3-D Simulation Codes for FEL Amplifiers

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Abstract

A fully 3-D simulation code for tapered FEL amplifiers has been developed at the ENEA Frascati Center. Particle equations include transverse and longitudinal motion, so finite emittance and energy spread effects are taken into account. Field equations, including higher harmonics, are solved by a finite-element method. Simulation results for the design of a FEL amplifier in the VUV region are presented.

1. THEORETICAL MODELS

FEL devices can be considered firmly established from the experimental point of view, so devices with high efficiency are now required in many applications. The need for realistic simulation codes to optimize the design of high-performance FELs is now widely recognized. In this framework we have developed a fully 3-D simulation code which also contains the undulator tapering design and higher harmonics evolution. The equations of the longitudinal electron motion are written in the KMR *classical* formalism [1].

$$\frac{d\gamma}{dz} = -\sum_n \frac{|e_n| a_u}{\gamma} \sin\psi_n f_B(n\xi)$$

$$\frac{d\psi_1}{dz} = k_u - \frac{k_1}{2\gamma^2}\left(1 + a_u^2 + \sum_n \left(\frac{|e_n|}{k_n}\right)^2 - \sum_n \left(2a_u \frac{|e_n|}{k_n} f_B(n\xi)\cos\psi_1\right)\right) + \frac{d\phi_1}{dz} \quad (1)$$

where

- k_n n-th harmonic laser wavenumber
- k_u undulator wavenumber
- $a_u = \dfrac{eB}{\sqrt{2}\, k_u mc}$ undulator parameter
- $\xi = \dfrac{1}{2}\left(\dfrac{a_u^2}{1+a_u^2}\right)$
- $f_B(n\xi) = (-1)^{n+1}\left(J_{\frac{n-1}{2}}(n\xi) - J_{\frac{n+1}{2}}(n\xi)\right)$
- J_n ordinary Bessel functions
- $|e_n|$, ϕ_n electric field amplitude and phase of the n-th harmonic
- $\psi_1 = (k+k_u)z - \omega t + \phi_1$ and γ are the electronic phase and its relativistic factor
- $\psi_n = (2n-1)(\psi_1 - \phi_1) + \phi_n$

Equations (1) are coupled to the laser-field equations written in the paraxial approximation [2]

$$\left(2ik_n \frac{\partial}{\partial z} + \nabla_\perp^2\right)e_n = -\frac{eZ_0}{2mc^2} a_u f_B(n\xi) \cdot J < \frac{e^{-i(\psi_n - \phi_n)}}{\gamma} > \quad (2)$$

where

- Z_0 vacuum impedance
- J current density

< > indicates particle average

and to the equations of the transverse motion

$$x'' = -\frac{v}{\dot{s}} \frac{e}{p} [y' B_z - (1+x'^2) B_y + x'y' B_x]$$
$$y'' = \frac{v}{\dot{s}} \frac{e}{p} [x' B_z - (1+y'^2) B_x + x'y' B_y] \quad (3)$$

where

$$\frac{v}{\dot{s}} = \sqrt{1+x'^2+y'^2}$$

Usually the transverse motion is approximated by an overlap of wiggling and betatron motions. The solution of system (3) permits us to simulate the full electron orbits without this approximation.

The magnetic field used is

$$B_x = B \sinh \frac{k_u x}{\sqrt{2}} \sinh \frac{k_u}{\sqrt{2}} y \cos k_u z$$

$$B_y = B \cosh \frac{k_u x}{\sqrt{2}} \cosh \frac{k_u y}{\sqrt{2}} y \cos k_u z$$

$$B_z = -\sqrt{2} B \cosh \frac{k_u x}{\sqrt{2}} \sinh \frac{k_u y}{\sqrt{2}} \sin k_u z$$

Undulator defects are simulated by random errors on the magnetic-field components.

2. NUMERICAL PROCEDURE AND RESULTS

The set of ordinary differential equations (3) representing the electron transverse motion is solved by an Adams-type method and is coupled to the other parts of the code only at the end of each integration step.

The partial differential equation system (2) of the laser field with harmonics, plus boundary conditions, is transformed into a mixed algebraic differential system by a Galerkin-type finite-element method and coupled to the ordinary differential equation system (1) of the longitudinal electron motion. The complete set of equations is then solved by a Gear-type stiff integrator.

The e-beam is simulated by 8192 electrons distributed according to the emittance and energy spread of a real beam.

As a test case we explore the dynamics of harmonics amplification in a FEL amplifier in the VUV region. The FEL parameters are summarized in Table 1. The evolution of the radiation power at the fundamental (a) and at 3rd harmonic (b) along the undulator, at different emittances, are shown in Figs. 1,2 and 3.

