Nonlinear Beam Dynamics Experiments at the IUCF Cooler *

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Abstract

The nonlinear beam dynamics of transverse betatron oscillations were studied experimentally at the IUCF Cooler Ring. Particles were kicked onto resonance islands and the properties of these islands were studied. The island tune was determined with high precision by Fourier analyzing the spectrum containing the island oscillations. The island width was estimated based on a single resonance model. The Hamiltonian of particle motion near a resonance condition was thus deduced. Future plans for nonlinear experiments will be discussed.

1 INTRODUCTION

The IUCF Cooler Ring provides an ideal environment for nonlinear beam dynamics experiments[1,2]. The Cooler ring is hexagonal with a circumference of 86.82 m. The relative FWHM momentum spread of the beam is about ±0.0001. The 95% emittance, or phase space area, of the proton beam is electron-cooled to less than 0.3 π mm-mrad in about 3 sec. The beam lifetime can be as long as hours.

The experimental procedure started with a single bunch of about 3×10^8 protons with kinetic energy of 45 MeV. The cycle time was 10 seconds. The injected beam was electron cooled for about 3 seconds. The bunch length was about 3.6 m (or 40 ns) with a revolution period of 969 ns at an rf frequency of 1.03168 MHz. The beam was kicked with various angular deflections, θ_K, by a pulsed deflecting magnet with a pulse width of 500 ns and rise and fall times of 100 ns. The electron cooling system was turned off 20 ms before kicking. The subsequent beam-centroid displacement was measured with two BPMs having an RMS position resolution of about 0.1 mm. The stability of the horizontal closed orbit were measured to be less than 0.02 mm. The turn-by-turn beam positions were digitized and recorded in transient recorders. A total of 4096 turns were recorded in the available memory buffer for each kick.

The resonance structure was investigated using different orbit deflector strengths. Transverse displacements $(x_1, x_2)_n$ were measured at the n-th turn using the two BPMs. The relative betatron amplitude functions and the betatron phase advance between the two BPMs were deduced from the turn-by-turn data of (x_1, x_2). The phase space coordinates were then transformed to the normal coordinates, $(x_1, p_{x1})_n$, where $p_{x1} = -\frac{1}{2}\frac{d\beta_x}{ds}x_1 + \beta_x x_1'$. For linear betatron motion, the phase space ellipse in the normal coordinates is a circle[3]. Fig. 1 shows the Poincaré map, where the betatron tune, ν_x, is 3.7578 for the left graph and 3.7500 for the right graph. The Poincaré map in the right part of the figure shows that particles were kicked onto the fourth order resonance islands. Within an island the particle trajectory traced out an ellipse around the corresponding stable fixed point. However due to the horizontal and vertical betatron coupling, the ellipse around the stable fixed point in an island is smeared.

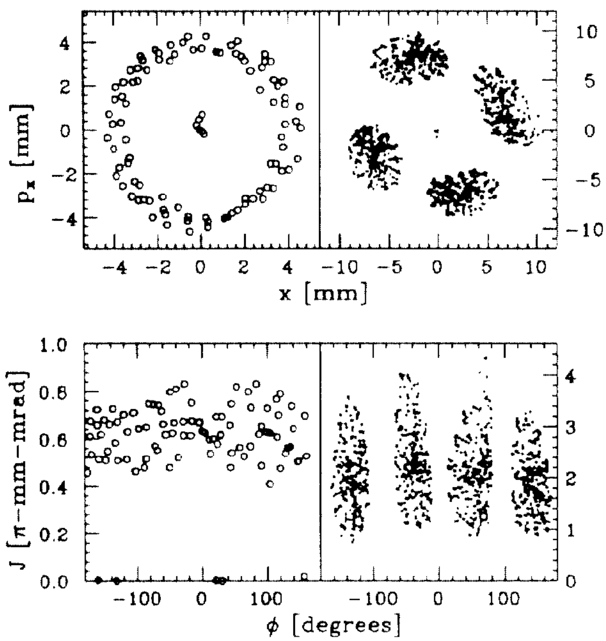

Fig.1 The Poincaré maps, (x_1, p_{x_1}), at the betatron tunes $\nu_x = 3.7578$ (left) and $\nu_x = 3.7500$ (right) are shown. The corresponding maps using the action-angle variables, (J_1, ϕ_1) are also shown in the lower part of the figure.

