Theoretical Investigation of Traveling Wave RF Gun

J. Gao
Laboratoire de L'Accélérateur Linéaire
Centre d'Orsay, 91405 Orsay cedex, France

Abstract
In this paper a traveling wave type rf gun (TW gun) is investigated theoretically. Analytical formulae concerning energy gain, energy spread, and transverse emittance are derived. Some numerical results are calculated to demonstrate further the behaviours of the TW gun and to compare with those from analytical formulae.

1 INTRODUCTION
As a potential high brightness electron injector for FEL and future $e^+$, $e^-$ linear colliders, rf gun have been extensively investigated theoretically and experimentally both for thermionic cathode [1][2] and photo-cathode [3][4][5]. Because of the historical reasons, almost all the experimental and theoretical works are confined within standing wave rf guns (SW guns). However, as pointed out by J. Le Duff [6], TW gun might be very interesting also to rf gun technology. In this paper TW gun will be theoretically investigated in a way different from that in ref. [7].

Before going on to the next section, it is assumed that the radius of the cathode is small, and the nonlinear force effects both from the space charge and the rf fields are neglected.

2 TW GUN THEORY
Quite different from a SW gun which is similar to a conventional DC gun except mainly that the accelerating electric field is oscillating with time, a TW gun is a little bit difficult to imagine at the first glance since there are almost no evident relations between a TW gun and a conventional DC gun. However, if one shifts his point of view from DC gun to a linear traveling wave accelerator, the confidence about a TW gun could be established without much difficulty. Now let's start with the "capture condition" of a linac [8] which says: if

$$E_x = E_0 \sin(\phi)$$

then

$$\cos(\phi_0) - \cos(\phi_f) = \frac{2 \pi m_0 c^2}{\lambda_g e E_0} \left( \frac{1 - \beta_{z0}}{1 + \beta_{z0}} \right)^{1/2}$$

where $\phi_0$ is the phase when an electron is injected, $\phi_f$ is the asymptotic phase when the electron's velocity reaches almost light velocity $c$, $\lambda_g$ is the wavelength in the accelerating structure, $E_0$ is the peak accelerating electric field strength, $\beta_{z0}$ corresponds to the initial injection electron's velocity, and $m_0 c^2$ is the electron's rest energy. If we assume that a cathode is just put at the end plate of the coupler cavity of a linac with $\beta_{z0} = 0$, and require that $\phi_f = \pi/2$, one can determine the necessary electric field strength $E_0$ with respect to the injection phase $\phi_0$

$$E_0 = \frac{2 \pi m_0 c^2}{\lambda_g \cos(\phi_0)} \left( \frac{1 - \beta_{z0}}{1 + \beta_{z0}} \right)^{1/2}$$

For example, if $\lambda_g = 10 cm$, $\beta_{z0} = 0$ and $\phi_0 = \pi/4$, from eq. 3 one gets $E_0 = 45MV/m$. It is obvious that $E_0 = 45MV/m$ is a practically possible value. It is now very easy to imagine that when a short laser pulse illuminates a photocathode at an initial rf phase $\phi_0 \leq \pi/4$ with $E_0 = 45MV/m$, the electron bunch will be accelerated continuously with its final asymptotic phase frozen at $\phi_f \leq \pi/2$. In the following section we will investigate in much more detail on the beam longitudinal and transverse motions.

2.1 Longitudinal Motion Due to Rf Fields
The longitudinal electric field inside a cylindrical symmetric TW structure is expressed as follows:

$$E_x(r, z, t) = E_x(r, z) \sin(\omega t - \beta_g z + \phi_f)$$

where $\omega = 2 \pi f$, $\beta_g$ is the fundamental wave number of this slow wave TW structure and $\phi_0$ is the emission phase of the center of the electron bunch. Since only linear term is kept, the electric field near the axis can be expressed as:

$$E_x(r, z, t) = E_x(0, z) \sin(\omega t - \beta_g z + \phi_0)$$

In the following analytical treatment, $E_x(0, z)$ has been chosen as a constant. From eq. 5 we have

$$\frac{d\gamma}{dz} = \frac{q E_x(0, z)}{m_0 c^2} \sin(\phi)$$

where

$$\phi = \omega t - \beta_g z + \phi_0 = k \int_0^z \left( \frac{\gamma}{(\gamma^2 - 1)^{1/2}} - 1 \right) dz + \phi_0$$

