Non-Linear Single-Particle Beam Dynamics in Circular Machines

W. Scandale
CERN, CH-1211 Geneva 23, Switzerland

Abstract

Nowadays, the studies of non-linear single-particle beam dynamics are devoted to two main fields: large superconducting hadron colliders, dominated by unintentional imperfections of the magnetic field-shape, and compact low-emittance lepton accelerators, governed by chromatic aberration correctors. The non-linear fields are at the origin of two effects: the betatron tunes change with the amplitude and the momentum of the circulating particles, and, for certain combinations of the horizontal, vertical, and synchrotron tunes, non-linear resonances are excited. These phenomena have a profound influence on the stability of the particle motion, and their destabilizing effect is sizably increased by the unavoidable ripple of power supplies which produces a modulation of the machine parameters. Analytical, numerical and experimental approaches have been used to evaluate and possibly compensate mechanisms which lead to particle losses.

1. INTRODUCTION

The motion of charged particles in circular accelerators is basically governed by the magnetic field of the guiding dipoles and the focusing quadrupoles. Intentional and non intentional non-linear fields are in general also present, the side-effect of which is to induce losses at large amplitude. Sextupoles are used to reduce the chromaticity and octupoles make the tune dependent on the amplitude, which is sometime exploited to improve the current-dependent behavior. In hadron accelerators, the destabilizing action of chromaticity sextupoles is self-compensated to a large extent due to the regularity of the lattice. However, usually, a strongly focused lattice is necessary to reduce the sensitivity to field errors, and this in turn increases the strength of chromaticity sextupoles. In electron machines, the chromatic aberration is even larger, due to the stronger focusing required to reduce the transverse beam size, consequently, sophisticated sextupole corrector schemes are used, including elements in zero dispersion region to partly compensate resonances. In addition, strongly non-linear insertion devices, like wigglers and undulators are often present.

Unintentional multipoles due to unavoidable imperfections of the guiding and focusing fields, especially relevant for hadron machines with superconducting magnets, introduce additional non-linearities, which represent the greatest hazard. However, compromises are to be found between making magnetic fields as uniform as possible and keeping the cost magnets low. This is a difficult achievement for superconducting magnets, whose quality depends on the mechanical tolerances of the coil geometry, rather than on the shape of the poles. Both in the Tevatron and in the Hera magnets, typical deviations from uniformity have been limited to about one part in ten thousand at 2.5 centimetres from the magnet axis. Similar values, properly extrapolated with the inner coil diameter and the superconducting filament size, are expected to be reached in the magnets of the SSC, the LHC, and RHIC. Smaller imperfections are present in the warm magnets of the low-emittance lepton machines.

The single-particle approach provides a sufficiently simple, reliable and coherent model of the real accelerator to investigate performances related to non-linear dynamics. The key issue is to estimate the stability of the motion over the operational cycle of the accelerator. In a linear machine with irrational tunes the motion is stable and regular all around the closed reference orbit near the magnetic axis. The non-linear fields add a tune dependence with the amplitude, which shift tunes to rational values, provoking resonant phenomena accompanied in the phase space by islands of finite area surrounded by thin chaotic layers. The islands and the chaotic layers exist through the entire phase space. However, at small amplitude, trajectories follow invariant surfaces, the KAM tori[1], and remain stable for indefinite time. As amplitude increases, the islands become larger until they overlap. Beyond that point, the chaotic layers become interconnected and the particle motion is no longer bounded. This is the domain in which the non-linear forces are able to provoke particle losses, that sometimes may occur after millions of turns. The border between regular and chaotic motion is called dynamic aperture. This is analytically well defined for $1\frac{1}{2}$ dimensional ($1\frac{1}{2}$-D) systems only[2]. With more degrees of freedom, particles in stochastic layers, even close to the origin, may escape through the entire phase space, due to the so called Arnold diffusion. However, for all practical purposes, the border between mostly regular and mostly chaotic trajectories can be used as the dynamic aperture.

Both analytical and numerical tools are used to estimate the dynamic aperture as a function of various machine parameters. Improvements of the linear lattice and correction schemes are studied to reduce the influence of the non-linear forces, and to specify upper limits for the magnet imperfections. The final validation is in general performed with numerical simulations in which the particle position is tracked element by element around the machine for large numbers of turns.

Simpler dynamical systems, such as the Hénon map[3], are advantageously used to investigate mechanisms of long-term losses.