2 DATA ANALYSIS

The fast Fourier transform (FFT) spectrum of the betatron motion at the fourth order resonance, $\nu_x = 3.7500$, is shown in Fig. 2. The vertical betatron tune is also observed at $\nu_y = 5 - 0.3024$ due to linear betatron coupling. The ratio of the betatron peaks in the Fourier spectrum is a measure of the amount of betatron coupling. We purposly moved the vertical betatron tune away from the horizontal tune to reduce the effect of the betatron coupling.

The frequency of oscillation about the island provides useful information. Using the data of Fig. 1, the FFT spectrum of oscillations in a single island, i.e. every fourth turn around the ring for the fourth order resonance, is

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shown in Fig. 3. Note that there are two dominant peaks: one located at $\nu_{coupling} = \nu_x - \nu_y + 1 = 0.0524 \pm 0.0007$ due to linear coupling and another corresponding to the island tune, ν_{island}, of 0.0013 ± 0.0007, where the accuracy of the island tune measurement is limited by the available memory in the transient recorders.

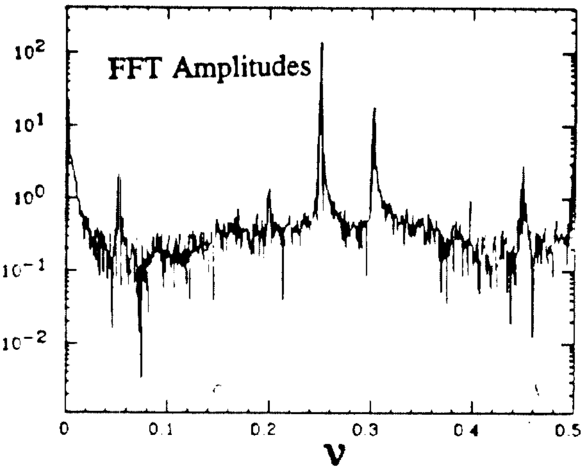

Fig.2 The FFT spectrum for the betatron motion at the resonance condition $4\nu_x = 15$ is shown.

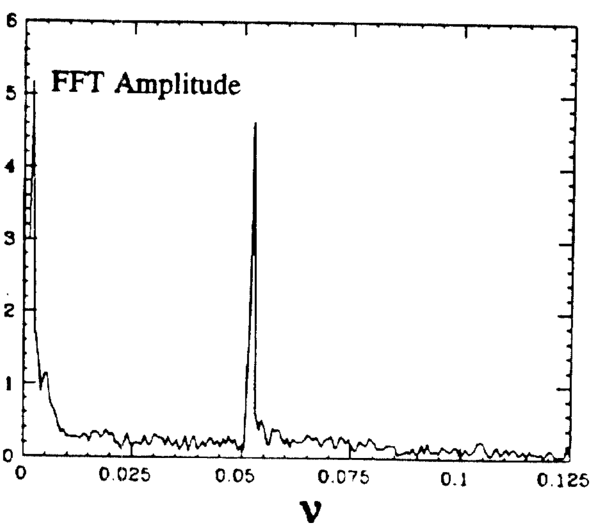

Fig.3 The FFT spectrum of the motion around a fixed point in an island is shown. The island tune is 0.0013.

The one dimensional resonance island ellipse shown in the right side of Fig. 1 is obscured by the linear coupling. Yet the island structure is retained. The motion is a superposition of the more rapid coupling oscillation and the slower resonance island oscillation. The phase space trajectory appears as the coupling oscillation winding around a resonance island ellipse. For the coupling tune of 0.0524 at the fourth order betatron resonance condition, it takes five island-turns (e.g. the 1st, 5th, 9th, 13th and 17th orbital turns for the first island etc.) for the particle to complete one loop around a centroid in the coupling ellipse. The size of the coupling loop depends on the betatron coupling strength. A five-island-turn moving average of the phase space coordinates will effectively eliminate this rapid coupling motion, revealing the slower resonance island oscillation. The moving average will trace out an ellipse around the stable fixed point of an island with a characteristic frequency of the island tune of 0.0013. This corresponds to a period of 800 orbital or 200 island turns. The small amplitude oscillation around the island fixed point is also an ellipse.

2.1 Hamiltonian

Near the single resonance, $m\nu \approx n$, the Hamiltonian can be approximated by[1] $H = H_0(J) + g(J)\cos(m\phi - n\theta - \chi)$. Here (J, ϕ) are the conjugate action-angle variables of the betatron motion, and χ is a phase factor determined by the distribution of nonlinear elements in the accelerator. The betatron tune is given by $\nu(J) = \frac{\partial H}{\partial J} \approx \nu_0 + \alpha J$, where we have used a first order Taylor series expansion in the action variable with ν_0 as the betatron tune at zero betatron amplitude, α as the coefficient of the first order expansion, $g(J)$ as the resonance strength and $\theta = s/\mathrm{R}$ as the orbital angle around an accelerator. For the present study, $m = 4$ and $n = 15$.

