Longitudinal Splitting of Bunches in a Cyclotron by Superposition of Different RF Harmonics

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Abstract

Cyclotron accelerating cavities with a radial voltage gradient introduce phase dependent magnetic forces that alter the longitudinal phase space distribution of the circulating beam. The addition of a higher harmonic cavity with sufficient amplitude and gradient, and phased to oppose the fundamental rf field could split each bunch longitudinally into two without loss of intensity. Additional cavities at still higher harmonics could further split the bunches leading to a time structure at extraction compatible with a much higher frequency than the fundamental accelerating field. Such an extracted beam could be used for injection into a superconducting cw linac, operating at several hundred MHz, thus preserving the 100% duty cycle of the cyclotron beam up to the final energy of the linac. The TRIUMF 500 MeV cyclotron and the PSI 590 MeV cyclotron are used as models to explore such an application. A beam experiment at TRIUMF verifying the phase-splitting principle is summarized.

I. INTRODUCTION

Cyclotrons typically operate in cw mode, with rf frequencies in the 10-50 MHz range and with beam bunches occupying up to 40° of each rf cycle. For certain applications, such as to match a cyclotron to a cw superconducting linac energy-booster, it may be desirable to increase the bunch frequency and decrease the bunch length. At linac operating frequencies of 200 MHz and above, a typical micro-pulse from a cyclotron would be much too large. For example a beam with 20° phase width at 25 MHz would occupy 160° of a 200 MHz linac bucket. In this paper we will show how, in principle, it would be possible to raise the frequency of the cyclotron bunches by a factor of 6-8 while simultaneously compressing the bunches longitudinally to a phase width compatible with linac acceleration.

II. PHASE COMPRESSION/EXPANSION

The phase compression/expansion effect in cyclotrons was introduced in 1970 [1] and summarized in 1974 [2]. A radial gradient in the accelerating field from an rf cavity gives rise to a time-varying magnetic field 90° out of phase with the accelerating field. Positive radial gradients focus or compress off-phase particles longitudinally, while negative gradients defocus or expand the bunch length. The amount of phase change per turn is given by

$$d\phi/dn = \phi_{ni}(E) - \frac{dE_{G1}}{dE}\cdot \sin\,\phi - \sum_m \frac{1}{m}\cdot\frac{dE_{Gm}}{dE}\cdot \sin\, m\cdot(\phi-\phi_m)$$

where E_{G1} is the peak energy gain per turn from the fundamental, E_{Gm} is the peak energy gain per turn from a cavity operating at the m^{th} harmonic, ϕ_m is the phase of the m^{th} cavity with respect to the fundamental and ϕ_{ni} is the phase slip per turn due to non-isochronism.

A. *Phase Expansion - an example*

Consider the case where an extra cavity of harmonic m is added at an outer radius and phased to oppose the fundamental (i.e. $m\phi_m = 180°$). For the perfectly isochronous case (i.e. $\phi_{ni} = 0$) the Hamiltonian is given by

$$H = E_{G1}(R)\cdot \sin\,\phi - \frac{E_{Gm}(R)}{m}\cdot \sin\, m\phi$$

with $d\phi/dn = -\partial H/\partial E$ and $dE/dn = \partial H/\partial\phi$. Since the Hamiltonian does not depend explicitly on turn number it is a constant of motion and at any one energy a particle's phase, ϕ_f, can be determined from the initial values of the peak energy gain per turn, $E_{G1}(R_o)$, and phase, ϕ_o, by solving

$$E_{G1}(R_o)\sin\,\phi_o = E_{G1}(R_f)\sin\,\phi_f - \frac{E_{Gm}(R_f)}{m}\cdot \sin\, m\phi_f.$$

When $E_{Gm}(R_f)/E_{G1}(R_f)$ is less than unity the results are straightforward; the added cavity by decreasing the overall energy gain per turn, expands the accelerated phase band.

B. *Phase-splitting*

For cases where the peak energy gain per turn from the cavity exceeds that from the fundamental an unstable fixed point occurs in phase space at the energy at which $E_{Gm}(E) = E_{G1}(E)$ and phase $\phi = 0$. Particles slightly off-phase are funneled past the fixed point skirting around a forbidden region defined by

$$\frac{m\cdot \sin\,\phi}{\sin\, m\phi} = \frac{E_{Gm}(E)}{E_{G1}(E)}.$$

The final phases for particles of $\phi_o = 0^+$ and $\phi_o = 10°$ are plotted for various harmonics and for various energy gain ratios in Fig. 1. The $\phi_o = 10°$ results assume $E_{G1}(R_o) = E_{G1}(R_f)$.

