Waveguide-Gap Tapering of a Far Infrared FEL

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Abstract

One of the most interesting features of a sub-millimeter waveguide-FEL is the potential tunability over almost one octave, obtained by varying the guide-gap. On the other hand, for a given operating frequency the resonant energy decreases as the waveguide gap is increased. A tapering of the waveguide gap to keep the resonant frequency constant as the electrons lose energy, might lead to an extraction efficiency close to 30% and would also help in launching the guided wave into the free-space. Considerations on the feasibility of such a tapering, and preliminary results are presented.

1. THEORETICAL MODEL

Theoretical models of very high efficiency FEL amplifiers have been experimentally confirmed [1]. The scheme used is the undulator field tapering, in which the field amplitude is changed according to

$$\gamma^2 = \frac{k}{k_u}(1+a_u^2) \tag{1}$$

where

k	laser wavenumber
k_u	undulator wavenumber
$a_u = \dfrac{eB}{\sqrt{2}\,k_u mc}$	undulator parameter
B	undulator field

in order to maintain electrons resonant when their energy is substantially decreased due to FEL interaction.

Operation at far infrared and millimeter wavelength regions requires radiation confinement by a waveguide structure. In this case relation (1) becomes

$$\gamma^2 = \frac{\omega/c\,(1+a_u^2)}{2\,(k_u+k-\omega/c)} \tag{2}$$

where $\omega = kc$

The modified resonance condition offers an exciting possibility to investigate a new tapering scheme, i.e., waveguide tapering, in which the waveguide gap is varied to maintain the resonance condition.

Therefore, we have developed a 3–D simulation code as a first check of this new tapering scheme. The code consists of two main blocks: tapering and simulation. The tapering block designs the gap profile by the same procedure used for magnetic field tapering, i.e., by requiring electron phase ψ_r to be held constant according to [2]

$$\frac{d\gamma_r}{dz} = -\frac{e_s a_u}{\gamma_r} f_B \sin\psi_r$$

$$\frac{d\psi_r}{dz} = k_u + k - \frac{\omega}{c} - \frac{\omega}{2c\gamma_r^2}\left[1 + a_u^2 + a_s^2 - 2a_u a_s f_B \cos\psi_r\right] + \frac{d\phi}{dz} \tag{3}$$

$$\frac{de_s}{dz} = \frac{eZ_0}{2\,mc^2} a_u \frac{2I}{ab} f_B \frac{\langle\sin\psi\rangle}{\gamma_r}$$

$$\frac{d\phi}{dz} = \frac{eZ_0}{2\,mc^2} a_u \frac{2I}{ab} f_B \frac{\langle\cos\psi\rangle}{e_s\gamma_r}$$

$$\frac{d\psi_r}{dz} = 0$$

where

- γ_r and ψ_r are the resonant energy and phase respectively
- e_s and ϕ the laser electric field and phase
- $a_s = e_s c/\omega$
- Z_0 vacuum impedance
- I e-beam current
- a waveguide width
- $b = \pi/\sqrt{(\omega/c)^2 - k^2}$ waveguide gap
- $f_B = J_0(1/2\,(a_u^2/(1+a_u^2)) - J_1(1/2\,(a_u^2/(1+a_u^2))$
- J_n the ordinary Bessel functions

and $\langle\ \rangle$ indicates particle average.

The code also simulates the dynamics of a real e-beam, characterized by a finite emittance and energy spread, utilizing typically 4096 electrons as well as the dynamics of the radiation field projection in the TE_{01} mode according to the equations

$$\frac{d\gamma_i}{dz} = -\frac{|e_\ell|\, a_u}{\gamma_i} f_B \sin(\theta_i + \phi)$$

$$\frac{d\theta_i}{dz} = k_u + k - \frac{\omega}{c} - \frac{\omega}{2c\gamma_i^2}$$

$$\left(1 + a_u^2 - \frac{2a_u\, |e_\ell|\, c}{\omega} f_B \cos(\theta_i + \phi)\right) \qquad (4$$

$$\frac{de_R}{dz} = \frac{e Z_0}{2\, mc^2} a_u f_B \frac{2I}{ab} < \frac{\sin\theta}{\gamma} >$$

$$\frac{de_I}{dz} = \frac{e Z_0}{2\, mc^2} a_u f_B \frac{2I}{ab} < \frac{\cos\theta}{\gamma} >$$

where

$$\theta = k_u z + \int k(z)\, dz - \omega t$$

$$e_\ell = \frac{\sqrt{2}\, mc^2}{e} E_\ell \cos\left(\frac{\pi y}{b}\right) e^{i(kz - \omega t)}$$

and e_R, e_I are the real and imaginary part of the electric field e_ℓ.