Table 1
FEL Parameters

Laser wavelength at the fundamental harmonic λ]nm]	240
3rd harmonic wavelength [nm]	80
undulator period λ_u [cm]	3
e-beam energy E [MeV]	300
Relative energy spread $\Delta E/E$	10^{-4}
Normalized horizontal, vertical emittance $\varepsilon_{x,y}$ [mm·mrad]	10π, 20π, 40π
e-beam current I[A]	300
Input power at $\lambda = 240$ nm P_{in} [W]	100

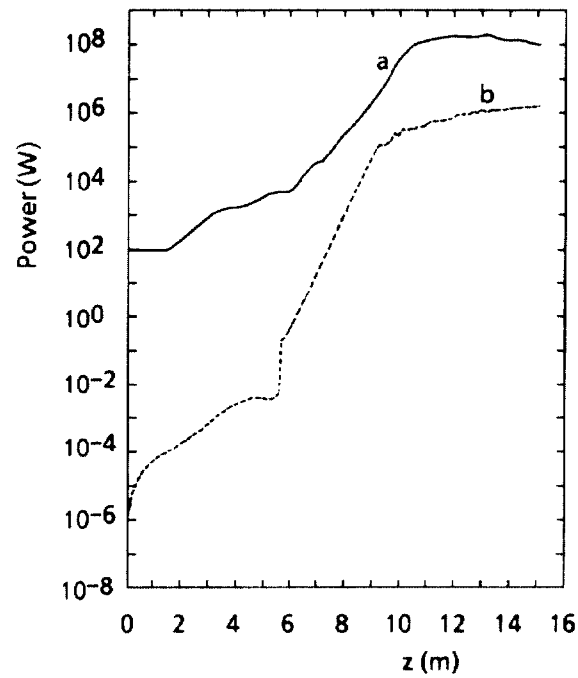

Figure 1. Evolution of the radiation power at the fundamental a) and at 3rd harmonic b) along the undulator at e-beam emittance $\varepsilon_{x,y} = 10\,\pi$ mm·mrad

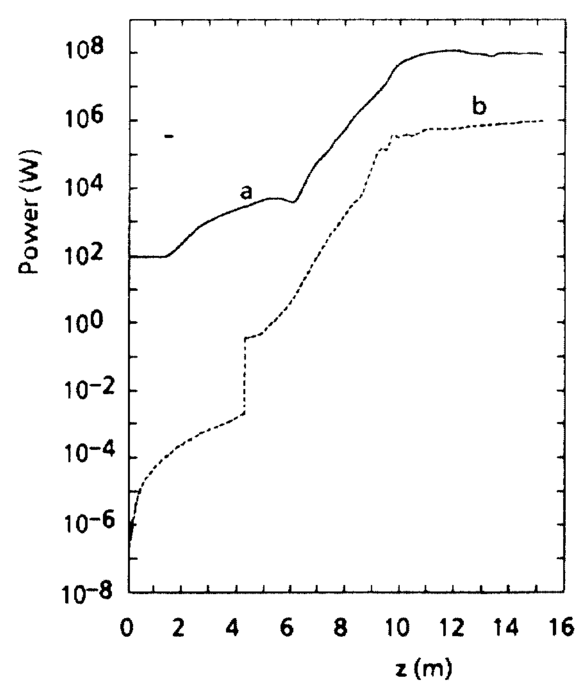

Figure 2. Evolution of the radiation power at the fundamental a) and at 3rd harmonic b) along the undulator at e-beam emittance $\varepsilon_{x,y} = 20\,\pi$ mm·mrad

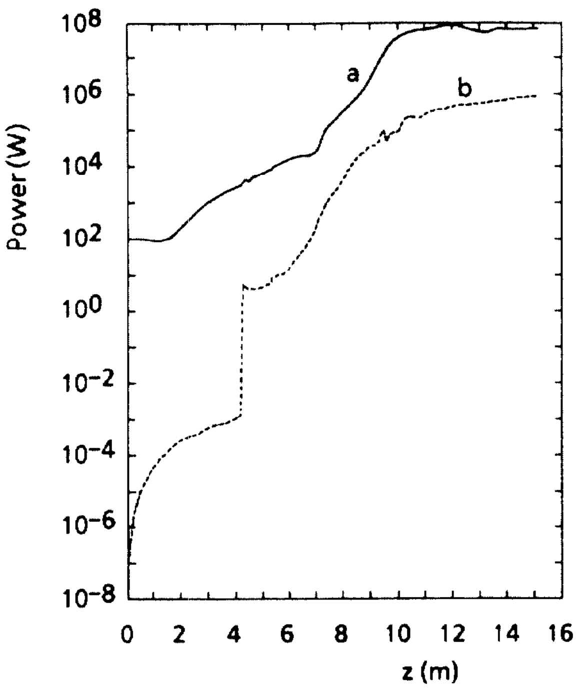

Figure 3. Evolution of the radiation power at the fundamental a) and at 3rd harmonic b) along the undulator at e-beam emittance $\varepsilon_{x,y} = 40\,\pi$ mm·mrad

3. REFERENCES

[1] N.M. Kroll et al., "Free-Electron Lasers with Variable parameter Wigglers", IEEE J. Quantum. Electron., Vol. QE-17, pp. 1436-1468 August, 1981

[2] W. M. Fawley et al., "Synchrotron-Betatron Resonances in Free-Electron Lasers", Phys. Rev. A, Vol. 30, pp. 2472-2481 November, 1984