Comparison of Theory and Experiment on Beam Impedances: The Case of LEP

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Abstract

The performance of LEP is presently limited by collective effects which restrict the amount of current which can be stored in a single bunch. These effects are due to the interaction of the particles in the beams with wake fields induced in the vacuum chamber. They can be calculated in the frequency domain if the impedances are known. Predictions and measurements of the impedances in LEP are discussed, as well as effects which bear on the machine performance.

1 INTRODUCTION

LEP is now in operation for over 2 years, and a number of machine development (MD) shifts have been spent to study the collective beam behaviour. In addition, many parasitic observations were made during physics runs in order to obtain the required data. Comparison with predicted behaviour[1] is generally quite good.

2 TRANSVERSE MODE COUPLING

TMC has been observed in most large electron storage rings: PETRA[2], PEP[3], TRISTAN[4]. It is driven by the transverse impedance which is proportional to the machine radius, and inversely to the cube of the chamber radius (the larger the machine, the smaller the chamber needs to be to keep magnet cost down). TMC was thus expected to be a dominant current limitation in LEP, by far the biggest electron storage ring ever built.

*with contributions by D. Brandt, K. Cornelis, J. Hagel, A. Hofmann, R. Pitthan, L. Rivkin (PSI), E. Rossa and many EIC's.

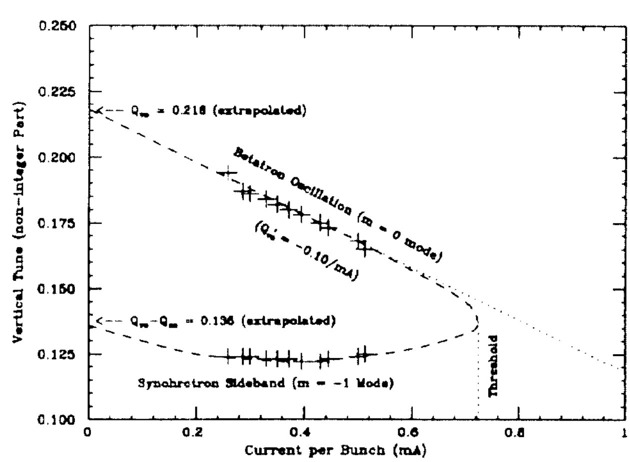

Figure 1: Vertical Tune Shift versus Current

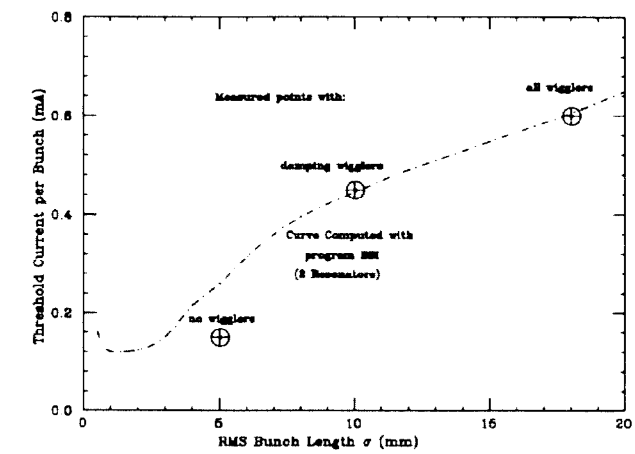

Figure 2: TMC Threshold versus Bunch Length

For short bunches in LEP, coupling of modes $m = 0$ and $m = -1$ has the lowest threshold. It is expected to occur first in the vertical plane, due to the larger vertical impedance of flat vacuum chambers (Fig.1). However, the threshold for horizontal TMC is only slightly higher[5].

For a transverse resonator with resonant frequency f_r and impedance $Z_\perp = R/Q$, the threshold current is given approximately by

$$I_{\rm thr} \approx \frac{2\pi f_s E/e}{\langle\beta\rangle f_r Z_\perp} F(\sigma_s) \approx \frac{8 f_s E/e}{\sum \langle\beta\rangle_i k_{\perp i}(\sigma_s)} \qquad (1)$$

The form factor F increases with bunch length σ_s, but is also large for extremely short bunches. If several impedances play a role, the second expression[6] with the transverse loss factor $k_\perp$ is more convenient. More reliable answers come from the computer program BBI[7] - which includes the TMC code MOSES[8] as subroutine. Fig.2 shows the TMC thresholds computed with BBI as well as three measured points with the 90-degree lattice.