2. NUMERICAL RESULTS

The text cases we have chosen are summarized in Table 1.

Table 1
FEL parameters

	Case 1	Case 2
Laser wavelength λ]mm]	1	1
Undulator period λ_u [cm]	8	8
e-beam energy E [MeV]	8	12
Relative energy spread $\Delta E/E$	10^{-3}	10^{-3}
Normalized Horizontal emittance ε_x [mm·mrad]	80π	80π
Normalized Vertical emittance ε_y [mm·mrad]	80π	80π
e-beam current I[A]	300	300
Horizontal waveguide length a[cm]	2	2
Vertical waveguide length b[cm]	0.5	0.35
Master oscillator power P_{in}[W]	50	50

Case 1,2 results are shown in Figs 1,2 respectively. Fig. 1a shows the evolution of the radiation power vs the undulator length, and Fig. 1b the waveguide gap tapering along the undulator. Figs 2a,b represent the same quantities referred to case 2.

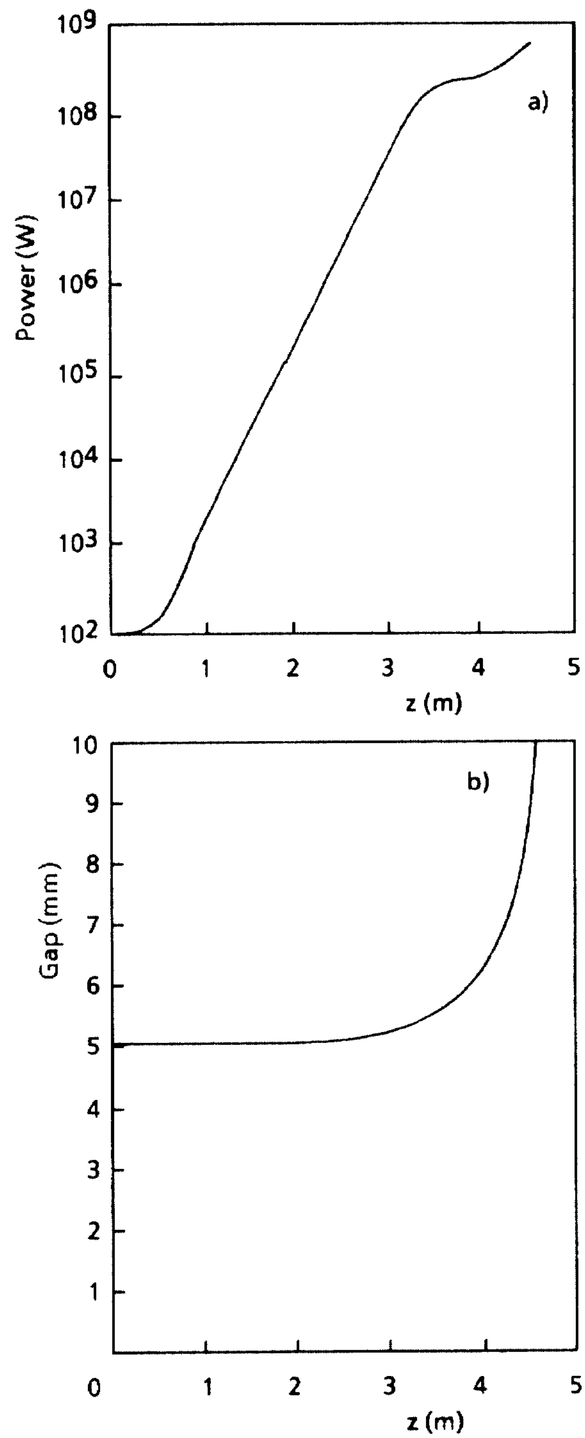
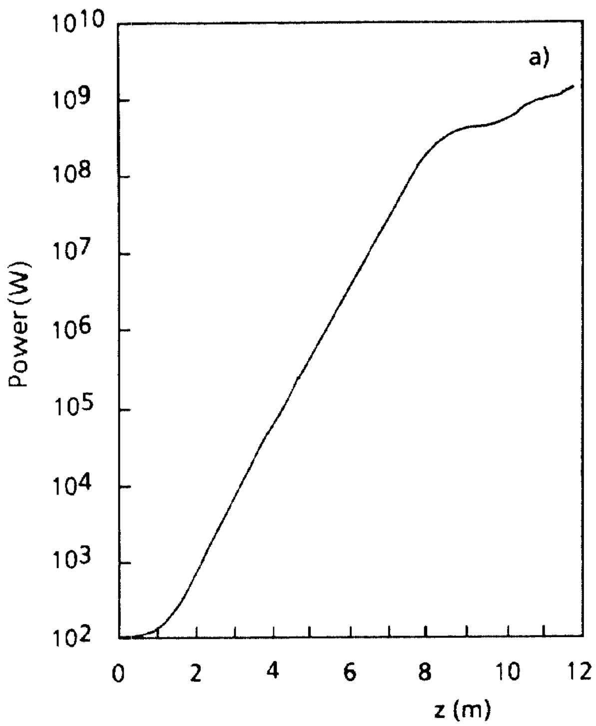


