Self-compensation of the LEP distributed skew gradient

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Abstract

The effect of the parasitic skew gradient in the LEP vacuum chamber is now well compensated. The principle was to select the integer parts of the betatron tunes (71/77) such that the betatron motion becomes mostly insensitive to the zero and first harmonics of the skew field. The residual coupling is corrected with the available skew quadrupoles. The drawbacks were to enforce tune values which were not optimal for performance and prevented polarization at the Z^0 peak. In 1991, the tunes were changed from 71/77 to 70/76 with a resulting performance improvement; the required tune difference caused however the vertical tune to become a multiple of the machine superperiod, making the machine rather sensitive to imperfections. In this paper, we describe an alternative solution which consists in departing from the equal betatron phase advances in the cells to make possible a cancellation of the betatron coupling over each arc. The tunes become free parameters that are optimized for highest performance and polarization at the Z^0 peak (78/78).

1 THE COUPLING SOURCE

LEP is operated at low magnetic field to reduce the energy loss by synchrotron radiation. It is thus sensitive to small field perturbations. The most noticeable one was identified during the commissioning. A thin nickel layer on the vacuum chamber of the dipoles was found magnetized in such a way as to create skew multipoles. Beam measurements [1] allowed to evaluate the skew quadrupole and sextupole components (figure 1). There were 5 measurements per arc in arcs 2, 3 and 4, and one in the others. Figure 2 shows the azimuthal harmonics of the skew gradient. The dominant components are the zero harmonic, and to some extent the first harmonic (modulo 8).

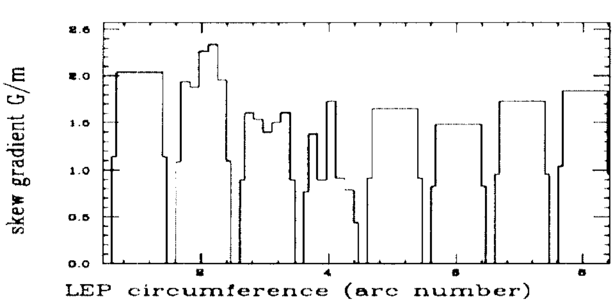

Figure 1: Azimuthal distribution of the skew gradient

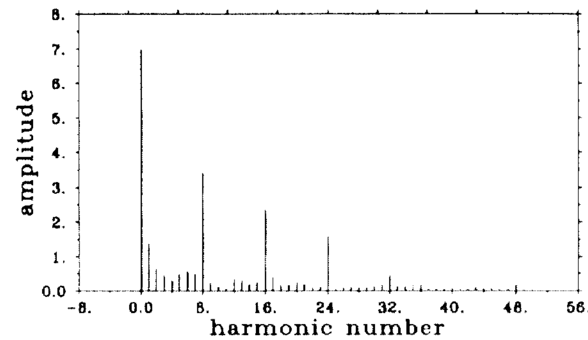

Figure 2: Spectrum of the parasitic skew gradient

2 STRENGTH OF THE COUPLING

The LEP working point being close to the linear difference resonance

$$\Delta e = Q_x - Q_y - p \leq .1\,, \tag{1}$$

the betatron coupling is well parameterized by its complex coefficient [2]:

$$c = \frac{1}{2\pi}\int_0^{2\pi} \sqrt{\beta_x\beta_y}\,K_s e^{i[(\mu_x - Q_x\theta)-(\mu_y - Q_y\theta)+p\theta]}\,d\theta \tag{2}$$

$$K_s = \frac{1}{2}\frac{R}{B\rho}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right)$$

R is the average machine radius, β, μ and Q the usual betatron parameters, θ the azimuthal angle and K_s the skew gradient.

Expression (2) may be simplified by observing that K_s is different from zero only in the arc FODO cells where the function $\beta_x\beta_y$ has the explicit form, for equal horizontal and vertical phase advances:

$$\beta_x\beta_y = \beta_F\beta_D\left[1 + 4\sin^2\left(\frac{\Phi}{2}\right)\tan^2\left(\frac{\Phi}{2}\right)\left(\frac{s}{L}\right)^2\left(\frac{s}{L}-1\right)^2\right] \tag{3}$$

Φ is the cell phase advance, β_F and β_D the maximum and minimum of the β function and L the half-cell length. For $\mu = 60°$, $\beta_x\beta_y$ is constant and equal to $\beta_F\beta_D$ to a very good approximation.