Machine experiments with existing accelerators, in which non-linear perturbations are deliberately introduced, allow comparisons with predictions of numerical simulations.

In the following sections three subject are reviewed: the tools by which predictions on beam stability are formulated, the applications on accelerator design, and finally the outcome of the experiments performed in various accelerators.

2. TOOLS FOR DYNAMIC APERTURE ESTIMATES

2.1. *Tracking Simulations*

Nowadays, simulations with thin-lens approximation and symplectic integrators are often considered as the master tool for quantitative estimates of particle behavior in circular accelerators with non-linear elements. Several computer codes are available to describe lattice models and compute particle trajectories of given initial conditions. They provide exact solutions of the equation of motion for a dynamical system approximating the accelerator. A sequence of linear transfer matrices interleaved with localized polynomial non-linearities is to be computed. Reliable results are easily obtained, since computer rounding errors can be kept under control[4]. However a vast computing power is required to get reliable estimates of the dynamic aperture as a function of various lattice and beam parameters. A fully realistic description of the accelerator structure is difficult if not impossible. Simplifications are also imposed by limitations in computing power. In large colliders, thin-lens description of guiding and focusing fields is frequently used, and do not imply relevant changes to orbit functions. Ignoring fringing fields, and representing non-linearities with equivalent thin-lenses is also considered acceptable. In small accelerators, empiric fitting with computed or measured fields are frequently used to represent fringing fields and insertion devices, and care is required in approximating the kinematic effect of curvature.

Examples of computer codes with tracking functionality are MAD[5], developed at CERN, SIXTRACK[6], developed at DESY, and TEAPOT[7], developed at SSC. All of them have vectorized versions to make use of modern parallel processors. Codes specifically oriented to simulations in hadron colliders, with imbedded model simplifications, like SSCTRK[8], are used to study unintentional random non-linear perturbations over large number of statistically different lattice samples.

2.2. *Maps*

In linear lattices, particle coordinates can be propagated along the accelerator azimuth by Twiss transfer matrices. The extension of maps to non-linear dynamics, originally motivated by the need to speed-up long-term tracking simulations in hadron colliders, provided a powerful tool to handle dynamical quantities and correlate them to non-linear coefficients. In presence of non-linearities, algebraic operators have to be used, expanded in Lie[9] or Taylor[10] series. The Lie map is always formally symplectic, regardless of the order to which it is extended, however in real applications terms are neglected and symplecticity is lost. The Taylor map, constructed very efficiently with Differential Algebra techniques, is also not symplectic because of the truncation of high orders. There are ways to restore symplecticity, whose physical meaning, however, is not fully understood.

A way is to replace the truncated map with a Normal Form[11], that is an integrable map, represented by a rotation of an angle depending on the amplitude of the orbit. In practice this is performed by a local change of coordinates in the phase space, which brings intricate orbits into circles. However, the normalized map has an optimized order of accuracy. Above it, the approximation is improved at lower amplitude and worsened at higher amplitude. The domain of convergency is limited by resonances of low order that are allowed by the truncated Taylor map. There are ways to handle the first limiting resonance, with Resonant Normal Forms, which are not yet made of practical use.

The Lie representation also results in the composition of linear rotations with symplectic non-linear matrices. The latter can be further factorized in a product of Lie operators applied to homogeneous polynomials of increasing degree. The factorization can be truncated at any stage, and the result is still symplectic. Terms with polynomials of degrees one and two can be handled exactly. Terms with polynomials of higher degrees have to be approximated by truncation, since they lead to infinite number of coefficients. Symplectic conditions can be recovered in an exact manner by the so called kick factorization, which gives the polynomial completion of the truncated terms. The algorithmic implementation of this is concise, but not straightforward, especially for concatenation. Simplified factorizations with reduced number of kicks interleaved with rotations are also available[12].

Conceptually, the Lie and the Normal Form approaches are equivalent and can be transformed into each other.

The one-turn map contains several dynamical quantities that can by extracted by simple analytical formulae, like the tune dependence with the amplitude and the momentum, the distortion functions and the smear, the high-order non-linear invariants, and finally the Fourier harmonic coefficients of the resonance driving terms. Within the same context it is possible to identify the dependence of dynamical quantities on parameters, by which, in many cases, one can suggest techniques to compensate the non-linear perturbations and to optimize the beam stability.

