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Abstract The LEP energy is obtained from a field measurement in a reference magnet which can be calibrated absolutely from the flux change in a loop embracing all LEP dipoles during magnetic cycling. To get the beam on the nominal orbit, the tunes were measured vs. RF-frequency for different sextupole settings. The frequency for which the tunes are independent of the latter gives the nominal orbit going through the centers of the sextupoles and, due to the relative alignment, also of the quadrupoles. The LEP energy calibration was improved by injecting protons, trapping them on a different harmonic number and adjusting the RF-frequency until they have the same orbit, and therefore the same momentum, as the electrons had. From this the revolution frequency, the velocity and the proton momentum were obtained in good agreement with the magnetic calibration.

1 Introduction

The energy calibration of LEP is important since it enters directly into the mass determination of the Z⁰ particle. The dipoles of LEP are iron-concrete magnets which undergo some aging. A reference magnet without concrete contains a flip coil for field measurements referred to as "field display" of "fd", [1]. The dipole magnets in the tunnel contain a flux loop mounted directly on the pole face, referred to as "flux loop" or "fl'. They are measured periodically by applying a symmetric current cycle between ± 2900 A and -2900 A and integrating the induced voltage in the flux loop. Annother method of energy calibration is the measurement of the revolution frequency of protons injected into LEP, [3] Since protons are not ultra relativistic the velocity obtained from the measured revolution frequency gives the momentum which is the same for protons and electrons for a fixed magnet setting and orbit. This energy calibration takes all effects into account but can only be done around the injection energy of $20 \,\,\mathrm{GeV}$

2 The LEP Field Display and Flux-loop System

A reference dipole magnet is connected in series with the LEP main dipole magnets. This reference dipole was made from a stack of standard dipole laminations. In order to ensure the best possible magnetic stability, it was not filled with mortar, but was mounted in a special frame. Measurements of the magnetic field are carried out with a flip coil mounted in the magnet gap along the position of central orbit. The flux is measured by a digital integrator triggered at the beginning and at the end of

each movement. A more detailed description of this field display system is given in [1]

The measurement in the reference dipole provides information about the field integral in the bending magnets, and thus the value of particle momentum at central orbit. The resolution of the measurement is about 20 ppm and a corresponding reproducibility has been confirmed.

The reference dipole is calibrated periodically by a direct measurement of flux variations in the flux loop mounted in the lower pole of each of the bending magnets. These loops are connected in series throughout each of the octants of LEP. The flux variation is measured by eight digital integrators placed in the even underground areas of the machine and which in turn are connected to the LEP control system. Polarity reversal permits a measurement of the remanent field which is of particular importance in these low field dipoles. This allows the beam momentum to be determined to an accuracy better than $5 \, 10^{-4}$. The calibration procedure is described in detail in [2] and the flux loop calibrations are shown in figure 1.

3 Determining the central orbit and the LEP circumference

From the field display and the flux loop calibration one obtains the energy of a beam on central orbit which goes in average through the center of the quadrupoles. This central orbit has a circumference $2\pi R$ which determines the central revolution frequency f_{oc} and the central RF-frequency f_{RFc}

$$f_{0c} = \frac{\beta c}{2\pi R} \quad , \quad f_{RFc} = h f_{0c} \tag{1}$$

where h is the harmonic number. The method used here to determine the central orbit and its circumference uses the fact that the sextupoles in LEP are very close to the quadrupoles and very well aligned with respect to them. If the beam goes through the center of the sextupoles the betatron tunes are independent of the excitation of these sextupoles. First the orbit is well corrected. Then the betatron tunes Q_x and Q_y are measured with the Q-meter [4] as a function of RF-frequency. This is repeated for different excitations of the sextupoles, i.e. for different chromaticities Q' = dQ/dp/p. Under ideal conditions the lines $Q(f_{RF})$ will cross at one point which determines the central RF-frequency f_{RFc} from which the circumference $2\pi R$ is obtained. The figure $2\,$ shows such a measurement and the results are tabulated in section 5. Due to the limited accuracy of the measurements the lines obtained for the different chromaticities don't all cross exactly in one point. An error ellipse for one standard deviation is shown on the figure. For the same reason the central frequencies obtained by measuring the two tunes is not exactly the same. We estimate that the error in the circumference measurement is of the order of 0.6 mm which corresponds to a relative error of about $2.3 \, 10^{-8}$. The accuracy of the frequency measurement is considerably better than this number and has not been considered. It does not enter into the energy calibration since only a ratio between frequencies is used as we will see later. The circumference has been measured in December 1989 and in May 1990. They differ by about 1.6 mm which we think is outside of the measurement errors. It is not impossible that the change is real and could be caused by temperature changes, tidal forces or earth motion. This has to be investigated further. The measured circumference is only about 10 mm different from the design value which indicates a relative error of only about $4 \, 10^{-7}$.

