FRINGING FIELD EFFECTS OF THE PLANE UNDULATOR ON BEAM DYNAMICS IN A STORAGE RING

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Abstract

This paper describes fringe effects in plane undulators using the methods of [1]. The undulator structure is practically a series of fringing fields. A change of the vertical field component along the azimuth is described by the sinusoid. Theoretical results are compared with the results, obtained at BESSY [2]. It is shown that these results are in good accordance. The questions of decreasing the nonlinear effects are discussed.

1. INTRODUCTION

The effects occurring due to the influence of the fringing fields of magnet elements, in particular, of dipole magnets, on the beam dynamics in cyclic accelerators may be significant for a small radius of curvature ρ [3]. These effects are usually investigated by the following procedure: first, on the basis of the known field (measured or calculated) the particle orbit is reidentified and then the field components in the beam concomitant natural coordinate system are expanded in a series about the degrees of deviation from this orbit, and the equations of motion are numerically integrated [3,4]. This classical approach is however rather cumbersome. It is particularly difficult to use it for the multielement systems as is the case with undulators. It has been proposed in [1] to apply the methods of the perturbation theory [5] for investigating the fringe effects. Here the results of applying these methods to the case of plane undulators are reported.

2. RESEARCH METHOD

At a small angle of particle deviation in the fringing field, $\alpha(s)$, the orbit displacement can be neglected, and the particle may be assumed to experience perturbations on each of the dipole magnet edges. These are caused by the rotation of the fixed coordinate system, in which the field is expanded relative to the orbit co-moving coordinate system. An expression has been derived for the tune shift of vertical betatron oscillations versus their amplitude. For small edge angles α this expression is written to an accuracy of α_0^2

$$d\nu_z = \frac{B_0 R^2}{2\pi B^2 \rho^2} \sum_{k=1}^{\infty} \sum_{m=1}^{2M} \frac{(-1)^{k+1}}{k!(k-1)!} |a_{zm}|^{2(k-1)} \times |V_m|^{2k} \int_0^{\infty} b_{0m}^{(2k-1)}(s) \Big(\int_0^{\infty} b_{0m}(s)ds - \frac{B}{B_0} \alpha_0 \rho \Big) ds, \qquad (1)$$

where B_0 is the magnet gap;

R - average machine radius;

$B\rho$ - magnetic rigidity of the particle;

a_{zm} - particle oscillation amplitude on the azimuth of the fringing field;

$|V_m| = \beta_{zm} / 2R$ - Floquet function modulus at the magnet edge;

$b_{0m}(s)$ is the vertical field component normalized to b_0;

$$b_{0m}^{(2k-1)}(s) = d^{2k-1} b_{0m}(s) / ds^{2k-1} .$$

The summation is taken over 2M edges of M magnets. Expression (1) is written for the magnets with infinitely wide poles under the assumptions that $|a_z| \ll \rho$ and β_{zm}=const over the fringing field length.

3. FRINGE EFFECTS IN PLANE UNDULATORS

The magnetic field components of a wide- pole plane undulator are described in the fixed coordinate system by the following expressions [6]:

$$\begin{aligned} B_x &= 0; \\ B_z &= B_0 \operatorname{ch}(2\pi z/\lambda) \cos(2\pi s/\lambda); \\ B_s &= -B_0 \operatorname{sh}(2\pi z/\lambda) \sin(2\pi s/\lambda), \end{aligned} \qquad (2)$$

where λ is the undulator period (Fig. 1).
After expanding the hyperbolic functions in terms of z as a thin-lens approximation, the perturbating field is described by the expression

$$B_x = \frac{B_0^2}{B\rho} \sum_{k=1}^{\infty} \frac{z^{2k-1}}{(2k-1)!} \Big(\frac{2\pi}{\lambda}\Big)^{2k-2} \sin\frac{2\pi}{\lambda} \times$$

$$\times\left(1 - \sin\frac{2\pi s}{\lambda} - \frac{2\pi B\rho\alpha_0}{B_0\lambda}\right). \qquad (3)$$

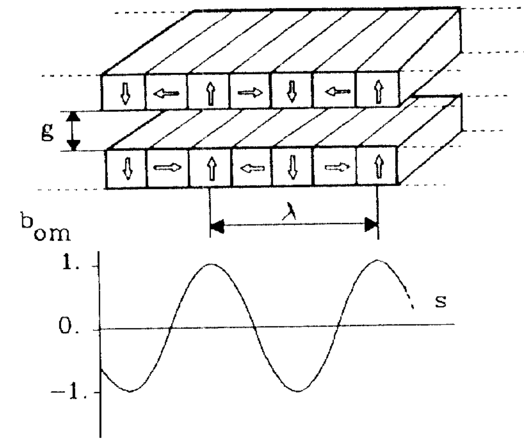

Fig.1. Schematic view of the plane undulator

This field causes the tune shift of vertical betatron oscillations

$$d\nu_z = -\frac{B_0^2 R}{2\pi B^2\rho^2}\sum_{k,m}\frac{1}{k!(k-1)!}|a_{zm}|^{2(k-1)}\times |V_m|^{2k}\left(\frac{2\pi}{\lambda}\right)^{2k-3}\left(1-\frac{\pi}{4}-\frac{2\pi\rho\alpha_0 B}{B_0\lambda}\right). \qquad (4)$$

It is seen that at $\alpha_0 = \lambda(4-\pi)/8\pi\rho$ for the monochromatic beam we have $d\nu_z=0$.