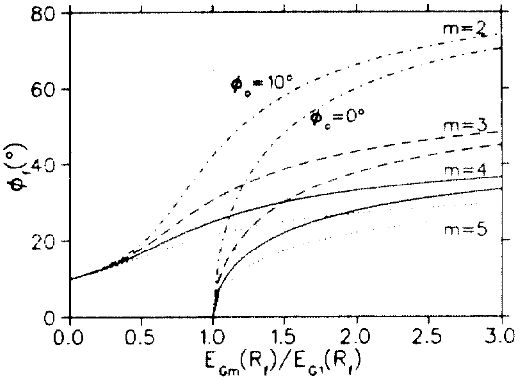

Figure 1: Final phase values for particles of initial phase $\phi_o = 0^+, 10°$ resulting from the addition of a higher harmonic cavity (m = 2, 3, 4, or 5) with peak energy gain E_{Gm} and phase opposing the fundamental. The $\phi_o = 10°$ results assume $E_{G1}(R_o) = E_{G1}(R_f)$.

Note that the separation of the positive bunch from the negative bunch increases, and the bunch width decreases for an increasing cavity voltage. Also as the cavity harmonic increases the bunch separation decreases.

C. *Multiple Cavities*

Once the phase band is split a new cavity of higher harmonic can be used to produce two other unstable fixed points, symmetric about $\phi = 0$, to further split the two bunches. For instance, two sub-bands separated by ~90° can be further split by a 4^{th} harmonic cavity phased to accelerate the 0° particle resulting in a total of four bunches for every initial bunch. With suitable choice of cavity voltage and harmonic it could be possible to achieve an output beam compatible with an accelerating frequency 4, 6, or 8 times that of the fundamental.

III. SIMULATIONS

A. *TRIUMF Model*

The TRIUMF 500 MeV H^- cyclotron accelerates five particle bunches per turn at an rf frequency of 23 MHz. For the present investigation a simple-impulse approximation code was developed to simulate the position of particles in longitudinal phase space. A perfectly isochronous, magnetic field was assumed. The dee voltage was chosen to be radially uniform at 80 kV giving a peak energy gain per turn, E_{G1}, of 320 keV. An initial study, with an additional 2^{nd} harmonic $\lambda/4$ cavity, confirmed the predictions of Fig. 1. The results, Fig. 2, show that the cavity, with a peak energy gain 1.76 times that provided by the fundamental and reverse phase, splits the ±10° phase band into two ~ 7° bands separated by 120°. The compressed bunches would be compatible with injection into a linac of frequency six times that of the cyclotron ($h_L = 6$), with two out of six buckets filled; the particles occupying ~40° of the linac bucket. (Also possible is a linac frequency three times the cyclotron frequency with two out of three linac buckets filled.)

In a similar way, other single cavities can be used to achieve different phase band separations compatible with other linac frequencies. From Fig. 1 a 2^{nd} or 3^{rd} harmonic cavity can give a separation of 90° and a 5^{th} harmonic cavity a separation of 45° both compatible with $h_L = 8$; while a 3^{rd} or 4^{th} harmonic cavity can be used to achieve a 60° separation compatible with $h_L = 6$. In all cases two bunches out of the eight or six buckets available during each cyclotron period would be filled.

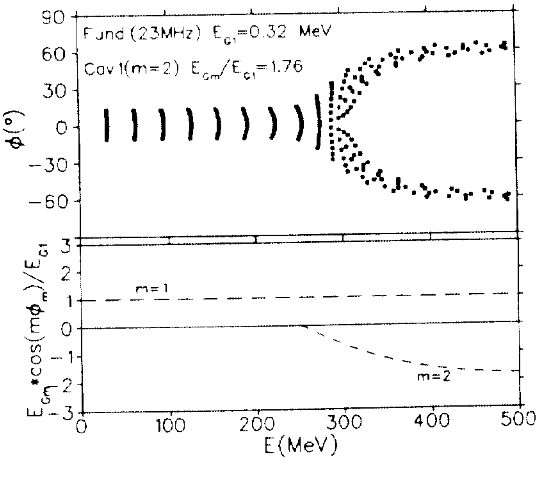

Figure 2: The bottom figure shows the effective energy gain per turn, normalized to the energy gain of the fundamental, for the fundamental, and an additional 2^{nd} harmonic $\lambda/4$ cavity as a function of energy. In the top figure particle trajectories in (E, ϕ) space, plotted every one hundred turns, show how the initial phase band of ±10° is split into two ~ 7° bands separated by 120°.

