Transition Crossing in the RHIC*

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Abstract

This report summarizes the study of various longitudinal problems pertaining to the transition-energy crossing in the proposed Relativistic Heavy Ion Collider. Scaling laws are provided for the effects of chromatic non-linearity, self-field mismatch, and microwave instability. It is indicated that the beam loss and bunch-area growth are mainly caused by the chromatic non-linear effect, which is enhanced by the space-charge force near transition. Computer simulation using the program TIBETAN shows that a "γ_T-jump" of about 0.8 unit within a time period of 60 ms is adequate to achieve a "clean" crossing, provided that the remnant voltage of the 160 MHz rf system is less than 10 kV.

1 INTRODUCTION

The transition-energy crossing of charged particles is characterized by a time scale T_c during which the particle motion is non-adiabatic,[1-2]

$$T_c = \left(\frac{\pi A E_0 \beta_s^2 \gamma_T^3}{qe\hat{V}|\cos\phi_s|\dot{\gamma}_s h\omega_s^2}\right)^{\frac{1}{3}}, \qquad (1)$$

where the subscript s represents the synchronous value, and,

qe = electric charge carried by the particle
$\hat{V}$ = peak voltage of the rf accelerating system
h = harmonic number of the rf accelerating system
$\eta_0 = 1/\gamma_T^2 - 1/\gamma_s^2$
γ_T = transition energy
ϕ_s = synchronous phase
ω_s = synchronous revolution frequency
$\beta_s c$ = synchronous velocity
$AE_0 = Am_0c^2\gamma_s$, synchronous energy of the particle.

Problems related to transition crossing can mainly be divided into two categories: single-and multi-particle. In the former category, we study the effect of chromatic non-linearities which impel particles of different momenta to cross transition at different times; while in the latter, we study the bunch-shape mismatch and microwave instability induced by low- and high-frequency self fields, respectively.

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2 THEORETICAL ESTIMATES

In this section, scaling laws are provided for the effects of chromatic non-linearity, self-field mismatch, and microwave instability in the absence of a γ_T-jump.

2.1 *Chromatic Non-Linear Effect*

Particles of different momenta traverse closed orbits of different lengths L. The difference may be expressed in terms of the momentum deviation ($\delta \equiv \Delta p/p$) as

$$\frac{L}{L_s} = 1 + \frac{\delta}{\gamma_T^2}\left[1 + \alpha_1\delta + O(\delta^2)\right]. \qquad (2)$$

The so-called "frequency-slip factor" η thus becomes

$$\eta = \eta_0 + \eta_1\delta + \cdots$$

where

$$\eta_0 = \frac{1}{\gamma_T^2} - \frac{1}{\gamma_s^2}, \quad \text{and} \quad \eta_1 \approx \frac{2}{\gamma_s^2}\left(\alpha_1 + \frac{3\beta_s^2}{2}\right).$$

The two terms in η_1 correspond[2-4] respectively to the differences in circumference and velocity for particles of different momenta at the first non-linear order. The effect of η_1 on the particle motion is important only near the transition when η_0 approaches zero. Define[2,5] the "non-linear time" T_{nl} during which $\left|\eta_1\hat{\delta}(\gamma_T)\right|$ is larger than $|\eta_0|$,

$$T_{nl} = \frac{\left|(\alpha_1 + \frac{3}{2}\beta_s^2)\right|\hat{\delta}(\gamma_T)\,\gamma_T}{\dot{\gamma}_s} \qquad (3)$$

where

$$\hat{\delta}(\gamma_T) = \frac{2^{1/2}\omega_s\left(hSqe\hat{V}|\cos\phi_s|T_c\right)^{1/2}}{3^{2/3}\pi^{1/2}\Gamma(2/3)AE_0\beta_s^2}$$

is the maximum momentum spread at transition, $\Gamma(2/3) \approx 1.354$, and S is the bunch area before transition. The effective increase in the bunch area during the crossing depends[2] on the ratio of T_{nl} to T_c

$$\frac{\Delta S}{S} \approx \begin{cases} 0.76\,\dfrac{T_{nl}}{T_c}, & \text{for } T_{nl} \ll T_c; \\ e^{\frac{1}{3}\left(\frac{T_{nl}}{T_c}\right)^{3/2}} - 1, & \text{for } T_{nl} \gtrsim T_c. \end{cases} \qquad (4)$$

Beam loss occurs if the effective bunch area $S + \Delta S$ after transition is larger than the bucket area.

It is assumed for the RHIC that $\gamma_T = 24.7$, $\omega_s = 4.91 \times 10^5$ s^{-1}, $h = 342$, $\hat{V} = 300$ kV, $\phi_s = 0.161$ rad., and $\alpha_1 = 0.6$. With these parameters, it can be evaluated that $\dot{\gamma}_s = 1.6$ s^{-1}, $T_c = 41$ ms, $\hat{\delta}(\gamma_T) = 4.3 \times 10^{-3}$, and $T_{nl} = 139$ ms. According to Eq. 4, the phase-space area enclosed by the trajectory of the particles near the edge of the bunch after transition is much larger than the bucket area. Therefore, beam loss is expected to occur in the absence of a γ_T-jump.