Generally, rectangular-pole undulators with $\alpha_0 = \phi/2$ are used, where ϕ is the bending angle in the magnet array. In this case the expression for the tune shift is

$$d\nu_z = \frac{B_0^2 R}{8B^2\rho^2}\sum_{k,m}\frac{1}{k!(k-1)!}|a_{zm}|^{2(k-1)}\times\left(\frac{2\pi}{\lambda}\right)^{2k-3}. \qquad (5)$$

The measurable parameter is z_m, i.e., the greatest particle deflection from the orbital plane (envelope). Putting $|V_m|' = 0$ over the fringing field length we can make the substitution: $|a_{zm}|^2 \simeq z_m^2\nu_{z0}^2|V_m|^2$. Then the expression for the tune shift of the monochromatic beam has the forms:

$$d\nu_{z1} = \frac{\lambda\sum_m \beta_{zm}}{32\pi\rho^2} \quad \text{(linear case)} \qquad (6)$$

$$d\nu_{z2} = \frac{\pi\nu_{z0}^2 z_m^2\sum_m \beta_{zm}^2}{32R\rho^2\lambda} \quad \text{(nonlinear case)} \qquad (7)$$

On measuring the beam envelope at an arbitrary point i, we carry out the change $z_m^2 \simeq z_i^2\beta_m/\beta_i$ in expression (7).

The $d\nu_{z1}$ values calculated by formula (6) were found to be 0.036 for the undulator SUPER ACO (0.03 in experiment[7]) and 0.043 for the wiggler DCI (0.042 in measurements [8]). It should be noted that one should be rather careful when comparing with DCI, since the trajectory distortion in a superconducting wiggler can be rather strong, and yet, the qualitative agreement between the results can be stated here with certainty.

Of most interest is the comparison with the results obtained with the installation BESSY [2], which is characterized by a relatively high value of the vertical amplitude function $\beta_z(\simeq 15m)$ at rather small R (9.9m) and ρ(16.4m at g=5cm; 6.5m at g=3cm). Figure 2 shows $d\nu_z$ versus ρ in the BESSY undulator.

The tune shift $d\nu_z$ in BESSY as a function of the vertical emittance ε_z ($z_m^2 \simeq \beta_{zm}\varepsilon_z$) is shown in Fig.3. It is seen that the calculations by the present method show rather good agreement with the available experimental data.

The expression corresponding to formula (4) and serving to take into account the effects under consideration in the computer codes that simulate the beam dynamics in the thin-lens approximation, has the form

$$\frac{dz}{ds} = \frac{B_0^2}{B^2\rho^2}\sum_{k,m}\frac{z_m^{2k-1}}{(2k-1)!}\left(\frac{2\pi}{\lambda}\right)^{2k-3}\times\left(1-\frac{\pi}{4}-\frac{2\pi\alpha_0 B\rho}{B_0\lambda}\right). \qquad (9)$$

4. RESULTS AND CONCLUSIONS

1. Analytical expressions are derived to describe nonlinear fringe effects in plane undulators.

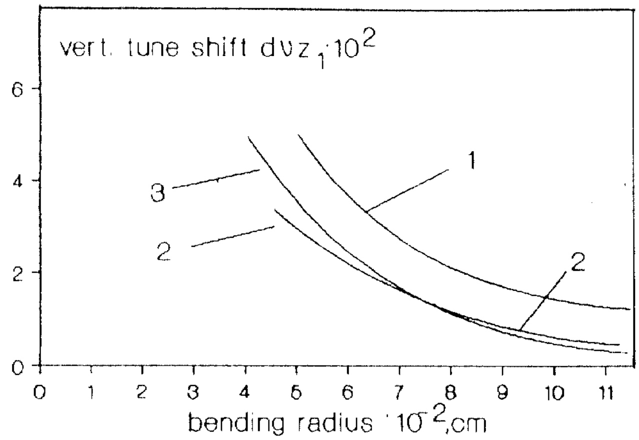

Fig.2. $d\nu_z$ versus $\rho(g)$ in the BESSY undulator. 1: experiment, 2: theory[2], 3: calculation by formula (6).

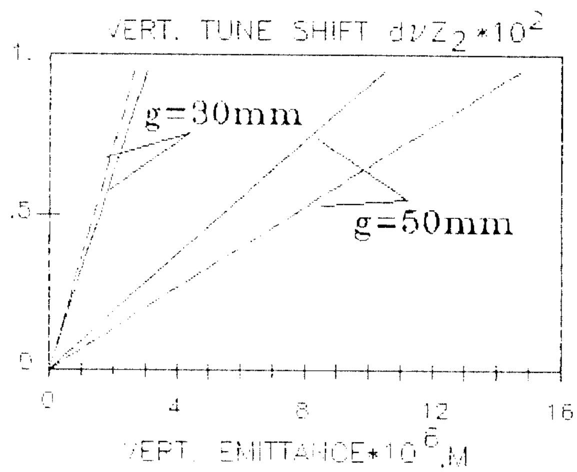

Fig.3. Dependence of $d\nu_z$ on ε_z, determined by the action of BESSY undulator fringing fields. The solid lines - approximation of the BESSY experimental data[2], the dashed lines - calculations by formula(7).

2. Calculations of fringe effects in plane undulators by the methods of the perturbation theory show rather good agreement with the experimental data.

3. The present method makes it possible to estimate the influence of undulator edges on the betatron tune shift, and also to represent rather simply the fringing field-excited perturbations as a thin-lens approximation for the use in the computer codes simulating the beam dynamics.

4. It is demonstrated that the effective octupole field component acting on the vertical betatron motion arises even at $\int B_x dx = 0$ (one of the standard requirements to the undulator), but it can be compensated by a proper choice of the angle $\alpha_0 = \lambda(4-\pi)/8\pi\rho$. In this case the undulator becomes transparent to a fixed beam energy.

5. REFERENCES

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