The current thresholds are strongly tune dependent, which might be explained by synchro-betatron resonances (SBR) due to wake fields induced in the cavities[9]. They are proportional to energy (E), with a minimum at injection (20 GeV), and to the synchrotron frequency (f_s), which is limited at injection by the available RF voltage if f_s is kept constant during ramping to high energy. f_s

could be higher if SBR's can be crossed without current loss which still needs to be demonstrated.

In the design of LEP the average beta function $\langle\beta\rangle$ was minimized at the RF cavities by optimizing the phase advance per cell in the RF sections to over 100°[10]. The transverse impedance of the bellows between elliptic chambers was reduced by careful shielding with sliding contacts. In general, the impedance of all elements visible to the beam was kept as low as possible.

3 TRANSVERSE BROAD-BAND IMPEDANCE

The transverse "broad-band resonator" parameters can be estimated from the dependence of the transverse loss factor on bunch length. This can be computed for axisymmetric structures with computer programs like TBCI[11] or ABCI[12]. The results obtained for LEP were[13]:

$R/Q = 1.51\,\mathrm{M\Omega/m}, f_r = 2\,\mathrm{GHz}$ (640 copper cavity cells)
$R/Q = 0.32\,\mathrm{M\Omega/m}, f_r > 8\,\mathrm{GHz}$ (2800 shielded bellows)
others (separators, pick-ups, large bellows etc) 5-10 %

The sum of all impedances can be found experimentally by measuring the tune shifts with current:

$$\frac{\Delta Q}{\Delta I} = \frac{R}{2\pi\sigma_s E}\sum\langle\beta\rangle_i Z^{(i)}_{\perp\mathrm{eff}} \qquad (2)$$

The "effective impedances", defined as overlap integrals of the impedance functions with the mode spectra, depend strongly on bunch length. For a Gaussian distribution one finds $Z_{\perp\mathrm{eff}} = 2\,(\omega_r\sigma_s/c)^2\,(R/Q)$ for short bunches ($\omega_r\sigma_s/c << 1$) and $Z_{\perp\mathrm{eff}} = R/Q$ for longer ones.

The impedances of the elements with circular beam pipes ("cavities") can be separated from those with elliptic ones ("bellows") with aspect ratio $r \approx 2$[5]. If one takes the computed resonant frequencies and assumes an "impedance ratio" $\kappa = Z_{\perp V}/Z_{\perp H} \sim r^{2-3} \approx 5$, one finds from the measured "slopes" (depending on bunch length) $(\frac{\Delta Q}{\Delta I})_H \approx 60 - 70A^{-1}$, $(\frac{\Delta Q}{\Delta I})_V \approx 100 - 130A^{-1}$:
$R/Q \approx 1.4\,\mathrm{M\Omega/m}$ (cavities with $f_r = 2\,\mathrm{GHz}$)
$R/Q \approx 0.6\,\mathrm{M\Omega/m}$ (bellows with $f_r = 12\,\mathrm{GHz}$).

However, estimating the impedance ratio from expressions for tune shifts in elliptic pipes, and from results of the 3-D program MAFIA[14], one finds only $\kappa \sim r \approx 2$. This created a problem: with a slope ratio also about 2, nothing would be left for cavities! A possible explanation - not yet verified - is a lower $f_{r,H}$ for elliptic chambers which increases the "effective impedance ratio" to 4-5.

4 HEAD-TAIL MODES

Definitions: Chromaticity $\xi = (\mathrm{dQ/Q})/(\mathrm{dp/p}) = Q'/Q$, chromatic frequency $\omega_\xi = Q'\omega_o/\alpha << \omega_r$ in LEP, chromatic phase shift $\chi = \omega_\xi\sigma << 1$ even for the LEP 90-degree lattice with $\alpha = 2.10^{-4}$.

It is well known that the $m = 0$ mode is unstable for negative chromaticity, hence natural chromaticity needs to be compensated. However, the higher head-tail modes

Figure 3: Head-Tail Oscillation on Streak Camera

with $m \geq 1$ can become unstable for positive $\xi > \xi_{\mathrm{thr}}$, although this was not seen before in short-bunch electron storage rings. In LEP these modes are clearly visible on a streak camera (Fig.3), and limit the current when their amplitude becomes too large.

