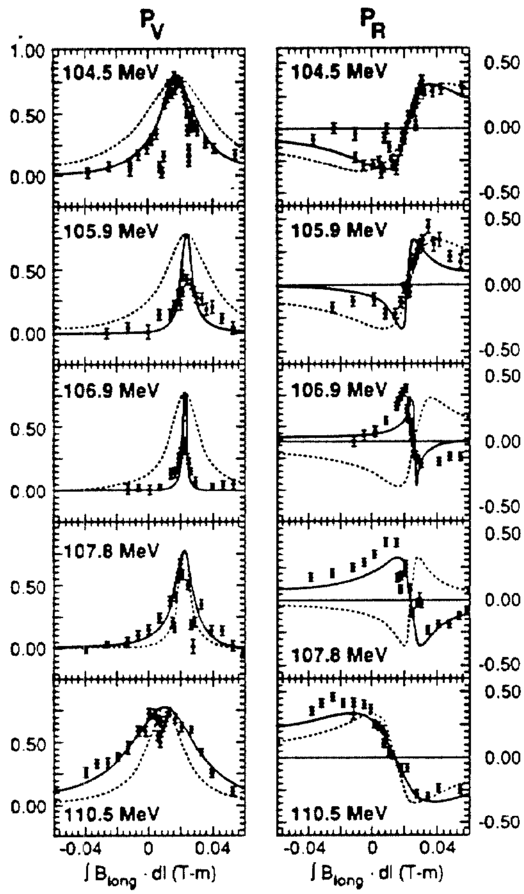


*Fig.1* Measured vertical and radial polarization at 104.5, 105.9, 106.9, 107.8, and 110.5 MeV plotted against the longitudinal magnetic field integral in the Cooler Ring solenoids.

## 3 THE IMPERFECTION RESONANCE

Along with the type-3 snake, the imperfection resonance may also shift the spin tune. In the single resonance approximation[3], the spin equation is given by

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} G\gamma & -\zeta \\ -\zeta^* & -G\gamma \end{pmatrix}\Psi, \quad \zeta = \epsilon \cdot e^{-iK\theta},$$

in which $\epsilon$ is the resonance strength, $K$ is the resonance tune, and $\theta$ is the particle orbital angle around the accelerator. The solution can be expressed as, $\Psi(\theta_f) = T(\theta_f, \theta_i)\Psi(\theta_i)$, with

$$T(\theta_f, \theta_i) = \begin{pmatrix} ae^{i(c-\frac{K(\theta_f-\theta_i)}{2})} & ibe^{-i(d+\frac{K(\theta_f+\theta_i)}{2})} \\ ibe^{i(d+\frac{K(\theta_f+\theta_i)}{2})} & ae^{-i(c-\frac{K(\theta_f-\theta_i)}{2})} \end{pmatrix} \quad (2)$$

where $b = \frac{|\epsilon|}{\lambda}\sin\frac{\lambda(\theta_f-\theta_i)}{2}$, $c = \arctan[\frac{\delta}{\lambda}\tan\frac{\lambda(\theta_f-\theta_i)}{2}]$, $a = \sqrt{1-b^2}$, $d = \arg\epsilon^*$, $\delta = K - G\gamma$, $\lambda = \sqrt{\delta^2 + |\epsilon|^2}$.

The spin tune can be obtained from the trace of the one turn transfer map, $T(\theta + 2\pi, \theta)$ of Eq. (2), i.e. $\cos\pi\nu_s = a\cos(c - K\pi)$. As $G\gamma$ approaches the resonance tune, the spin tune is shifted from $G\gamma$ by $\Delta\nu_s = -|\epsilon|$ below the resonance and by $\Delta\nu_s = |\epsilon|$ above the resonance; i.e. the spin tune is always shifted away from the resonance tune.

