LONGITUDINAL DYNAMICS SIMULATION OF THE HIGH INTENSITY BEAM IN THE MMF STORAGE RING

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Abstract

The one-dimensional multi-particle tracking code including space charge forces for study of the high current beam accumulation in the proton storage ring has been developed. The study of momentum broadening and bunch edges distortion has been carried out in the fast extraction operation mode of the MMF storage ring.

This paper presents the physical assumptions used for the simulation and some results.

Introduction

The proton storage ring of the Moscow Meson Factory (MMF) is being designed to store $2.4\cdot10^{13}$ particles per pulse in the fast extraction operation mode (FEM) at 100 Hz repetition rate. For such an intense beam the space charge effect is of a major concern. The main parameters of the storage ring and beam are:

orbit circumference	$L = 106.7$ m
injection time	$T = 100\ \mu s$
revolution period	$T_R \cong 450$ ns
transition energy	$\gamma_{tr} = 1.640$
relativistic parameters	$\gamma = 1.636$, $\beta = 0.7915$
linac rf frequency	$f_{rf} = 198.2$ MHz

There is not rf cavities in the MMF storage ring and it is planed to maintain the accumulated beam bunch structure in FEM operating closely to the izochronous mode [1] (momentum slip factor $\eta = (1/\gamma^2 - 1/\gamma^2_{tr}) = 0.17\cdot10^{-2}$).

It will be one bunch up to 350 ns on the ring circumference in FEM and the main goal of the longitudinal painting is to avoid the rf microbunch structure from linac. This can be done adjusting the nominal particle energy to fulfill the relation $T_R/T_{rf} \neq n$, where n is an integer.

In the present case the nominal particle energy is below the transition energy. Therefore the capacitive behaviour of the beam-environment interaction in the smooth vacuum chamber is expected. However the motion nonlinearities is more expressed close to transition energy. Therefore the stability of the particle motion near the transition energy is a problem. The nonlinearities can lead to the bunch dilution and microwave instabilities. In condition of the strong capacitance and small resistance of the beam-environment interaction it is possible to achieve sufficiently long growth time for the instabilities [2]. Moreover the self-excited electromagnetic field can change the particle momentum which in turn influences on the radial and longitudinal position of the particles. As a result the bunch is elongated and skewed and the cross section of the stored beam is changed as a whole.

The effects mentioned above have been investigated on the computer model. It has allowed to explore the nonlinearities of the longitudinal beam dynamics and to consider the instabilities in a self-consistent manner.

Model assumptions

The tracking procedure used in code consists of the step-by-step iteration of a differential equations describing longitudinal particle motion. Near the transition energy the motion equations are [3]

$$\frac{d\theta}{dt} = \omega_o\cdot\delta\cdot(\eta_o - \eta_1\delta - \ldots) \tag{1}$$

$$\frac{d\delta}{dt} = \frac{eE_z}{\beta c m_o} \tag{2}$$

where $E_z(\theta)$ is an excited electromagnetic field; θ is a particle azimuth; ω_o is a revolution frequency of the particle with nominal momentum p_o ; $\delta = \Delta p/p_o$;

$$\eta_o = 1/\gamma^2 - 1/\gamma^2_{tr} ;$$

$$\eta_1 = \eta_o\alpha_o + 3\beta^2/2\gamma^2 + \alpha_1\alpha_o \tag{3}$$

The coefficients α_o, α_1 ... are obtained from the expansion of the momentum compaction factor $\alpha = \alpha_o(1 + \alpha_1\delta + \ldots)$, where $\alpha_o = 1/\gamma^2_{tr}$. In our computer model we suppose two terms (η_o and $\eta_1\delta$) only in eq.1 are important to the particle motion and the ring chromaticity is zero ($\alpha_1 = 0$).

To solve the self-consistent eqs.1-2 we have applied the Monte-Carlo numerical calculation using macroparticle technique. Knowing the longitudinal particle distribution one can construct an induced field in eq.2 as follows [4]

$$E_z(\theta) = -\frac{e\omega_o}{2\pi R_o}\sum_{n=-\infty}^{\infty}\rho_n Z(n\omega_o)e^{in\theta} \tag{4}$$

where ρ_n represents the Fourier spectrum of the longitudinal beam density and $Z(\omega)$ is a complex longitudinal coupling impedance depending on frequency. The total longitudinal impedance can be presented in the form

$$Z(\omega) = Z_{SC}(\omega) + Z_W(\omega) + Z_{BB}(\omega)$$

where

$$Z_{SC}(n\omega_o) = i\,\frac{Z_o g}{2\beta\gamma^2}\,n$$

is the longitudinal space charge impedance ;

$$Z_W(n\omega_o) = (1-i)\,\frac{Z_o\beta}{2b}\,\delta^* n^{1/2},\quad \delta^{*2} = \frac{2}{\mu_o\omega_o\sigma}$$

is the resistive wall impedance due to finite conductivity σ of the wall ;

$$Z_{\parallel\parallel}(n\omega_o) = \frac{R_{sh}}{1 - iQ\cdot(n/n_r - n_r/n)} \qquad (5)$$

is the broad-band parasitic resonator impedance. At the above formulas $Z_o = 337\Omega$ is the vacuum impedance; $g = 1+2\ \ln(b/a)$, where b is the half pipe diameter and $a(\theta)$ is a smallest beam transverse size growing as the square root of the time; R_{sh}, Q, n_r are respectively the shunt resistance, quality factor and the resonance frequency harmonic number of the broad-band resonator which reproduces the combined effect of the cross section variations in the real vacuum pipe.