A further simplification arises from the symmetry of the betatron phase advance. Although LEP is 4-fold symmetric, it was designed with a natural symmetry of 8 for the phase advances. Taking advantage of this symmetry and combining (2) and (3) yields:

$$c = \frac{\sqrt{\beta_F \beta_D}}{2\pi} \sum_{j=0}^{7} e^{-ij\frac{2\pi}{8}p} \int_0^{\theta_{arc}} K_s(\theta + j\frac{2\pi}{8}) e^{i(\mu_x - \mu_y - \Delta e \theta)} \, d\theta \tag{4}$$

Since K_s has a relatively simple spectrum, it is worth developing it in azimuthal harmonics K_n:

$$c = \frac{4}{\pi}\sqrt{\beta_F \beta_D} \sum_{n=-\infty}^{n=+\infty} K_n \, III_8(p-n) \cdot \left(\int_{arc} e^{-in\theta} e^{i(\mu_x - \mu_y)} e^{-i\Delta e \theta} \, d\theta \right) \tag{5}$$

with

$$III_8(p-n) = \sum_{n=-\infty}^{n=+\infty} \sum_{j=0}^{7} e^{ij\frac{\pi}{4}(p-n)},$$

a function which vanishes except for $(p-n) = 8k, \quad k \in \mathbb{Z}$.

3 MINIMIZATION OF THE COUPLING

3.1 Minimization over one turn

The betatron phase advances between arcs are chosen to create overall compensation. This is achieved by selecting p in such a way that $III_8(p-n)$ vanishes when K_n is large. This method, chosen during the LEP commissioning, is economical as it involves rematching only the non-experimental straight-sections of LEP. To avoid K_0, K_1 and their harmonics, we had selected $Q_x - Q_y = -6$ instead of the canonical -8. This compensation has been extremely efficient. The purpose of another strategy is to achieve the same result, but without constraining the tunes. A difference of 6 entails either odd tunes, which are not optimal for beam-beam, or one of the tunes being a multiple of the LEP periodicity. With the advent of polarization and the installation of a pretzel scheme, the tunes are even more constrained.

3.2 Minimization in one arc

Apart from arc 4, the parasitic skew gradient is measured or expected to be rather constant in each arc, though at different levels. The decoupling conditions may be found using equation 4 where the term in $\Delta e \theta$ is neglected ($\Delta e \approx 0.1$).

$$N_c(\Phi_x - \Phi_y) = 2k\pi, \quad k \in \mathbb{Z} \tag{6}$$

with N_c, number of FODO cells in the arc.

4 IMPLEMENTATION

Departing from the canonical 60° or 90° phase advance per cell perturbs the arrangement of the sextupole families in achromats and may thus reduce the dynamic aperture. The chromatic aberration is dominated by the vertically focusing insertion quadrupoles. The vertical betatron phase advance of 60° was thus kept and the horizontal phase advance adjusted to satisfy (6).

4.1 Optimal betatron phase advances

LEP departs from the simple model used so far by the dispersion suppressors cells, which are not periodic and where the parasitic skew gradient is weaker. The optimal phase advance difference was found by an optimization process (table 1). The distributed skew gradient was represented by two thin skew quadrupoles in each half-cell, i.e. in total over 1000 elements. The horizontal phase advance was varied to yield the smallest tune approach, used as a criterion of decoupling. This procedure involved rather heavy calculations to match the varying arc to the insertions, which were conveniently carried out with MAD [3]. The gain with respect to the design optics, which, due to its betatron tunes of 70 and 78, was sensitive to the zero harmonic of the parasitic skew gradient, is of two orders of magnitude. Optimizing beyond the value of $|c| \approx 0.03$ is actually not significant, as the precision in the knowledge of the skew gradient is not better than 5% .

Table 1: Coupling versus optics and phase advances

Optics	Q_x	Q_y	Φ_x	Φ_y	$\|c\|$
design	70	78	60°	60°	0.60
production	70	76	60°	60°	0.050
optimized	78	78	70°	60°	0.086
			71°	60°	0.022
			71.5°	60°	0.006
			72°	60°	0.024

4.2 Parasitic vertical dispersion

The spurious vertical dispersion, which drives the synchrobetatron resonances, is not affected, as the vertical betatron phase advance is not changed (table 2).

Table 2: Calculated parasitic vertical dispersion

Optics	$< D_y^2 >^{1/2}$ cm
physics	3.00
optimized	2.68

4.3 Betatron tunes

With the increased horizontal phase advance, tunes of 78/78, equivalent to the design tunes 70/78, can easily be produced by adjusting the low-β and high-β insertions. These tunes have the further advantage that the effect of the systematic spin resonances [4] on the polarization at the Z^0 is minimized.

4.4 Chromaticity correction

The LEP sextupoles are cabled in 6 families to provide the necessary parameters to correct the chromaticity on the 60° lattice. To avoid recabling, the correction of the chromaticity was attempted with the present scheme.