High order transfer maps are also used to estimate the long term dynamic aperture in a faster way than with the usual element by element tracking: however this approach is still controversial. Taylor maps are inherently not symplectic, therefore inappropriate to preserve volume in phase space during tracking. By increasing the order of the map the error can be made arbitrarily small, but the map size grows exponentially and the computing speed decreases accordingly. Alternatively, one can restore symplecticity by high order completion of the map, however it is not clear that this will not affect the estimate of the dynamic aperture. In comparative studies, long-term simulations are performed either with thin-lens models and symplectic integrators, or with Taylor maps obtained by standard tracking codes with Differential Algebra techniques. The former approach is usually assumed to give a reference result, the latter requires less computing power at least if the truncation is not in excess of 12th to 14th order. However, in these conditions residual artefacts due non-symplecticity are still evident in the vicinity of the stability border[13]. Symplectification of the Taylor map different orders. Systematic errors are larger at injection due to persistent currents. Large low-order (3^{rd} to 5^{th}) values provoke a sizeable detuning with the amplitude and the momentum, which can be corrected either locally or, more economically, using a clever cancellation of the detuning terms by means of Sympson rules[28]. In the latter case, Normal Form techniques have been shown to be perfectly suited to establish a robust minimization procedure, beneficial to the long-term stability[29]. Large high-order (7^{th} and 9^{th}) systematic multipoles destabilize off-momentum particles and have to be minimized by design. Random imperfections, which vary from magnet to magnet due to manufacturing tolerances, are the main source of non-linear resonances and distortion functions. Statistical distributions can be easily predicted, but are insufficient for a complete knowledge of the non-linear optics, since resonance strengths depend on the specific sequence of the random errors around the ring rather than on statistical properties. Therefore, criteria for magnet design are to be studied on several non-linear lattices, with different sequences of random multipoles. Additional parameters to be considered are residual closed orbit, linear coupling due to imperfections, and synchrotron motion.

Strategies of magnet sorting have been invented, by which the magnets are installed in such a sequence in the machines as to minimize the combined non-linear effects. For practical and theoretical reasons, the sorting scheme should be as local as possible and must refer to a limited kind of multipoles. Different solutions have been proposed[30]. By introducing a quasi-periodicity of multipoles every two betatronic wavelengths, the harmonic content of non-linearities can be shifted away from harmful frequencies. Alternatively, small groups of magnets, typically ten, are ordered in such a way to minimize a broad band of non-linear driving terms computed to 2nd perturbative order, contributing to resonances up to order 12th.

Beam stability is influenced by linear lattice parameters like tune, residual linear coupling, peak-ß values in the insertion quadrupoles[31]. Residual closed orbit associated to magnet misalignment strongly interferes with beam stability only in lepton low-emittance accelerators[32]. In hadron colliders the dynamic aperture is in general contained in the vacuum pipe, however, sophisticated collimation systems are located close to the stable orbit to protect superconducting magnets from losses. For a safe operation, careful matching of physical aperture and stability border is to be performed[27].

Crude simplifications of the lattice structure itself have dramatic effects on non-linear performances. Cell lattice models with only regular cells and no interaction regions show a regular azimuthal pattern of the orbit functions and in particular of the betatronic phase advance leading to unrealistic enhancement of the particle stability. They are in general used for numerical studies of simple dynamical systems as the Hénon map. A more realistic way to drop the insertions is to replace them with equivalent rotation matrices. Part of the chromatic aberration and some unintentional field errors are disregarded in this way. However, relevant informations can be gained with less computing power and complexity, especially for hadron colliders, dominated, during the injection plateau, by non-linear perturbations in the arcs.

Diffusion with steady state increase of the amplitude has never been detected in numerical simulations, even in presence of external tune modulation, contrary to what it is usually observed with beam-beam interaction. Due to resonance crossing and non-linear coupling, migration of particles in the tune diagram and mixing of horizontal and vertical oscillations are well visible in long-term tracking results. However large increases in amplitude and particle losses are sudden and unpredictable, even if they occur after a large number of turns.