2.2 Bunch-Shape Mismatch

Both reactive and resistive impedances cause mismatch[6-7] in the nominal bunch shape at the time the synchronous phase is jumped at transition.

A reactive impedance changes the focusing force of the rf system differently below and above transition. The amount of mismatch is then proportional to the ratio of the self field to the rf field provided by the accelerating cavities. For a parabolic distribution, the effective increase in the bunch area due to the mismatch, induced by a coupling impedance $|Z_L/n|$ at low frequency range,[2] is

$$\frac{\Delta S}{S} = \frac{2h\hat{I}(\gamma_T)\,|Z_L/n|}{\hat{V}|\cos\phi_s|\,\hat{\phi}^2(\gamma_T)} \tag{5}$$

where

$$\hat{I} = \frac{3hN_0qe\omega_s}{4\hat{\phi}} \tag{6}$$

is the peak current, and

$$\hat{\phi}(\gamma_T) = 3^{1/6}\Gamma(2/3)\left(\frac{2hS}{\pi qe\hat{V}|\cos\phi_s|T_c}\right)^{1/2}$$

is the maximum phase spread of the bunch at transition.

The effective impedance of the space charge below the cutoff frequency corresponds to a capacitive impedance of about 1.2 Ω at transition. With an intensity of $N_0 = 1 \times 10^9$ $^{197}Au^{79+}$ ions per bunch, the increase of bunch area due to the corresponding bunch-shape mismatch is about 60% in the absence of a γ_T-jump.

In addition to the mismatch, a resistive impedance causes energy dissipation which partly cancels the energy gain from the rf acceleration. Because this cancellation induces a shift in the synchronous phase (ϕ_s), the amount of synchronous phase ($\pi - 2\phi_s$) to be jumped at transition is changed accordingly. If the resistive impedance at low frequency is $\mathcal{R}$, the shift in synchronous phase can also be shown as[2]

$$\Delta\phi_s = \frac{\hat{I}\mathcal{R}}{\hat{V}|\cos\phi_s|} \tag{7}$$

for $\Delta\phi_s$ to be much smaller than 1. Compared with the mismatch due to the space charge, the effect of the resistive impedance is estimated to be small for the transition crossing in the RHIC.

2.3 Microwave Instability

Near the transition energy, the revolution-frequency spread which provides Landau damping vanishes along with the vanishing phase stability and the decreasing synchrotron-oscillation frequency. Both the reactive and the resistive components of the coupling impedance are likely to induce instability. However, since particles cross transition with a non-zero acceleration rate, the synchrotron-oscillation frequency defined by the time derivative of the angle variable (canonically conjugate with the action variable) of the system Hamiltonian, is also non-zero at transition. Consequently, the threshold for microwave instability to occur at transition is[2] for the parabolic distribution

$$D_0 \equiv \frac{8h\hat{I}(\gamma_T)\,|Z_H/n|}{3\hat{V}|\cos\phi_s|\,\hat{\phi}^2(\gamma_T)} \geq 1, \tag{8}$$

where $|Z_H/n|$ refers to the coupling impedance at microwave frequency range.

In the case that the beam current is below the threshold (Eq. 8), the bunch crosses the transition without experiencing microwave instability. On the other hand, in the case that this threshold is exceeded, the time period T_{mw} during which the instability occurs can be estimated,

$$T_{mw} \approx \frac{2\pi}{3^{5/6}\Gamma^2(2/3)}(D_0 - 1)T_c \quad \text{for } D_0 - 1 \ll 1. \tag{9}$$

The amount of growth of the density amplitude, which is defined as the ratio of the amplitude increment of the density disturbance at the end, to the amplitude at the beginning of the time interval T_{mw}, is found to be

$$\frac{\sqrt{3}n}{4h}\hat{\phi}(\gamma_T)(D_0 - 1)^2. \tag{10}$$

With a beam intensity of 1×10^9 ions per bunch and a capacitive impedance of 1.2Ω due to the space charge, it can be calculated that $D_0 \approx 0.9$. Therefore, in the absence of a γ_T-jump the beam is close to the microwave-instability threshold.

The theoretical estimates indicate that the primary concern at transition is due to the chromatic non-linear effect. The development of the "non-linear tails" after the synchronous-phase jump is further enhanced by the space-charge force.

