Experimental Beam Dynamics at the Third Integer Resonance*

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Abstract

The nonlinear beam dynamics of transverse betatron oscillations have been studied experimentally at the IUCF Cooler Ring. Motion in one dimension was measured for betatron tunes near the third integer resonance. The Hamiltonian for nonlinear particle motion near the third integer resonance has been experimentally deduced and is compared with the results from a simple model.

1 INTRODUCTION

Nonlinear magnetic fields in an accelerator can produce limiting dynamic aperatures, but they have also been found useful in applications such as slow extraction of beam from a high energy accelerator, or for beam manipulations in phase space [1]. Theoretical studies[2] of nonlinear fields have been used to predict both the long and the short term behavior of orbiting particles in an accelerator and several nonlinear beam dynamics experiments have been performed in the past[3].

Since individual particle motion cannot be tracked experimentally, these studies typically track the motion of the beam centroid after collectively perturbing the beam. The degree to which the beam motion accurately represents the motion of a single particle is a function of the emittance of the beam; the smaller the emittance of the beam, the more accurate its representation of single particle motion. In this respect, the IUCF Cooler Ring provides an ideal environment for nonlinear beam dynamics experiments. The 95% emittance, or phase space area, of the proton beam is electron-cooled to about 0.3 π mm-mrad with a resulting relative momentum spread for the beam of about ± 0.0001.

This article describes a nonlinear beam dynamics experiment performed at the IUCF Cooler Ring, in which particle motion near the third integer resonance was studied. The experimental methods used in this study are described in Section II. The data and the analysis are discussed in Section III, where we determine the Hamiltonian for the particle motion near the third integer resonance. Section IV contains a summary and concluding remarks.

2 EXPERIMENTAL METHODS

The IUCF Cooler Ring is hexagonal with a circumference of 86.82 m. The experiment was done with a stored 45 MeV proton beam, injected in a 10 s cycle. The stored beam consisted of a single bunch, typically with 3×10^8 protons and a bunch length of about 3.6 m (or 40 ns) full width at half maximum, FWHM. The revolution period in the accelerator was 969 ns with bunching produced by operating an rf cavity with frequency 1.03168 MHz at harmonic number, $h = 1$. The motion of the beam centroid was tracked using two beam position monitors, BPMs[5]. Each BPM measured the displacement of the beam from the stable closed orbit in the horizontal plane. Because the signals from the BPMs are only about 40 ns in duration, digitization without further processing was deemed impractical. Instead, a peak detecting circuit was used in conjunction with a sample-and-hold circuit to produce an analog signal with a level proportional to the peak value of the amplified position, R, and intensity, Σ, signals. Further details on the analog electronics can be found in Ref. [6].

Once processed, R and Σ signals were digitized with a transient recorder having 8192 channels. However, the number of available transient recorders was limited which required that the R and Σ signals be multiplexed in each transient recorder. Thus only 4096 turns were tracked, with 512 of these turns occuring before the beam was kicked.

To produce the coherent transverse motion, the beam was kicked with a pulsed magnetic kicker whose duration was about 500 ns. The kick occurred in conjunction with a triple coincidence between a signal from the data acquisition system, the rf system which was providing the beam bunching, and a seven second delay from the beginning of the injection cycle. Electron cooling has a very small effect in the time a measurement is made (4096 turns), nevertheless it was turned off 20 ms before the beam was kicked to avoid any damping of the betatron oscillations by the electron cooling.

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3 DATA AND ANALYSIS

For particle motion in a circular accelerator, the horizontal deviation from the closed orbit, x, satisfies Hill's equation:

$$\frac{d^2x}{ds^2} + K(s)x = \frac{\Delta B_z}{B\rho}. \qquad (1)$$

Here $K(s)$ is a function of the quadrupole strength, $B\rho = p/e$ is the magnetic rigidity, and s is the longitudinal particle coordinate, which advances from 0 to C, the circumference, as the particle completes one revolution of the cyclic accelerator. The anharmonic term, $\frac{\Delta B_z}{B\rho}$, which arises from higher order multipoles, coupling terms, or quadrupole and dipole errors, is normally small. Oscillations about the closed orbit due to the linear focussing force of quadrupoles, $K(s)$, are called betatron oscillations. The number of oscillation periods in one revolution is the horizontal betatron tune, ν_x, which can be adjusted by varying the quadrupole strength within the accelerator. Both $K(s)$ and the anharmonic term, $\frac{\Delta B_z}{B\rho}$, are periodic functions of s with period C.