The growth rate depends almost linearly on current and - for small ξ - on chromaticity (Fig.4), but in a more complicated manner on bunch length. Stability is lost when it exceeds the damping rate. At injection energy, radiation damping in LEP is rather weak ($\tau = 0.4s$), and Landau damping by tune spread is dominant. A value of $\Delta Q/Q \approx 5.10^{-4}$ is compatible with the growth rate of the $m = 0$ mode at negative chromaticities.

The stable chromaticity range $0 < \xi < \xi_{\mathrm{thr}}$ is thus quite limited, and reduces inversely with current. In order to avoid frequent adjustment during accumulation, it may actually be preferable to work with a slightly negative chromaticity and stabilize with feedback.

The resistive wall impedance becomes quite large at the lowest betatron frequency $f_{\mathrm{min}} = qf_{\mathrm{rev}}$ (already 30 MΩ/m

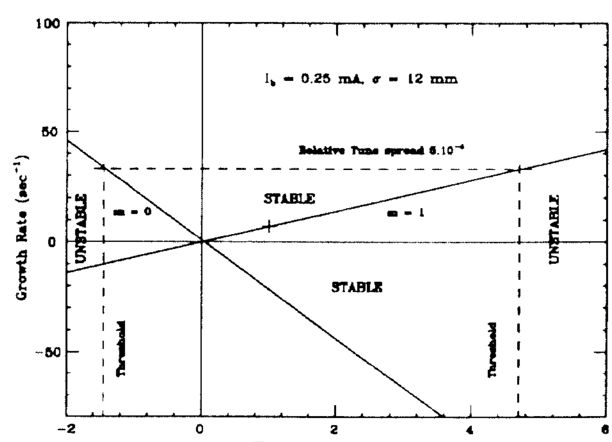

Figure 4: Head-Tail Growth Rate vs. Chromaticity

at $f_{rev} = 11\text{kHz}$), but falls off proportional to $1/\sqrt{f}$. It thus becomes quite negligible in the GHz range, where the spectra of modes with $m \geq 1$ have their peaks for short bunches. However, the spectrum of the dipole ($m = 0$) mode is centered around ω_ξ. For typical values of $Q' \leq 10$, the chromatic shift is still small compared to the width of the spectrum $\Delta\omega \sim c/\sigma$. The (odd) real part of the transverse impedance produces a negligible contribution to the growth rate, while the (even) imaginary part increases only the tune shift.

5 LONGITUDINAL BROAD-BAND IMPEDANCE

Many measurements of bunch length have been performed in LEP. They were limited in accuracy due to lack of instrumentation for measuring very short bunch lengths until a streak camera gave more consistent results.

The threshold current for turbulent bunch lengthening can be used to obtain the longitudinal broad-band impedance. The stability criterion for Gaussian bunches can be written as

$$\left(\frac{Z}{n}\right)_{\text{eff}} \leq \frac{hV_{RF}\cos\phi_s}{\sqrt{2\pi}I_{\text{thr}}}\left(\frac{\sigma_s}{R}\right)^3 F(Z) \qquad (3)$$

where the form factor F=1 for a capacitive impedance. First measurements of bunch length were done with signals from a bandwidth limited pick-up connected to a sampling scope. The observed "raw data" needed a large correction as shown in Fig.5[15]. The "turbulent threshold" is clearly visible near $I_{\text{thr}} = 100\,\mu A$. For a corresponding RMS bunch length of $\sigma_s = 5\,mm$, the (effective) impedance is about 30 $m\Omega$!

The low-frequency impedance $\left|\frac{Z}{n}\right|$ can be calculated from the effective impedance for a given mode spectrum. For a Gaussian distribution $\left(\frac{Z}{n}\right)_{\text{eff}} \approx 2\,(\omega_r\sigma_s/c)^2\left|\frac{Z}{n}\right|$ for short bunches ($\omega_r\sigma_s/c << 1$). Assuming a resonant frequency of 2 GHz (the same as in the transverse plane), one finds a longitudinal impedance of $Z/n = 0.25\Omega$. However, this value is model dependent: for $f_r = 1.4$ GHz one would get $Z/n = 0.5\Omega$. In any case, the longitudinal impedance in LEP is very low compared to earlier machines.