In spite of the sextupoles of the same family not being well in phase with respect to the off-momentum β-beating, an acceptable solution could be found owing to the smaller chromatic perturbation of the horizontal motion. It was further improved by increasing the β_x^* by a factor of 2, changing the optimal coupling from 4% to 2%. The $Q(\delta)$ characteristics are rather similar to that of the production optics (figure 3) and so are the dynamic apertures expressed in beam rms width (figure 4).

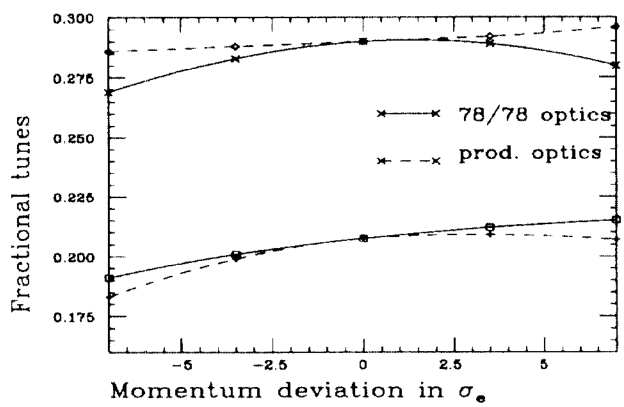

Figure 3: Tunes variations with δ on the new and reference optics

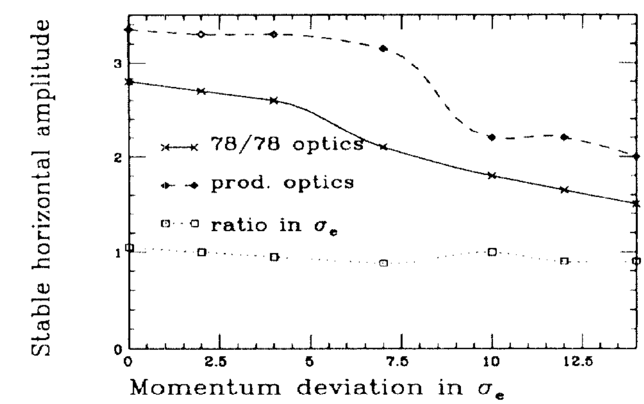

Figure 4: Dynamic apertures of the new and reference optics

5 EXPERIMENTATION

Within a few hours, a circulating beam on a well corrected orbit was obtained. Without much optimization of the tunes, chromaticities, ..., a beam current of 550 μA in four bunches could be accumulated, very close to the performance of the production optics.

The residual betatron coupling is expected to stem not only from the parasitic skew gradient, but also from the tilt of the quadrupoles and the vertical orbit deviations in the sextupoles. For the observed vertical orbit (y_{rms} = 1.16 mm), the expected coupling contributions are:

- 0.0019 from an rms tilt of 0.1 mrad of the quadrupoles
- 0.0040 from the orbit displacement in the sextupoles,
- 0.006 from the parasitic skew gradient.

The total betatron coupling should thus be $|c| = 0.0074$

A standard closest tune approach was performed to measure the actual value of $|c|$ (figure 5).

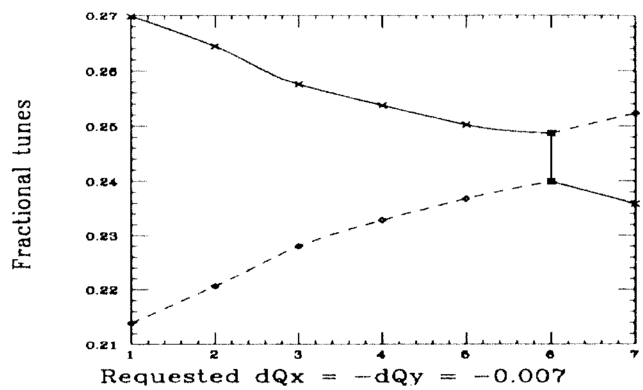

Figure 5: Closest tune approach

The result of 0.0076 is equal to the expectation within the measurement errors. The closest tune approach was further minimized to 0.0027 by exciting the standard skew quadrupoles of LEP (solenoid compensators).

6 CONCLUSION

The split of the horizontal and vertical phase advances of the cells very efficiently weakened the effect of the parasitic skew gradient. In the mean time, new requirements have appeared for the LEP optics (pretzel scheme, energy increase) which all require stronger focusing to decrease the horizontal emittance. Phase advances of 90°/45° or 90°/60° which satisfy (6) are being studied.

7 ACKNOWLEDGMENTS

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8 REFERENCES

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