The Fourier spectrum of the longitudinal particle density in eq.4 is calculated numerically using standard binning technique [5]. The ring circumference is binned into N_b subintervals to receive a histogram representation of the longitudinal density, then the discrete fast Fourier transformation is used in order to determine ρ_n in eq.4. In our case N_b equals 2048 that means the harmonic series in eq.5 is truncated to value $N_h = N_b/2 = 1024$.

The microwave cutoff frequency consideration [5] leads to an upper limit on the number of harmonics to be included in eq.5 (n_{cutoff}=600). Therefore we have used n_r = 512 and over-pessimistic value $R_{sh}/n_r = 50\Omega$ in eq.5.

To evaluate the eqs.1-2 we used a symmetric 2^{nd} order symplectic mapping [6]: the electromagnetic field is calculated at the midpoint of the time step. Each injected linac microbunch is represented by 16 macroparticles according to the Gaussian distribution in the momentum space. The total number of macroparticles in the FEM simulation is $\approx$ 250000. In parallel with integration of the eqs.1-2 the horizontal displacement of the macroparticles has been calculated using the storage ring transfer matrices as well as the equilibrium orbit displacement due to particle momentum loss.

Due to the large value of the storage ring dispersion function and in order to avoid the particle loss the maximum particle momentum spread must not exceed $\pm 0.7\cdot 10^{-2}$ whereas the initial particle momentum spread is expected as $\pm 0.2\cdot 10^{-2}$.

To estimate the growth times of the possible instabilities we have used the cubic spline approximation of the macroparticle momentum distribution histogram and then analytically calculated the dispersion integrals in the linearized instability theory [4]

Simulation results

The Fourier spectra of the longitudinal particle density are shown in fig.1. Only low frequency part of spectrum is essentially excited in FEM. Every spectrum line can be excited because only one intense long bunch is circulated in the machine, but the harmonic amplitudes behaviour is similar to function of (sin x)/x. Hence a few first harmonics have maximum amplitudes.

In fig.2 horizontal beam projection is presented without and with space charge influence just before the extraction. On the part of the storage ring with high dispersion function one can see the skewed head and tail of the stored bunch. The simulation has shown that to maintain the particle momentum in the bunch head and tail at the permissible value it is necessary to have a drop of the longitudinal particle density in the bunch edges not less 10 ns. The extraction septum is placed in the ring azimuth where the dispersion function is zero. It means the skewing will not be observed at the extraction point.

In fig.3 and fig.4 the normalized momentum distribution function and rms momentum spread are shown. Using the analytical representation of the momentum distributions it can be calculated by the standard linearized theory [4] that the growth time of any simulated Fourier harmonic of the longitudinal particle density is greater than 10 ms, that is much more than the accumulation time ($\approx$ 113 μs) for the fast extraction operation mode of the MMF storage ring.

Conclusion

The simulation has shown an optimistic results to accumulate the required number of protons in the FEM without great problems with space charge effects. The proper longitudinal particle distribution must be realized during linac beam injection. However the particles in the edges of the stored bunch can have essentially different momentum with respect to the particles located in the bunch centre that must be taken into account to transport the extracted beam without losses in the MMF experimental area.

References

[1] Yu.P.Severgin and I.A.Shukeilo, "Time Compression of Moscow Meson Factory Macropulse", in Proceedings of the XIII International Conference on High Energy Accelerators, 1986, vol.1, pp. 277-279.

[2] I.Hofmann,"Suppression of microwave instabilities", Laser and Particle Beams, vol.3, part 1, pp. 1-8, 1985.

[3] S.Y.Lee and J.Wei, "Nonlinear synchrotron motion near transition energy in RHIC", in Proceedings of the European Particle Accelerator Conference, 1988, vol.1, pp. 764-766.

[4] A.Hofmann, "Single-beam collective phenomena - longitudinal", CAS, CERN 77-13, 1977, pp. 139-174.

[5] J.A.MacLachlan, "Longitudinal Phase Space Tracking With Space Charge and Wall Coupling Impedance", FN-446, FNAL, 1987.

[6] S.R.Koscielniak, "The LONG1D Simulation Code", in Proceedings of the European Particle Accelerator Conference, 1988, vol.1, pp. 743-745.