4 Energy calibration with protons

The energy calibration with protons [3] uses the fact that these particles are not ultra relativistic at the energy of 20 GeV used for our experiments. The velocity $v = \beta c$ is therefore measurably different from the speed of light and can be used to determine the momentum. In contrast the speed of the electrons and positrons has at this energy a relative difference of about $3 \, 10^{-10}$ from the speed of light which could not be measured with our methods.

To carry out the energy calibration one first sets the machine to the injection energy of 20 GeV and injects positrons. After an orbit correction the central RF-frequency $f_{RF:e}$ is measured with the method described in the last section which gives the circumference

$$2\pi R = \frac{h_e \beta_e c}{f_{RFce}} \approx \frac{h_e c}{f_{RFce}} \tag{2}$$

After this protons are injected into the same magnetic settings and trapped with the RF-system on a different harmonic number h_p . The tunes of the protons are then measured as a function of RF-frequency for different chromaticities as for the positrons. This determines the central RF-frequency f_{RFcp} for protons. The velocity of the protons divided by the speed of light is

$$\beta_p = \frac{2\pi R f_{RFcp}}{h_p c} = \beta_e \frac{h_e f_{RFcp}}{h_p f_{RFce}} \approx \frac{h_e f_{RFcp}}{h_p f_{RFce}} = \frac{f_{0cp}}{f_{0ce}}.$$
 (3)

From the proton velocity one gets its momentum p_p which has an error multiplied by γ^2 with respect to the velocity error

$$p_p = m_0 c \beta_p \gamma_p = m_0 c \frac{\beta_p}{\sqrt{1 - \beta_p^2}} , \quad \frac{\Delta p}{p} = \gamma^2 \frac{\Delta \beta}{\beta}. \tag{4}$$

The harmonic number of the electrons is $h_e = 31320$ and is well known because one unit in this number corresponds to a change of circumference of 0.8 m which is well outside the accuracy. The machine energy is known from the field display and the flux loop calibration to sufficient accuracy to determine the proton harmonic number h_p which is 31358 at 20 GeV/c. For a change of one unit in this number the momentum would change by more than 1 %.

5 Results

The results of the flux loop calibrations done between October 1989 and May 1990 are shown in figure 1 for 45 GeV/c. The numbers give the deviation of the raw calibration from the field display. An exponential fit has been made through the points. There are a few corrections to be applied to this data. The aging of flux loop itself gives a relative correction of -310^{-4} at 45 GeV and at 20 GeV. The earth magnetic field, not included during the cycling of the magnets, makes a correction of $-0.5 \, 10^{-4}$ at 20 GeV and -0.210^{-4} at 45 GeV. Finally the presence of some ferromagnetic Ni in the vacuum chamber makes a small field change not seen by the flux loop which is outside the chamber. The corresponding correction is $-5 \, 10^{-4}$ at 20 GeV and negligible at 45 GeV as obtained from field measurements in the laboratory with the vacuum chamber in place. Since we don't know how equal the nickel is distributed among the chambers of the ring this correction has a large uncertainty of about 210^{-4} . This, together with some other uncertainties results in a relative error of $2.4 \, 10^{-4}$ for the scaling between 20 and 45 GeV/c. With all this corrections one obtains the momentum of a beam on central orbit. The results are listed in table 1 for December 11, 1989 and May 21, 1990 which were the dates a calibration was done with protons.

Date	11.1	11.12.89		21.05.90	
Momentum GeV/c	20	45	20	45	
$(p(fd)-p(fd))/p(fd) = 10^{-4}$	0.8	-2.1	-1.5	-7.2	
fl aging 10 ⁻⁴	3.0	3.0	3.0	3.0	
Ni 10 ⁻⁴	-5.0	0.0	-5.0	0.0	
earth field 10^{-4}	-0.5	-0.2	-0.5	-0.2	
$(p(0)-p(fd))/p(fd) = 10^{-4}$	-1.7	0.7	-4.0	-4.4	

Table 1: Flux loop calibration

With this flux loop a calibration is made with respect to the field display for the momentum p(0) on the central orbit. To get the momentum of a physics run we have to know by how much the corresponding orbit differs from the central one. All the physics runs were done with an $f_{RF} = 352254220$ Hz. The RF-frequency f_{RFce} of the central orbit is determined with the method explained in section 3. The results are shown if Fig. 2 and in table 2.