A canonical transformation with generating function $F_2(\phi, J_1) = (\phi - \frac{n}{m}\theta)J_1$ can be performed easily to yield a new Hamiltonian, $\tilde{H} = H_0(J_1) - \frac{n}{m}J_1 + g(J_1)\cos(m\phi_1 - \chi)$, where (J_1, ϕ_1) are the new conjugate action-angle variables with $J_1 = J$. Note here that the new Hamiltonian $\tilde{H}$ is a constant of motion; the particle tractory follows a constant contour of $\tilde{H}$. Fixed points of the Hamiltonian are given by $\partial\tilde{H}/\partial J_1 = 0$ and $\partial\tilde{H}/\partial\phi_1 = 0$, i.e. $\nu_1(J_1) - \frac{n}{m} + g'(J_1)\cos(m\phi_1 - \chi) = 0$ and $\sin(m\phi_1 - \chi) = 0$.

Let J_r be the corresponding action such that the betatron tune satisfies a resonance condition; i.e. $m\nu(J_r) = n$. The Hamiltonian can then be expanded around the resonant action:

$$\tilde{H} = \frac{\alpha}{2}(J_1 - J_r)^2 + g(J_r)\cos(m\phi_1 - \chi) + \cdots. \quad (1)$$

Thus the equation of motion in the resonance region satisfies the pendulum like equation of motion. The island tune is given by $\nu_{island} = m\sqrt{|\alpha g|}$. Hence the resonance strength is given by $g = \nu_{island}^2/(m^2\alpha)$. The island width, or the maximum difference in the action variables between the stable fixed point and the separatrix, is given by $\Delta J = (J_1 - J_r) \approx 2\sqrt{\frac{g(J_r)}{\alpha}} = \frac{2\nu_{island}}{m\alpha}$.

The parameter α could be obtained from the slope of the betatron tune as a function of the kicked betatron amplitude J. Alternatively, the ellipses of particle motion around the stable fixed point can be described by the invariant Hamiltonian of Eq.(1). Substituting $\nu_{island}^2/(m^2\alpha)$ for g in Eq.(1), the parameter α can be obtained through matching the particle trajectory with the contour of the Hamiltonian. Fig. 4 shows a (J, ϕ) plot of the data of the island ellipses of Fig. 1 after making a five island-turn moving average in each island in order to remove the coupling motion. Using the Hamiltonian in Eq.(1), we obtain $\alpha = 0.00048 \pm 0.0001$ $(\pi\ \mathrm{mm} - \mathrm{mrad})^{-1}$. The corresponding separatrix is also shown in Fig. 4.

2.2 *Linear Coupling and Resonance Island*

The linear coupling coefficient due to skew quadrupoles and solenoids is given by

$$C_{-} = \frac{1}{4\pi} \oint \sqrt{\beta_x \beta_z}\Big[K + \frac{1}{4}M\Big(\frac{\beta_z'}{\beta_z} - \frac{\beta_x'}{\beta_x}\Big) - i\frac{1}{2}M\Big(\frac{1}{\beta_x} + \frac{1}{\beta_z}\Big)\Big]\times \times e^{i[\psi_x - \psi_z - (\nu_x - \nu_z + 1)\theta]} ds$$

with $K = \frac{1}{2}\big(\frac{\partial B_x/\partial x}{B\rho} - \frac{\partial B_z/\partial z}{B\rho}\big)$ and $M = \frac{B_{sol}}{B\rho}$. Using the skew quadrupoles located in the high dispersion function region, we have corrected the linear coupling coefficient from 0.030 to less than 0.001. After the linear coupling correction, particles kicked onto resonance islands remaining coherent for more than a few seconds. Fig. 5 shows the Poincaré map after the resonance correction. The correcponding FFT spectrum shows clearly a single betatron tune with resonance island sidebands. Using these phase space maps, the detailed functional form of $g(J)$ in the Hamiltonian can be obtained[4].