## 4 EFFECT OF THE TYPE-3 SNAKE AND THE $G\gamma = 2$ IMPERFECTION RESONANCE

The periodic solution to the spin equation of motion in the presence of both the electron cooling system's type-3 snake and the $G\gamma = 2$ imperfection resonance may be expressed using the spin precession matrix of the cooling section, $C$, defined in Eq. (1) and the transfer matrix, T, defined in Eq. (2). The solution may be calculated by expanding

$$e^{-i\pi\nu_s(\vec{n}\cdot\vec{\sigma})} = T_2(\theta_0 + 2\pi, \theta_0 + \frac{\pi}{3})\ C\ T_1(\theta_0 + \frac{\pi}{3}, \theta_0).$$

To first order in $|\epsilon|$, we obtain the radial and vertical polarizations as

$$P_R \approx -P_{inj}[\sin\tfrac{\psi-\psi_c}{2}\sin\tfrac{2\pi G\gamma}{3} + |\epsilon|\pi\cos\tfrac{\psi-\psi_c}{2}\cos(d + 2\theta_0)]\tfrac{\cos\alpha_c}{\sin\pi\nu_s}.$$

$$P_V \approx P_{inj}\{\cos\tfrac{\psi-\psi_c}{2}\sin(\pi G\gamma - \tfrac{\psi_c\phi}{2}) + \tfrac{2\pi|\epsilon|}{3}\sin\tfrac{\psi_c-\psi}{2}\cos(d + 2\theta_0 + \tfrac{2\pi}{3})\}^2/\sin^2\pi\nu_s.$$

The asymmetry in both the radial and vertical polarization arising from the imperfection resonance is linear in the longitudinal error field, $\psi_c - \psi$.

The imperfection resonance strength at the IUCF Cooler Ring is expected to be on the order of 0.001. Thus the effects of the imperfection resonance should be important only in the immediate vicinity of the $G\gamma = 2$ resonance; i.e. at energies ranging from 105 to 108 MeV. Experimental data exists at 105.9, 106.9, and 107.8 MeV.

### 4.1 Analysis for 105.9 MeV Data

Using the three parameters that govern the spin motion, $\phi$, $|\epsilon|$, and $\arg\epsilon^*$, one can fit the data at 105.9 MeV. Fig.2 shows the experimental data along with model calculations with no type-3 snake (dashed curve), the type-3 snake only (dotted curve), and the type-3 snake along with the $G\gamma = 2$ imperfection resonance (solid curve).

With the type-3 snake turned off ($\phi = 0$), the width of the vertical polarization peak (measured full width at half maximum) is predicted to be too wide and full beam polarization is predicted to be maintained for a fully compensated longitudinal field. With the type-3 snake turned on ($\phi = -25$ mrad) in the absence of the $G\gamma = 2$ imperfection resonance, the polarization profile is predicted to be slightly too narrow and again peaks at full polarization given a fully compensated field. With the type-3 snake turned on ($\phi = -25$ mrad) in the presence of the nearby $G\gamma = 2$ imperfection resonance ($|\epsilon| = .0015, \arg\epsilon^* = 4.5$ rad), both the vertical and the radial polarizations are well fit by the model calculation. The fit to the radial polarization exhibits the asymmetry contained within the data.

Also the magnitude of the vertical polarization is seen to be reduced. This is due to the fact that the stable spin direction is tilted away from the vertical direction thereby resulting in a smaller projection of the injected polarization onto the stable spin direction.

### *4.2 Analysis for 106.9 MeV and 107.8 Data*

In one interesting situation, the polarization was measured at two incident beam energies, 106.9 and 107.8 MeV, during the same run. It is expected that the machine conditions did not change. We therefore attempt to study these two energies by using the exact same three parameters, $\phi, |\epsilon|$, and $\arg\epsilon^*$.

The experimental data at 106.9 and 107.8 MeV are shown in Fig.3 along with the type-3 snake analysis and the results of the present analysis which include both the effects of the type-3 snake and the nearby imperfection depolarizing resonance. The 106.9 MeV data was first fit to determine the values of the type-3 snake and resonance parameters. They are $|\epsilon| = 0.0008$, $\arg\epsilon^* = 1.0$ rad, and $\phi = -34$ mrad. These parameters were then used to predict the expected behavior of the polarization at 107.8 MeV. The theoretical prediction agrees quite well with the experimental data. Both the peak in the measured vertical polarization and the asymmetry in the measured radial polarization are reproduced using the strength parameters extracted from the 106.9 MeV data.