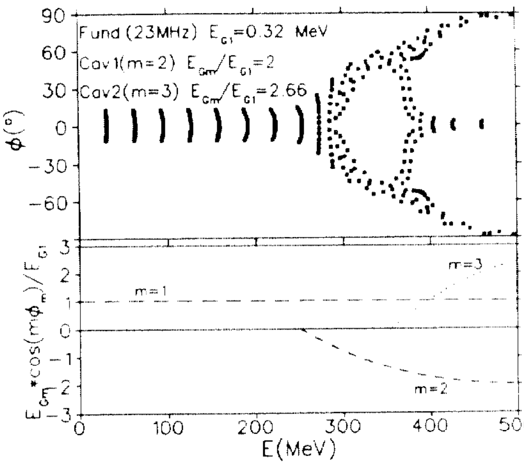

Figure 3: Similar to Fig. 2 only a positively phased 3^{rd} harmonic $\lambda/4$ cavity is added to split the initial phase band of ±10° into three ~ 4° bands separated by 90°.

A second cavity of proper strength and harmonic can be added to split the two sub-bands into four with the two new bands closest to the central phase forced together to eventually join yielding three bunches. In Fig. 3 we show an example where the output bunches are separated by 90° compatible with $h_L = 4$ or $h_L = 8$.

Similarly four phase bands separated by 60° that would fill four linac buckets out of six for each cyclotron pulse are produced by adding a third 6^{th} harmonic cavity as shown in Fig. 4. Finally, four phase bands separated by 45° that would fill four linac buckets out of eight for each cyclotron pulse are produced by adding a third 8^{th} harmonic cavity. In this case the resultant

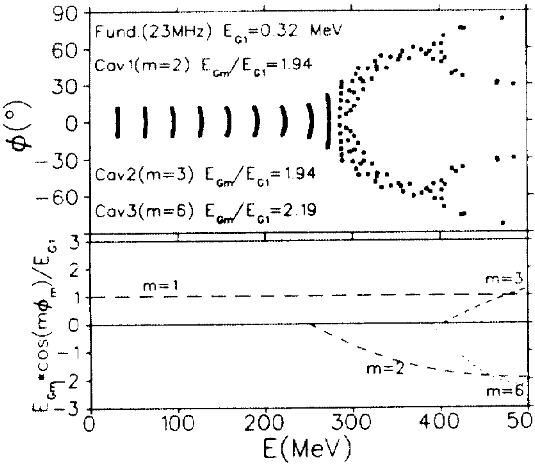

Figure 4: Similar to Fig. 3 except a 6^{th} harmonic $\lambda/4$ cavity phased negatively is added and the initial phase band is split into four ~ 2° bands separated by 60°.

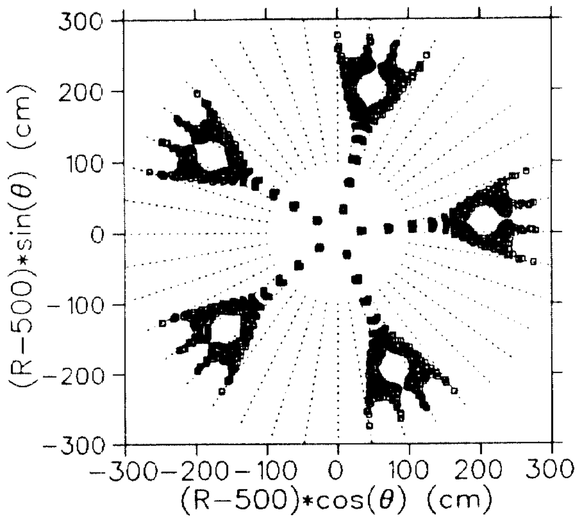

Figure 5: Shown are the five particle bunches per turn in TRIUMF, starting from 500 cm (E =140 MeV), and splitting into four bunches per spoke with the influence of the higher harmonic cavities; a 2^{nd}, 3^{rd}, and 8^{th}. Superimposed on the plot are the linac buckets, eight times the cyclotron frequency.

particle trajectories are plotted in (R,θ) space in Fig. 5. Shown are the five particle bunches per turn in TRIUMF, starting from 500 cm (E = 140 MeV), and splitting into four bunches per spoke with the influence of the higher harmonic cavities. Superimposed on the plot are the linac buckets, eight times the cyclotron frequency. The results of the simulation studies using the TRIUMF cyclotron as a model are summarized in Table 1. Both h_L = 6 and h_L = 8 cases are documented. In all cases the cavity radial positions are altered to achieve the optimum result. In an actual application the extracted population densities in phase space would depend on the details of isochronism and on the stability of the rf and magnetic fields.