Neglecting the small anharmonic term in the Hamiltonian, the betatron motion is linear. Hill's equation (Eqn. 1) can be solved using the Floquet transformation[4] to obtain the solution

$$x = \sqrt{2\beta_x J}\cos\phi, \qquad (2)$$

where J and ϕ are action-angle variables. Here $2J$ is the phase space area (called the Courant-Snyder invariant or the emittance) of the betatron motion and β_x is the betatron amplitude function of the Floquet transformation (β_x is periodic in s with period C). After an appropriate canonical transformation, linear betatron oscillations appear as circles in a Poincaré map. The deviations from a circle in a Poincaré map can be used to study the anharmonic term of the Hamiltonian.

The Poincaré map deviates from a circle most strongly at a resonance. Particle motion around stable fixed points (a stable solution to the equation of motion) in phase space bounded by invariant surfaces may occur for nearly integrable Hamiltonian systems. These stable phase space ellipses around fixed points, called islands, are separated by the unstable fixed points. The particle phase space trajectory passing through unstable fixed points is called the separatrix.

Motion near the third integer resonance at $3\nu_x = 11$ was studied. The available phase space, or dynamic aperature, was not large enough in the current study to allow the observation of any stable fixed points beyond the one at the origin. Consequently, no island structure is observed in this case. However the effect of the nonlinearity on motion is easily seen. In Fig. 1 the Poincaré maps for five different kick amplitudes are shown. In this figure it can be seen that the largest kick has placed the beam just beyond the separatrix, and the beam intensity falls below detection threshold in about 70 turns after the kick.

The lowest order nonlinear field error which can account for this motion can be found in the expansion of the vector

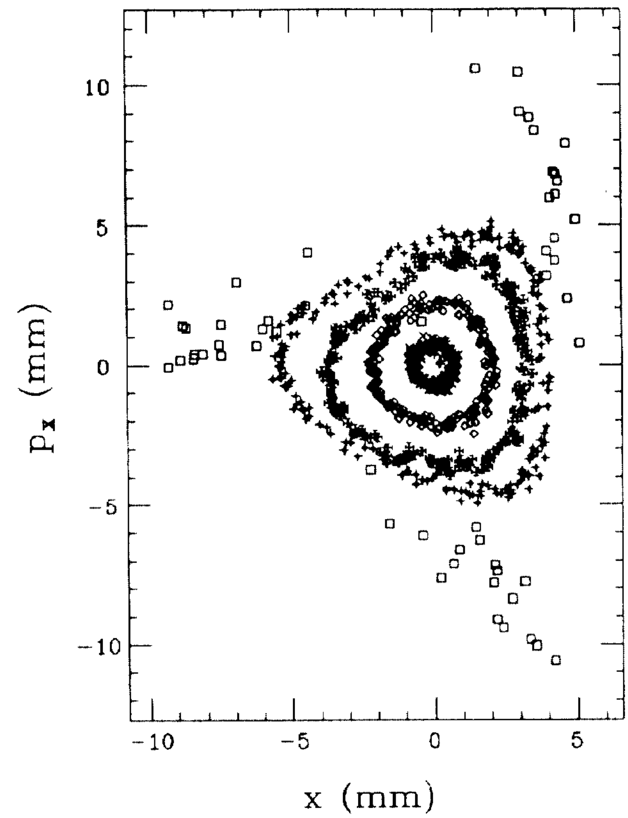

Figure 1: Poincaré maps for motion near the third integer resonance for five different kicker amplitudes: 200, 300, 400, 450, and 470 in arbitrary units, increasing kicker amplitude corresponding to increasing large J contours.

potential. In the median plane of the accelerator, the fields have a relatively simple expansion,

$$A_s = \sum_{n=0} \frac{B^{(n)}}{(n+1)!}x^{n+1}, \qquad (3)$$

where $n = 0$ is the dipole term, $n = 1$ the quadrupole term, $n = 2$ the sextupole term, etc. When the sextupole term is placed in the Hill's equation, motion similar to what is observed will result.

It can be shown the Hill's equation with a sextupole nonlinear field term can be derived from the following Hamiltonian,

$$H = \frac{(2J)^{3/2}F}{24}\cos(3(\phi+\xi)) - 2\pi J\delta \qquad (4)$$

where δ is the difference between the nearby third integer tune and the tune for particles with a betatron amplitude of zero, in this case $\delta = \nu_x - 3\frac{2}{3}$. The factors F and ξ are related to the strength and location of the sources of the third order nonlinearities.