Figure 1. a) Case 1 - Evolution of the radiation power along the undulator. b) Case 1 - Waveguide gap profile along the undulator.



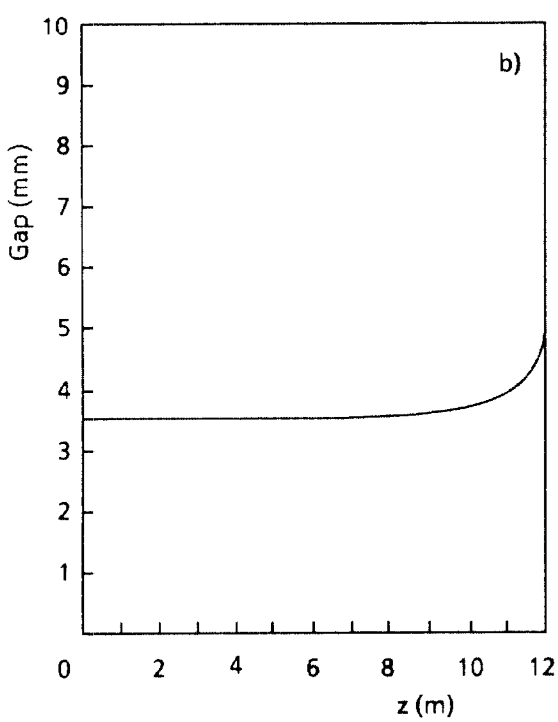


Figure 2. a) Case 2 - Evolution of the radiation power along the undulator. b) Case 2 - Waveguide gap profile along the undulator

The analogy with the *classical* undulator tapering is evident: the waveguide gap tapering technique increases the amount of energy which can be extracted from the electron beam. The schematization of the radiation field by the TE_{01} mode alone is a rather poor approximation when the waveguide gap changes rapidly [3]. To maintain the validity of approximation in the worse case (case 1) we can stop the gap tapering at a level of 30% of variation.

It is worth noticing that the proper choice of the cut-off frequency affects the energy extraction efficiency. In case 1 an efficiency $\eta=12\%$ can be reached, while in case 2 we can obtain an efficiency $\eta\sim40\%$.

3. CONCLUSIONS

The 3-D simulation code we have developed shows the possibility of obtaining very high efficiency, keeping in mind the main code limitation: schematization of the radiation field by the TE_{01} mode alone is a rather poor approximation when the waveguide gap changes rapidly [3], so the very high efficiency found can be a little bit optimistic.

4. REFERENCES

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