B. PSI Model

The PSI main cyclotron accelerates protons from 72 MeV to 590 MeV at an rf frequency of 50 MHz and an energy gain of 2 MeV/turn. Alterations similar to those proposed above would yield bunches compatible with linac frequencies twice those found with the TRIUMF model but demanding cavity voltages a factor of six higher! This, coupled with a shortage of available space for the new cavities leads one to suggest a new ring, from 590 MeV to say 1500 MeV, similar to ASTOR [3], where higher harmonic cavities could be added to achieve the required phase splitting, followed by a cw linac to take the beam to several GeV. Since the turn structure is smeared out at extraction, the current is limited to something like 50 μA.

Table 1: Specifications for TRIUMF Phase Splitting Cavities

			Cavity 1		Cavity 2	
h_l	Buckets Filled	$\phi_{extracted}$	$\frac{E_{Gm}}{E_{G1}}$ [a]	m	$\frac{E_{Gm}}{E_{G1}}$	m
6	2/6	(±30°)	1.41*	3	-	-
	2/6	(±60°)	1.76*	2	-	-
	2/6	(±90°)	2.19*	2	2.69	3
	3/6	(0,±60°)	1.25*	2	1.13	5
	4/6	(±30°,±90°)	1.94*	2	1.94	3 [b]
8	2/8	(±22.5°)	1.38*	5	-	-
	2/8	(±45°)	1.25*	2	-	-
	2/8	(±67.5°)	2.13*	2	-	-
	2/8	(±90°)	2.19*	2	2.69	3
	3/8	(0,±45°)	1.50*	3	1.31	6
	3/8	(0,±90°)	2.0*	2	2.66	3
	4/8	(±22.5°,±67.5°)	1.69*	2	1.25	3 [c]

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[a] A * denotes a reverse phase cavity

[b] A third cavity $\frac{E_{Gm}\cos(m\phi_m)}{E_{G1}} = -2.19$, $m = 6$ is used.

[c] A third cavity $\frac{E_{Gm}\cos(m\phi_m)}{E_{G1}} = -1.90$, $m = 8$ is used.

IV. TEST RESULTS

A fourth harmonic $\lambda/4$ cavity [4] installed in the TRIUMF cyclotron to augment the fundamental energy gain per turn, and hence reduce losses, from 350 MeV to 500 MeV, was used to test the phase-splitting principle. The cavity was phased to oppose the circulating beam and powered to its maximum giving a ratio of energy gain at extraction of $E_{G4}(R_f)/E_{G1}(R_f) = 0.88$. To produce the phase-splitting effect the isochronism, over the radial range of the added cavity, was shifted from the ideal, reducing the effective energy gain from the fundamental. The time spectrum of the resulting extracted beam compared to the time structure with the cavity off is shown in Fig. 6. Complete phase-splitting is evident with the two sub-bunches separated by 7.6 ns.

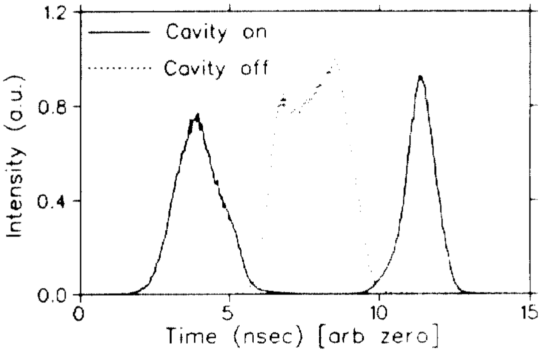

Figure 6: Experimental result showing the time spectrum of beam extracted from the TRIUMF cyclotron with (solid line) and without (dotted line) an additional 4^{th} harmonic cavity ($E_{G4}(R_f)/E_{G1}(R_f) = 0.88$) phased to oppose the circulating beam.

V. ACKNOWLEDGEMENTS

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VI. REFERENCES

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