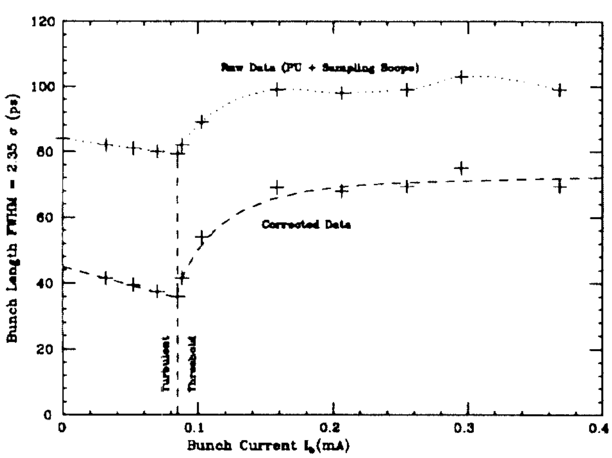

Figure 5: Bunch Length versus Current

The impedance can also be found from the shift of the longitudinal quadrupole mode

$$Im\left(\frac{Z}{n}\right)_{\text{eff}} = -\frac{hV_{RF}\cos\phi_s}{2f_{so}}\left(\frac{\sigma_s}{R}\right)^3 \frac{f_{s2} - 2f_{so}}{\Delta I} \qquad (4)$$

The measured slope 400 Hz/mA yields $\left(\frac{Z}{n}\right)_{\text{eff}} \approx 40m\Omega$ in reasonable agreement with the former result. Similarly, the approximate relation $Z_\perp \approx (2R/b^2)(Z/n)$ yields with $Z_\perp = 2\text{M}\Omega/\text{m}$, about $Z/n \approx 0.4\Omega$.

The tune difference of the $m = \pm 1$ head-tail modes is proportional to the longitudinal impedance[16]. The $m = +1$ mode is only visible horizontally, for bunch currents above $I_b = 200\mu A$ (see Fig.6). The incoherent synchrotron frequency there has increased by 5-10 %, and remains constant indicating that the current is already above the turbulent threshold.

A direct measurement of the small shift of the synchronous phase angle - which would be a model independent method - was unsuccessful so far.

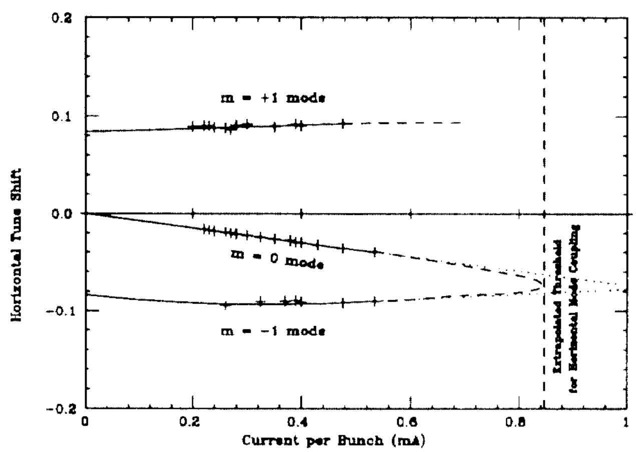

Figure 6: Horizontal Tune Shift versus Current

6 COUPLED BUNCH MODES

Narrow resonances - with long memory - can excite other bunches (or the same bunch on subsequent turns). The bunch spacing is very large in LEP (7 km for 4 bunches!). Nevertheless, if some HOM frequencies in all cells are equal (and at unlucky value), the longitudinal threshold is only about 1/3 of design current (3 mA in 4 bunches).

However, there is a large number of (copper) cavities in LEP (640 cells). The HOM frequencies are distributed due to fabrication tolerances, and an exact overlap is rather unlikely. This can be taken into account by an equivalent reduction of the quality factor to Q_{eff}. Then the total impedance is reduced to $R = N_{\text{cells}}Q_{\text{eff}}(R/Q)_{\text{cell}}$.

Transverse coupled bunch modes have even lower thresholds, with a minimum at 1/10 of the design current. Transverse feedback was therefore installed in LEP from the beginning. Despite this, longitudinal bunch motion was observed during commissioning, and limited the current which could be stored. Hence longitudinal feedback had to be added. In order to act on both beams, we used the fact that the synchrotron frequencies for electrons and positrons were slightly different. This is caused by different phases of the RF voltage envelope due to the oscillation of RF power between accelerating and storage cavities.