The Hamiltonian of Eqn. 4 define contours of constant H in J-ϕ space. The data shown in Fig. 1 are plotted in J-ϕ space in Fig. 2. The solid lines drawn correspond to lines of constant H/F. The values of δ/F and ξ used in

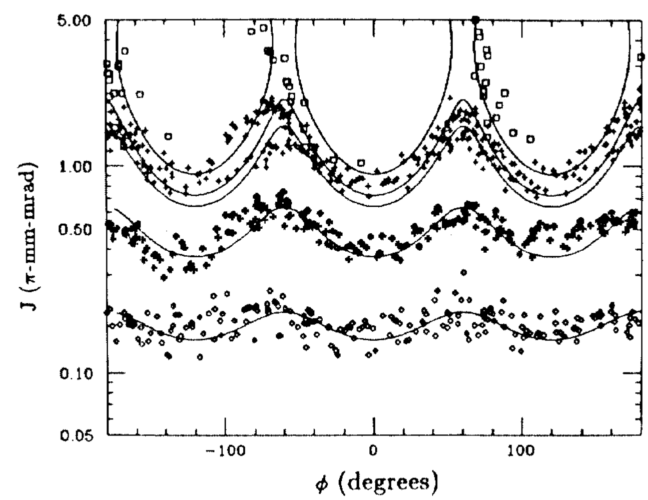

Figure 2: The data from Fig. 1 shown in J–ϕ space. The contours shown are calculated using Eqn. 4.

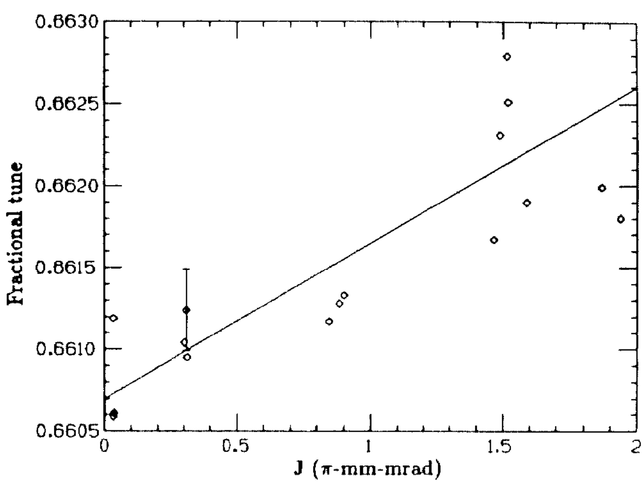

Figure 3: A plot of the measured tunes for data taken near the third integer tune, of which the data shown in Fig. 1 is a subset.

these calculations were determined empirically, the value of δ/F was $-0.050\sqrt{\pi\text{mm-mrad}}$ and the value of ξ was 0°. The uncertainity in this determination of δ/F is estimated to be 5%. In Fig. 3 the tune shift for the data pictured in Fig. 1 is shown. From this figure it can be seen that the value of δ is -0.0060. From this and the empirically determined value of δ/F above, the experimental value of F can deduced to be about 120 $\text{m}^{-\frac{1}{2}}$.

Assuming that the third order resonance is driven by sextupole contributions only, the parameters F and ξ in Eqn. 4 are given by

$$Fe^{3i\xi} = \oint (\beta(s))^{3/2} \frac{B''}{B\rho} e^{3i\nu_x\phi(s)} ds. \qquad (5)$$

The value of F can be found by integrating the sextupole strengths for the different components of the ring. The major contributors of sextupole strength, $S = \frac{B''}{B\rho}$, are the chromaticity correcting sextupoles and the fringe fields of the 12 main dipole magnets. The expected contribution to the integral from the sextupole magnets has been evaluated and has a magnitude of about 82 $\text{m}^{-\frac{1}{2}}$. Assuming the sextupole strength at the ends of each dipole magnet, S_d, to be the same, then Eqn. 5 can be written for the current case as

$$Fe^{3i\xi} = 82\text{m}^{-\frac{1}{2}} e^{-i26^\circ} + S_d(20\text{m}^{\frac{5}{2}} e^{+i109^\circ}) \qquad (6)$$

The sextupole strength of the dipole magnets can be determined from the measured chromaticities, C_x and C_z. Using the program MAD, the value of S needed to produce the measured chromaticities is deduced to be about 0.4m^{-3}. Thus the calculated value of F is 88 $\text{m}^{-\frac{1}{2}}$ and ξ is $-10°$. The discrepency of these calculated values from the experimental values is a topic of continuing investigation.

4 CONCLUSION

In conclusion, we have experimentally identified and measured the properties of third order nonlinear motion of a beam bunch in an accelerator. The experimentally determined parameter F in the Hamiltonian is within 30% agreement with the value calculated from a simple model.

Experimentally derived Hamiltonians, including additional higher order terms, may allow more reliable predictions of particle motion. These experimental nonlinear beam dynamics studies may prove to be useful in an effort to understand the dynamic aperture and the long term behavior of particle motion for future colliders, such as the SSC and RHIC.

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