$k = 2 \pi / \lambda$ and $\lambda$ is the electromagnetic wavelength in free space. $\beta_g$ has been chosen equal to $k$ (that is to say the phase velocity of this traveling wave equals to the speed of light). $\gamma$ is the ratio between electron's relativistic energy and the rest energy $m_0 c^2$. As the first order approximation of $\gamma$, eq. 6 can be integrated as follows:

$$\Gamma = 1 + \alpha \sin(\phi_0 + \delta \phi) k z$$

Note: The content is from a scientific paper on the theoretical investigation of a traveling wave RF gun, which includes the abstract, introduction, and some theoretical derivations. The paper is focused on the analytical and numerical results of the TW gun's behavior and is likely to be of interest to researchers in the field of accelerator physics.
\[ \alpha = \frac{qE_z(0,0)}{m_0c^2k} \]  

where \( E_z(0,0) \) is the peak electric field on the cathode surface. \( \Gamma \) is an approximate expression of \( \gamma \). In eq. 8 a very important parameter \( \delta \phi \) is introduced. The physical reason of there existing this parameter is due to electron's low initial longitudinal velocity and the rapid increasing of \( \phi \) in the cathode region. The introduction of \( \delta \phi \) is to improve the accuracy of this first order approximation. It should be pointed out that for the theory of SW gun, this parameter should be introduced also. By using eq. 8, eq. 7 can be integrated as follows:

\[ \psi = \frac{1}{\alpha \sin(\phi_0 + \delta \phi)} \left( \left( \frac{\Gamma^2 - 1}{2} \right)^{1/2} (\Gamma - 1) \right) + \phi_0 \]  

If \( \Gamma > 1 \) then \( \psi \) will be frozen at its asymptotic value

\[ \psi_f = \frac{1}{\alpha \sin(\phi_0 + \delta \phi)} + \phi_0 \]  

where \( \delta \phi \) can be calculated from an empirical formula which is given as follows:

\[ \delta \phi (\text{degree}) = 19E_z(0,0)^{-0.9} \]  

where \( E_z(0,0) \) is in MV/cm. It is obvious that \( \delta \phi \approx 0 \) corresponds to \( E_z(0,0) \) approaching infinity. From eq. 8 and eq. 11 one can get the second order approximation of the final energy gain of a TW gun

\[ \Gamma_f = 1 + \alpha \sin(\phi_f)kL \]  

where \( L \) is the length of the TW gun structure. For an electron bunch emitted from a cathode with the emission phase of the bunch center being \( \phi_0 \), and the bunch length being \( \Delta \phi_0 \), the energy spread of this electron bunch can be calculated readily from eq. 13

\[ \Delta \Gamma_f = \alpha kL \cos(\phi_f)(1 - \frac{\cos(\phi_0 + \delta \phi)}{\alpha \sin(\phi_0 + \delta \phi)^2}) \Delta \phi_0 \]  

If \( \phi_f = 90^\circ \)

\[ \Delta \Gamma_f = \frac{1}{2} \alpha kL(1 - \frac{\cos(\phi_0 + \delta \phi)}{\alpha \sin(\phi_0 + \delta \phi)^2})^2 (\Delta \phi_0)^2 \]  

where \( \phi_0^* \) corresponds to \( \phi_f = 90^\circ \). From eq. 11 the asymptotic bunch length can be calculated

\[ \Delta \phi_f - \Delta \phi_0 = \frac{\cos(\phi_0 + \delta \phi)}{\alpha \sin(\phi_0 + \delta \phi)^2} \Delta \phi_0 \]  

where \( \Delta \phi_0, \Delta \phi_f \) are the initial and asymptotic bunch length respectively.