Date	Dec. 89	May 90
f_{RFct} from Q_x Hz	352254149.4	352254172.9
	± 2.3	$\pm 4.7.0$
f_{RFee} from Q_y Hz	352254156.0	352254177.1
	± 7.6	± 4.7
f_{RFce} average Hz	352254151.7	352254172.9
	± 5	± 9
circumference mm	26 658 873.7	26 658 872.1
	± 0.4	± 0.7

Table 2: Central RF-frequency and circumference

To correct for the difference $\Delta f_{RF} = f_{RF} - f_{RFcc}$ between the two frequencies one uses the relation

$$\frac{\Delta p}{p} = -\frac{1}{\alpha} \frac{\Delta f_{RF}}{f_{RF}}.$$
(5)

where we used the momentum compaction factor being $\alpha = 3.866 \, 10^{-4}$ for the optics used presently in LEP. This give a relative correction of $-5.0 \, 10^{-4}$ for December 1989 and $-3.5 \, 10^{-4}$ for May 1990.

Two calibrations using protons were carried out at the LEP injection momentum of 20 GeV/c. The first one done in December 1989 has a relatively large error for the observation of the horizontal tune Q_x probably due to a small tune change occurring during the experiment. The one done in May 1990 worked better and is shown in figure 3. The results are summarized in table 3.

Date	11.12.89	21.05.90
f_{RFet} from Q_x Hz	352249435.3	352249247.5
	\pm 190.6	± 19.0
f_{RFce} from Q_y Hz	352249478.7	352249237.7
	± 42.3	\pm 18.0
$f_{RFc\epsilon}$ average Hz	352249471.5	352249242.6
	\pm 70	± 20
β_p	0.9989025	0.9989018
	$\pm 2.1 \ 10^{-7}$	$\pm 5.7 10^{-8}$
p(x=0) GeV/c	20.0102	20.0038
	$\pm 1.0\ 10^{-4}$	$\pm 0.310^{-4}$
p(field display) GeV/c	20.007	20.009
(p(0)-p(fd))/p(fd)	$(1.6 \pm 1.0) 10^{-4}$	$(-2.6\pm0.3)10^{-4}$

Table 3: Calibration with protons

Comparing the results of the proton calibration with the flux loop results one finds a discrepancy of about $3.3 \ 10^{-4}$ in December and $1.4 \, 10^{-4}$ in May which is not inconsistent considering all the errors. We think the protons calibrations are slightly more accurate and take them for values at 20 GeV/c. For the scaling to 45GeV/c we use the differences between the flux loop calibrations at these two energies, which we take from the fit shown in the lower part of figure 1 rather than from table 1. There are some other effects which have to be considered. The horizontal orbit correctors provide some bending and could have a slight effect on the energy. They are taken into account by the calibration with protons. However the powering of these correctors can be different for different runs. By looking at different orbit configurations the error is estimated to be smaller than $0.5 \, 10^{-4}$. There are two effects which could make slight differences in the center of mass energy at different interaction points. The distributed loss of energy due to synchrotron radiation in the bending magnets is replaced by localized RF-stations which gives an azimuthal energy dependence of each beam. The sum of the two energies is constant around the ring but this is not exactly true for the center of mass energy. One RF-station could give less acceleration to one beam compared to the other if the spacing between the cavities is not exactly a multiple of the RF-wave length and, at the same time, the phasing is not correct. Such an effect will be compensated in average around the ring but could lead to an azimuthal center of mass energy dependence. Both of these effects are estimated to be negligible compared to the other energy errors. The results are summarized in table 4.

Date		11.12.89	21.05.90
$\frac{p(0)-p(fd)}{p(fd)}$, 20 GeV/c	10^{-4}	1.6 ± 1.0	-2.6 ± 0.3
effects of correctors etc.	10^{-4}	0.0 ± 0.5	0.0 ± 0.5
scaling to 45 GeV/c	10^{-4}	2.3 ± 2.4	-0.7 ± 2.4
p(0)-p(fd)/p(fd), 45 GeV/c	10^{-4}	3.9 ± 2.6	-3.3 ± 2.5
p(phys.)-p(0)/p(0)	10^{-4}	-5.0 ± 0.4	-3.5 ± 0.7
p(phys.)-p(fd)/p(fd), 45 GeV/	'c 10 ⁻⁴	-1.1 ± 2.7	-6.8 ± 2.6

Table 4: Calibration for physics conditions

6 Conclusions

The energy calibration based on magnetic measurements combined with the flux loop calibration is in good agreement with the momentum determination with protons at 20 GeV/c. For the scaling to 45 GeV/c the flux loop is used. The relative energy error at 45 GeV is about 310^{-4} .



Figure 1: Flux loop calibration



Figure 2: Finding central orbit with positrons



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