However, the sou^rce of the longitudinal motion is still under investigation:

- Coupled bunch oscillations are unlikely, even a single bunch moves under certain conditions
- Magnet power supply ripple is small, and higher frequencies are filtered by chamber walls
- RF noise at synchrotron sidebands is quite low (90 dB below carrier)
- Adding RF noise at these sidebands increases motion
- Reducing the bandwidth, by keeping the beam with the SC cavities alone, did not seem to help.

7 LEP FUTURE: "PRETZELS" AND LEP 200

In the LEP "Pretzel" scheme, the beam intensity - and hence luminosity - will be increased by having more than 4 bunches in each beam. The electrons and positrons will be separated everywhere except at the desired intersections. In a first phase, to be tested next year, 2 × 8 bunches will be stored (the highest number possible is 2 × 36 bunches).

The required additional electrostatic separators lead to larger impedances, but mostly at lower frequencies. The increased total current pushes the beams further above the minimum thresholds, but a new longitudinal feedback system using 1 GHz cavities will be available soon.

For LEP200, some 192 4-cell SC cavities will be added to the existing copper cavities. The SC cavities have large beam ports (24 cm dia), since it is no longer necessary to optimize R/Q for high shunt impedance when Q is very large. Also the smoother shape (elliptic cross section, no nose cones), required to avoid field-emission and/or multipactor, reduces the transverse impedance. The broadband resonator impedance for all 192 SC cavities is only $R/Q = 0.48$ MΩ/m at $f_r = 0.7$ GHz. With SC cavities, the TMC thresholds are not much reduced, and actually increased when copper cavities are removed (see Fig.7).

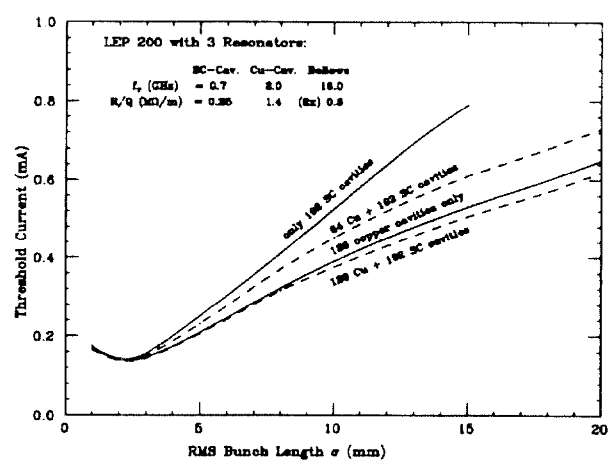

Figure 7: TMC thresholds for LEP200

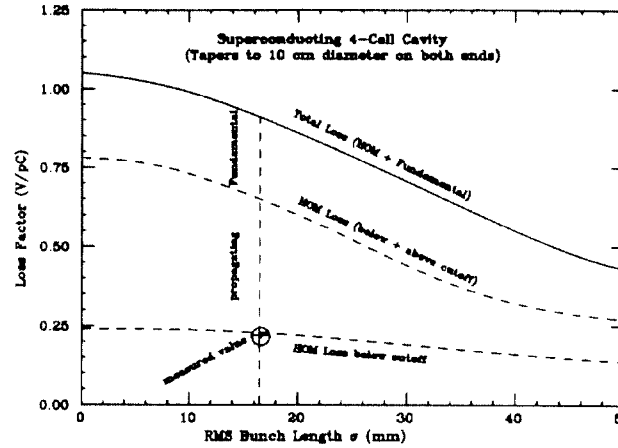

Figure 8: HOM Loss factors in SC Cavities

Narrow resonant impedances of the SC cavities are damped by HOM couplers: for all important resonances the quality factors are expected to be below those of the copper cavities. The only real problem comes from the energy deposited by HOM losses in the SC cavities: they will increase liquid He consumption, and may even damage sensitive elements (couplers, tapers, bellows etc). The HOM losses have been computed and compare quite well with measurement (Fig.8). The limited power handling capability of the HOM couplers made a redesign necessary.

8 REFERENCES

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