4. EXPERIMENTS

Routine operation of existing accelerators has to deal with non-linear single-particle phenomena, like exciting low-order resonances for controlled slow extraction, or compensation of them to enlarge the dynamic aperture. Recently, experiments have been devised to study the effect of high-order resonances under controlled conditions. The hope is that by comparing theoretical or numerical results with experimental ones, it may be possible to define suitable criteria for beam stability and validate their predictive power. This is an ambitious goal, since real machines are much more complicated than models used in tracking codes or analytical evaluations. There are many phenomena, like collective instabilities, synchro-betatron resonances, linear imperfections affecting the orbit functions, the linear coupling, the closed orbit, non-linear imperfections due to fringing fields and saturations, which may take a long time to be quantified in order to disentangle single-particle effects from measurements. However, analysis of operating situations provides a wealth of informations which can be exploited to bridge the gap between models and reality. These experiments have been performed in the CERN-SPS and the FNAL-TEVATRON which are already well understood, so that clear conditions could be defined, spurious effect eliminated and phenomena under study carefully isolated, with reasonably small effort. Having repeated similar measurements in different accelerators is invaluable to help in distinguishing results of general interest from those which are just a property of the machine used. Common motivations of the two experiments are related to the refinement of aperture and field quality criteria for future large hadron accelerators, like LHC or SSC. A common procedure consists in exciting already existing sextupoles in order to introduce in a controlled fashion non-linearities in an otherwise linear lattice. To probe large amplitudes, a pencil beam with small emittance and momentum spread is used, to which a large enough coherent deflection is applied. In a few hundred turns, a 'hollow' distribution of charges is created around the central orbit due to nonlinear filamentation, whose behavior is observed with several instruments: current transformers record lifetime, Schottky noise detectors give tune and tune-spread, flying wires provide transverse profile, and orthogonal pairs of position monitors are able to produce a phase space portrait.

In the experiment E778 at FNAL[33], sextupoles were exciting strongly the third integer resonance together with the by linear scaling transformation to the particle coordinates at each turn is under investigation[14].

2.3. Early indicators of chaos

Early indicators of chaotic motion are often used to speed up the estimate of the dynamic aperture. The exponential divergence of two initially very close trajectories is a criterion for chaos, a linear growth indicating regular motion. The exponential coefficient, called Lyapunov exponent[15], can be used to localize stochastic layers in the phase space and eventually to identify the stability border below which its value is zero. The routine way to evaluate the Lyapunov coefficient is to track simultaneously two particles with a slightly different initial amplitude, and to compute periodically and plot their mutual distance[16]. An equivalent method is based on the analytical evaluation of the Jacobian in the phase space domain of interest[17]. A third possibility is to compute the slow change of an action invariant[18]. The predictability of all three methods is enhanced when the non-linear deformation of the phase space is removed by Lie algebraic or Normal Forms type change of coordinates. It is currently admitted that thorough early indicators of chaos a conservative estimate of the dynamic aperture can be obtained with about ten times less computing power than for standard tracking.

2.4 Figure of merit and data processing

The linear aperture, based on smear and tuneshift with the amplitude, and the short-term dynamic aperture were widely used in the past[19] to estimate non-linear effects, since threshold values for detuning and amplitude distortion were considered sufficient to ensure long-term stability. However, the validity of this extrapolation has not been confirmed by deeper studies. Therefore, intensive tracking and sophisticated data processing are preferred nowadays to estimate the dynamic aperture, after a preliminary selection of rather few significant cases, on the basis of short-term simulations. Results are presented in graphical form: survival plots depict the maximum number of stable turns as a function of starting amplitude[20]. These plots and early indicators of chaos provide a practical estimate of the stable region. Dense survival plots are ragged and show a large spread in the survival time close to the chaotic border, rapidly decreasing at larger amplitudes. Such an irregular shape reflects the local origin of the particle instability: at moderate amplitude in presence of weaker perturbations, the escape time is largely influenced by microscopic changes of initial coordinates; at large amplitude, instead there are only fast losses. Under the influence of non-linearities, particles migrate across different nests of resonances, which can be at least phenomenologically correlated to average lifetime. The loss mechanism is in general sudden: the particle may stay confined even for millions of turns and then diverge in a few thousand turns.

2.5 Estimate of diffusion in Hamiltonian systems

A technique has been proposed to establish long-term bounds on amplitude of non-linear Hamiltonian systems for any initial condition in large regions of phase space[21]. It consists in computing the maximum change in action during n turns for any orbit starting in a given area, and in extrapolating diffusion speed in smaller area for larger number of turns. Approximate invariant tori are found by fitting tracking data by spline in action and Fourier series in angle variable. Such a method has not yet been fully exploited in practical applications.

3. APPLICATIONS IN ACCELERATOR DESIGN

Sound evaluations of performance limitation related to non-linearities are essential in the design of new accelerators. The present fields of interest are twofold: low-emittance lepton machines with sophisticated lattice, irregular orbit functions and strong sextupoles, which have to be studied for only about 10^3 turns, thanks to the existence of radiation damping, and superconducting hadron machines which must operate with negligible loss for long periods, up to 10^8 turns, in spite of the unavoidable field shape imperfections.