## 5 CONCLUSION AND DISCUSSION

The spin motion in the presence of the electron beam confinement elements is analyzed using the Thomas-BMT equation. In the limit that the precession angles due to the steerers is small, the spin tune is predicted to be shifted by an amount $\frac{\psi\xi}{2\pi}$. The effective steerer angle of -30 mrad corresponds to an orbital angle of -10 mrad, which is about the expected steerer rotation angle.

Along with the spin tune shift introduced by the type-3 snake, the calculations indicate that the proton spin tune is always shifted away from the imperfection resonance tune: the spin tune is shifted from $G\gamma$ by $\Delta\nu_s = -|\epsilon|$ below the resonance and by $\Delta\nu_s = |\epsilon|$ above the resonance.

Using two data sets at 106.9 and 107.8 MeV taken during the same running period, the imperfection resonance strength parameters are deduced. The data set at 106.9 MeV is first fit with the model calculation to determine the three parameters that characterize the measured polarization curves: the steerer angle due to the type-3 snake, the resonance strength, and the resonance phase. Since we anticipate that the machine conditions did not change (i.e. the closed orbit remained the same), we then use these parameters to predict the measured polarization at 107.8 MeV. The prediction agrees with the experiment in two major respects: the amount of depolarization and the amount of asymmetry in the radial polarization.

## 6 REFERENCES


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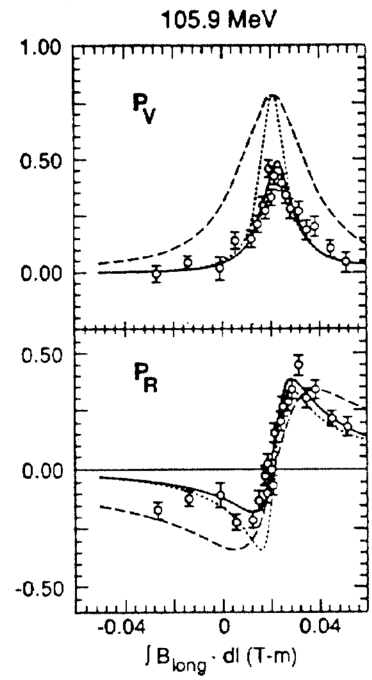


*Fig.2* Comparison of calculations for 105.9 MeV injected protons. The dashed curve omits the effects of both the type-3 snake and the imperfection resonance ($\phi = 0, \epsilon = 0.$). The dotted curve is calculated with a type-3 snake. The solid curve is the prediction with both the type-3 snake and the $G\gamma = 2$ imperfection resonance.

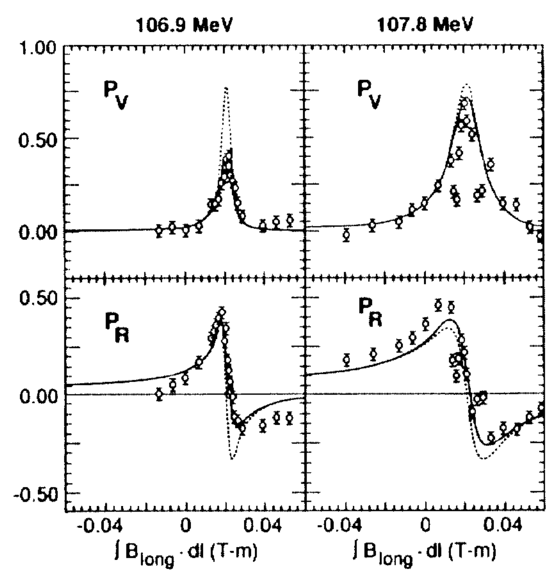


*Fig.3* Comparison of calculations and experimental data at 106.9 and 107.8 MeV. The dotted curve is calculation with a type-3 snake. The solid curve is the prediction including both the type-3 snake and the $G\gamma = 2$ imperfection resonance.