In order to understand more precisely the various mechanisms and to quantitatively determine the crossing efficiency, the computer program TIBETAN has been[2] developed. The program simulates the longitudinal motion of a particle beam by tracking a collection of macro-particles in phase space. It constructs the self fields directly in the time (phase) domain. The bin length used for the construction of the self fields is chosen in accordance with the cutoff frequency of the vacuum pipe. Before transition, the bunch of $^{197}Au^{79+}$ ions is assumed to have a truncated Gaussian-like distribution in longitudinal phase space with an area of 0.3 eV·s/u. With $\alpha_1 = 0.6$ and 10^9 ions per bunch, the chromatic non-linear effect, enhanced by the self-field mismatch, results in a beam loss of about 70% in the absence of a γ_T-jump.

3 CROSSING WITH γ_T-JUMP

An effective way to cure both the beam-induced and the chromatic non-linear effect is to increase the transition-crossing rate of the beam. This can be accomplished either by temporarily adjusting the lattice to achieve a γ_T-jump, or by manipulating the synchronous phase to achieve a larger acceleration rate. Previous studies, however, indicate that the latter results in a non-negligible mismatch at transition.

The method of γ_T-jump, which has been successfully used in many accelerators, provides a large crossing-rate enhancement without causing severe mismatch at transition. In the case that the non-linear effect is dominant, the amount $\Delta\gamma_T$ of jump needed to eliminate the un-desired beam loss and bunch-area growth is

$$\Delta\gamma_T \approx 4\dot{\gamma}_s T_{nl}, \tag{11}$$

with both $\dot{\gamma}_s$ and T_{nl} taking the original values; in the case that the self-field effect is dominant, the amount of jump should satisfy

$$\frac{\Delta S}{S} = \left.\frac{2h\hat{I}(\gamma)\,|Z_{L,H}/n|}{\hat{V}|\cos\phi_s|\;\hat{\phi}^2(\gamma)}\right|_{\gamma=\gamma_T\pm\Delta\gamma_T/2} \ll 1, \tag{12}$$

where $\gamma_T \pm \Delta\gamma_T/2$ correspond to the instants just after and before the γ_T-jump, $\hat{I}$ is given by Eq. 6, and the phase spread at the adiabatic regime is

$$\hat{\phi} = S^{1/2}\left(\frac{-2h^3\omega_s^2\eta_0}{\pi A E_0\beta_s^2 qe\hat{V}\cos\phi_s}\right)^{1/4}. \tag{13}$$

Furthermore, the jump should be accomplished in a short time period so that the effective crossing rate is much larger than the original acceleration rate.

It is obtained that a jump of 0.8 unit performed in about 60 ms is required to eliminate the chromatic non-linear effect. With such a jump, the growth in the bunch area due to the self- field mismatch is about 10%.

Fig. 1 shows the bunch area $S_f = S + \Delta S$ after transition as a function of the area S before transition. For small bunch area S the effect of the non-linearity is reduced, while that of the self-field mismatch and microwave instability becomes dominant. On the other hand, for an area larger than 0.3 eV·s/u, the area growth increases due to the fact that a γ_T-jump of 0.8 unit is not enough to completely eliminate the chromatic non-linear effect.

With the γ_T-jump, the area growth is approximately a linear function of the net coupling impedance. If measures are taken so that the effective space-charge impedance is properly compensated by the inductive wall impedance, the growth will be minimized accordingly.

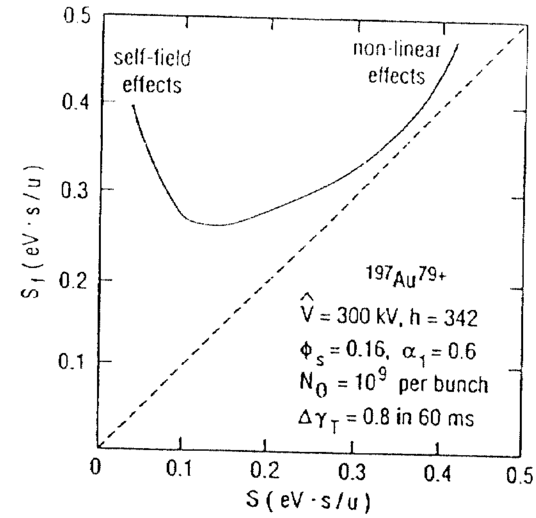

Figure 1: Longitudinal bunch area after transition as a function of the area before transition.

4 TOLERANCE ON STORAGE SYSTEM

The transition energy is crossed with the 26.7 MHz rf system. Despite various measures, a remnant voltage still remains on the 160 MHz rf system (for storage). Previous studies indicate that for a given phase ϕ_{2s} of the 160 MHz system relative to the center of the bunch, the bunch-area growth is linearly proportional to the ratio of the voltage $\hat{V}_2$ to the accelerating voltage. On the other hand, for a given amplitude $\hat{V}_2$, the most severe bunch distortion occurs when $\phi_{2s} = 180°$. Since the bunch length is comparable to the bucket width of the 160 MHz rf system, the bunch appears S-shaped after transition if the remnant voltage is significant. Computer simulation shows that with the γ_T-jump, the growth in area will be less than 10% if $\hat{V}_2$ is less than 10 kV.

5 REFERENCES

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