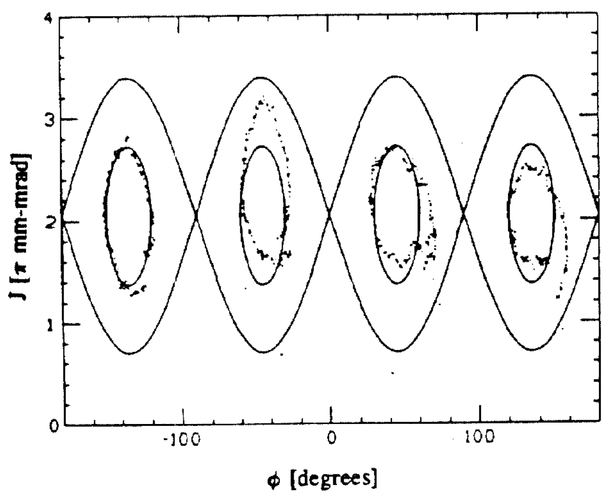

Fig.4 The stable ellipse around island fixed points in the action angle variable is fitted by the Hamiltonian of Eq.(1).

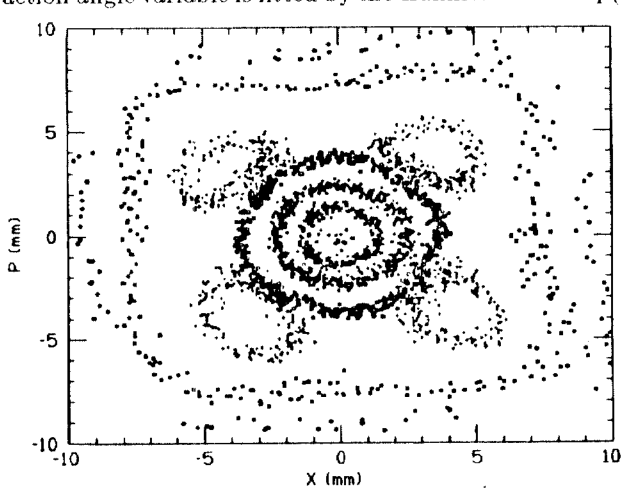

Fig.5 The Poincaré maps around the fourth order resonance islands after linear coupling correction.

2.3 *Longitudinal Phase space tracking*

We mapped the longitudinal phase space by digitizing the bunch phase coordinate relative to the rf phase, and the momentum deviation obtained from the digitized position from a BPM located in a high dispersion region. Fig. 6 shows an example of the longitudinal phase space tracking and its FFT spectrum. The synchrotron tune as a function of the synchrotron amplitude was thus measured.

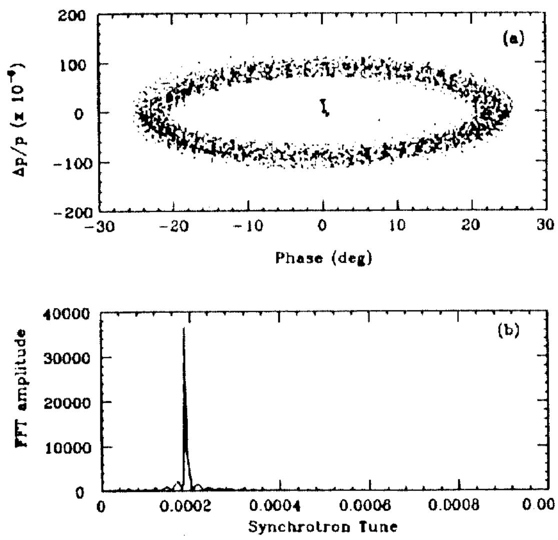

Fig.6 The longitudinal Poincaré map and the corresponding FFT spectrum.

3 CONCLUSION & FUTURE PROSPECTS

We studied properties of fourth order nonlinear resonance islands. One interesting feature is that the betatron coupling does not destroy the structure of one dimensional resonance islands. Experimental data were used to determine resonance island parameters, ν_{island}, J_r and α. The Hamiltonian for the particle motion was derived near the resonance region for the first time.

Currently, we are expanding our particle tracking system into two transverse degrees of freedom with memory buffer of 256K turns. With much improved hardware systems, our nonlinear experiments will study (1) Poincaré maps at 2D resonances, e.g. $2\nu_x + 2\nu_z = n$ or $\nu_x + 2\nu_z = n$, (2) the survival plots with known prescribed multipole elements, (3) the effect of cooling on the phase space motion, (4) the induced tune modulation on the particle motion near a resonance condition and possibly (5) synchro-beta coupling resonances.

4 REFERENCES

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