For lepton machines the problem is compensating linear and non-linear chromatic terms, while preserving an adequate dynamic aperture. Local corrections, implying a wide-band cancellation of non-linear resonances, is only suited for large accelerators with regular orbit functions. In small machines global methods of correction are in general used, which require several separate powering circuits for quadrupoles and/or sextupoles, and lead to easy compensation of resonant driving terms only. Standard perturbative analysis of one-turn map and Hamiltonian has been shown to be helpful. Quality factors of general use are the size of the distortion function[22], leading to ten integrals depending on sextupole strengths, or alternatively the strength of resonant harmonics together with the width of second order tune shift with the amplitude driven by sextupoles[23]. With an adequate number of independent sextupolar families, the above quantities can be minimized or cancelled which results in a dramatic improvement of the stability range. It is easy to include in the procedure an appropriate optimization of the linear orbit functions[24]. Compensation at large amplitude based on the minimization of the high order harmonics and their combined effect, computed with Hamiltonian perturbation techniques, have also been tried with some success[25].

In hadron colliders, an upper limit to unintentional field-shape imperfections and practical compensation strategies have to be devised for a safe operation. This implies a thorough understanding of the influence of the non-linearities on the long-term behavior of particle trajectories. Analytical methods available are not yet fully exploited. Numerical simulations are too heavy and time consuming for an exhaustive overview of all the possible situations. Nevertheless, remarkable progress has been made through heuristic approaches proposed in various laboratories[26, 27].

The field-shape imperfections are equivalent to multipoles up to large order, which can be expressed as the sum of two parts, one systematic and the other random. The general agreement is to stop at order 11th in the multipolar expansion and to neglect correlations between random multipoles of higher-order ones. Smear, injection efficiency and short-term dynamic aperture were measured and compared with tracking. The agreement is good, however long-term losses could not be quantitatively reproduced. The existence of stable nonlinear resonance islands was demonstrated experimentally by observing coherent persistent signals of particles captured into them. Tune modulation effects were explored and compared with those of 1-D forced pendulum.

Similar results have been obtained at the SPS[34], with sextupolar excitation such as to suppress the third integer resonance. Detuning compensation was experimentally tested by using existing octupoles. A 30% increase of dynamic aperture resulted from a factor ten reduction of tune-spread. This provides experimental guidance in devising correction schemes for large hadron accelerators. However most of the emphasis was put on the study of slow diffusion induced by power supply ripple, and controlled modulation of a special quadrupole. The diffusion coefficient was measured as a function of the amplitude, the modulation frequency and depth, and the tune. It was obtained by scraping the beam tail with horizontal and vertical collimators, retracting them suddenly by a few mm, and observing the beam lifetime to estimate the time taken by the particles to fill the gap created by the retraction. Diffusion immediately sets in when tune modulation is turned on, and there is evidence that a ripple which leads to tune modulation of 10^{-3} cannot be tolerated in a machine with strong non-linearities. A simultaneous tune modulation at two frequencies is by far more destructive than a modulation a single frequency for the same overall depth. The agreement with numerical simulations is of the order of 20%, however the strong dependence of diffusion on modulation depth and the dependence on frequency are not yet understood.

Recent experiments at FNAL[35] have also addressed the problem of ripple induced diffusion. Intensity and transverse profile were recorded and used to deduce a phenomenological dependence of the diffusion coefficient on amplitude. The proposed model assumes a threshold amplitude, below which there is no diffusion, and above which the diffusion speed increases as a polynomial of the amplitude. A steady reduction of beam size was pointed out which was never observed at the SPS. This is likely to be typical of the regime of large losses explored at FNAL.

The phenomenological model of FNAL for diffusion has been found to be in contradiction with some results of the CERN experiment as well as with the sudden manifestation of fast amplitude growth in tracking[36]. More general models of diffusion based on Markov process with jumps, using master equations on status transition probability are under investigation.

5. TRENDS

Studies of theoretical aspects related to non-linearities are pursued in several laboratories, to develop new tools for stability estimate, and to understand diffusion in 1-D and 2-D Hénon map models. However, the main tools widely used in new accelerator conception are tracking simulations and maps associated to Differential Algebra methods, whose potentiality seems to be far from being fully exploited. Experiments are likely to be vigorously pursued, to compare predictions with real world in a controlled fashion, and to clarify features of slow diffusion in presence of external modulation and non-linear fields.