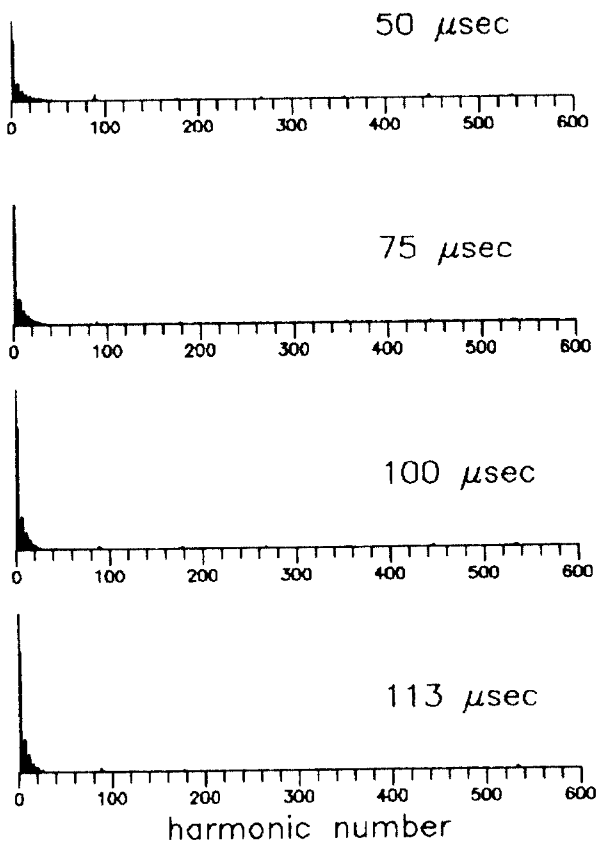

Fig.1 Fourier spectrum of the longitudinal density

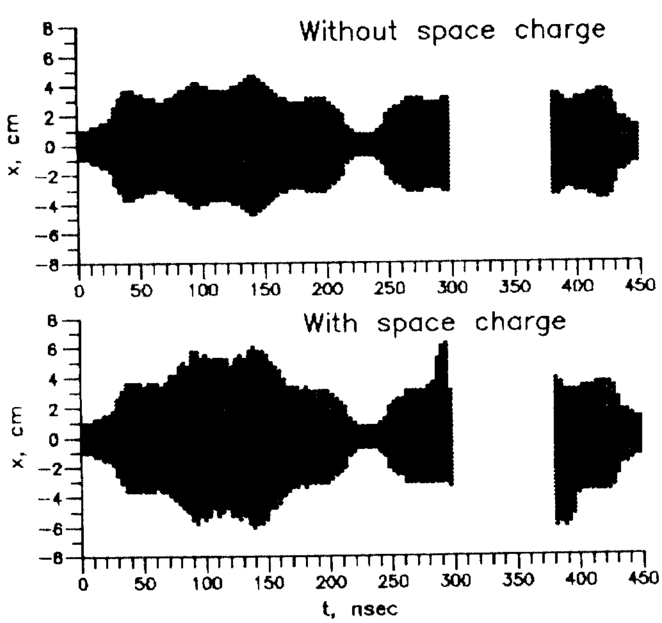

Fig.2 Horizontal beam projection ($t= \Theta R/\beta c$)

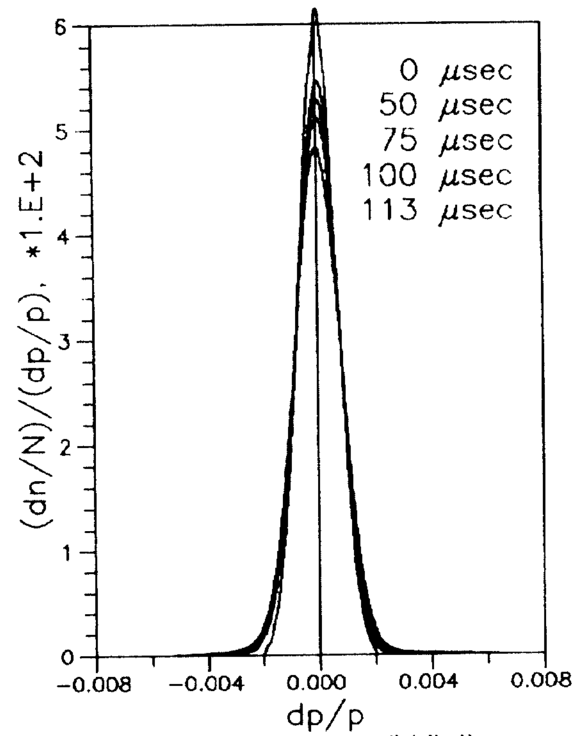

Fig.3 Normalized momentum distribution function

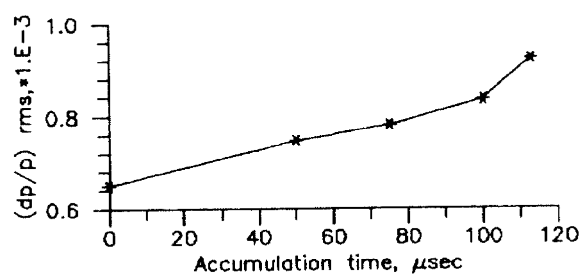

Fig.4 Rms momentum spread