2.2 Transverse Motion Due to Rf fields

From \( \nabla \cdot E = 0 \) and \( E_x = 0 \)

\[ \frac{1}{r} \frac{\partial (rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 0 \]  

From eq. 18 one can get the transverse momentum gain due to rf field

\[ P_{rf} = \int_{r_0}^{r} F_r dt \]  

where \( r' = \frac{dr}{dt}, P_r \) is normalized transverse momentum. If we assume that at the cathode surface \( F_{r0} = 0 \), during the acceleration \( r \) keeps constant value, and at the exit of the TW gun \( \gamma \gg 1, \beta_z \approx 1 \) and \( E_z(0, z) \) from constant value \( E_z(0,0) \) in any manner drops to \( E(0, L) = 0 \), by using eqs. 13 and 22 we get

\[ P_{rf} = \frac{\alpha k L}{2} \left( \sin(\phi_f) + \frac{1}{\alpha} \cot(\phi_f) \right) \]  

Since electrons emitted from cathode around emission phase \( \phi_0 \) have an initial phase spread \( \Delta \phi_0 \), at the exit of the TW gun there is also a transverse momentume spread \( \Delta P_{rf} \) which can be derived from eq. 25

\[ \Delta P_{rf} = \frac{\partial P_{rf}}{\partial \phi_0} \Delta \phi_0 + \frac{1}{2} \frac{\partial^2 P_{rf}}{\partial \phi_0^2} (\Delta \phi_0)^2 \]  

\( \Delta P_{rf} \) is the source of the electron bunch's transverse emittance produced by the linear rf field. From eq. 25 one can derive the analytical formula for the transverse r.m.s. emittance

\[ \epsilon_r (\pi m \cdot \text{rad}) = 4(<p_r^2> - <r^2> - <r^2>)^{1/2} \]  

where \( r \) denotes \( x \) or \( y \) components. Since this analytical formula is complicated, it is omitted here. On the contrary we use the definition that the transverse emittance \( \epsilon_r \) is the
area occupied by the electrons in the phase space divided by π, we have
\[ \frac{\partial p_{rf}}{\partial \phi_0} \neq 0 \] (28)
\[ \xi_{\phi}^f = \frac{a r_c^2}{4\pi} \frac{1}{\alpha \sin^2(\phi_f)} \left( 1 - \frac{\cos(\phi_0 + \delta \phi)}{\alpha \sin^2(\phi_0 + \delta \phi)} \right) (\Delta \phi_0) \] (29)
\[ \xi_{\phi}^f = \frac{a r_c^2}{4\pi} \frac{\sin(\phi_f) - 2 \cos(\phi_f)}{\alpha \sin^2(\phi_f)} \left( 1 - \frac{\cos(\phi_0 + \delta \phi)}{\alpha \sin^2(\phi_0 + \delta \phi)} \right)^2 (\Delta \phi_0)^2 \] (31)
where \( r_c \) is cathode radius.

3 DISCUSSIONS

The main advantage of TW gun is that it is suitable for providing high electron beam current. Limited by the length of this paper the space charge effects and the comparisons between TW gun and SW gun have been omitted [9]. Figs. (1-4) from theoretical formulae and numerical calculation are used to show the behaviour of a TW gun.

As for the practical TW structure design it has been suggested that a backward wave TW gun structure as shown in Fig. 5 could be used in order to avoid the problem of the unsymmetric field perturbation caused by the waveguide-TW structure coupling aperture in the case of a forward traveling wave structure [10].

4 ACKNOWLEDGEMENTS

The author would like to show his thanks to J. Le Duff for his constant interests in TW gun research and the precious encouragements and discussions. He thanks also B. Mouton for the help of using EPAC LaTeX.

5 REFERENCES