Many thanks to J. Gareyte and F. Schmidt for suggestions.

6. REFERENCES

[1] V.I. Arnold, "Mathematical Methods of Classical Mechanics", Springer-Verlag, 1978.
[2] B.V. Chirikov, Physics Report 52 N. 5, p. 263 (1979).
[3] M. Q. Hénon, Quarterly of Appl. Math. 27, 291 (1969).
[4] E. Lohrmann, P. Wilhelm, Particle Acc. 19, p. 99 (1986).
[5] H. Grote, C. Iselin, CERN/SL/90-13 (AP) Rev.2 (1991).
[6] F. Schmidt, CERN/SL/AP 91-52 (1991).
[7] L. Schaschinger, R. Talman, Particle Accel. 22, 35 (1987).
[8] D. Ritson, in "Nonlinear Problems in Future Particle Accelerators", p. 304, Word Scientific (1991).
[9] A. Dragt, J. M. Finn J. Math. Phys. 17,p. 2215 (1976), A. Dragt, AIP Conf. Proc. 57, 143 (1979).
[10] M. Berz, Particle Accel. 24, 109 (1989).
[11] A.Bazzani, et al., Il Nuovo Cimento, Vol. 102 B, N. 1 (1988).
E. Forrest, J. Irwin, M. Berz, Part. Acc. 24, p. 91 (1989).
[12] J. Irwin, SSC 228 (1989).
[13] R. Kleiss, F. Schmidt, Y. Yan, F. Zimmermann, joint DESY, CERN, SSC Report, CERN SL/92-02 (AP) (1992).
[14] R. Kleiss, F. Schmidt, "Experience with a simple Method to Symplectify Differential Algebra Maps", to be published.
[15] A. M. Lyapunov, "Stability of Motion", Ac. Press (1966).
[16] F.Schmidt, F. Willeke, F. Zimmermann, Part. Accel. 35, p. 249 (1991).
[17] F. Zimmermann, Dipl. Thesis, DESY-HERA 91-03 (1991).
[18] J.Irwin, Y. Yan, IEEE PAC 89, p. 1340 (1989).
[19] A. Chao et al., CERN 88-04, p. 152 (1988).
[20] F. Galluccio, F. Schmidt, "Slow Particle Loss in Hadron Colliders", These Proceedings.
[21] W. Gabella, R. Ruth, R. Warnock, IEEE PAC 89, p. 1310 (1989).
[22] B. Autin, KEK int. 89-12 (1989).
[23] A. Ropert, CERN 88-04, p. 223 (1988).
[24] A. Bengtsson, E. Forrest, to be published (1991).
[25] R. Nagaoka et al., in "Non-Linear Problems in Future Particle Accelerators", p. 336, Word Scientific (1991).
[26] T. Garavaglia et al., SSCL Report, SSCL 268 (1990), Y. Yan, SSCL Report, SSLC 500 (1991).
F. Zimmermann, F. Willeke, DESY HERA 91-08 (1991).
J. Milutinovic, A.G. Ruggiero, BNL AD/AP-34 (1992).
[27] Z. Guo, T. Risselada, W. Scandale, "Dynamical Behavior of the CERN-LHC", these proceedings.
[28] D. Neuffer, CERN 88-04, p. 179 (1988).
[29] W. Scandale, F. Schmidt, E. Todesco, Part. Accel. 35, p. 53 (1991).
[30] R. Gluckstern, S. Ohnuma, IEEE Trans on Nucl. Science Vol NS32, p. 2315 (1985).
F. Willeke, DESY-HERA 87-12 (1987).
W. Scandale, L. Wang, CERN SPS 89-41 (AMS) (1989).
[31] G. Galluccio et al., IEEE PAC 91, p. 1669, and p. 1672 (1991).
[32] H.Bizek et al., IEEE PAC 91, p. 210 (1991).
[33] L. Merminga et al., IEEE PAC 89, p. 1429 (1989).
[34] D. Brandt et al., EPAC 90, p.1438,Edition Frontière (1990), X. Altuna et al., CERN SL/91-41 (AP) (1991).
[35] T. Chen et al, "Measurements of a Hamiltonian System and their description by a Diffusive Model", to be pub. (1991).
[36] A. Gerasimov, Private Communication (Dec. 1991).