COHERENT BEAM-BEAM OSCILLATIONS IN ASYMMETRIC COLLIDERS

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In this paper we study specific properties of coherent beam-beam oscillations in asymmetric colliders. The spectra of eigenfrequencies as well as possible ways of damping of unstable modes and limitations of the luminosity are discussed.

Recently, several projects for B-factories have been proposed [2] with different energies for e^+ and e^- beams. It was recognized that such a configuration can be more suitable for studying CP violation process in B-decay. Some of these projects assume to use storage rings with different circumferences for beams with different energies. As revolution frequencies for e^+ and e^- are not equal in these rings, each bunch of one beam meets with every bunch of a counter-moving beam. This circumstance changes the character of development of the coherent beam-beam instability, since it becomes a multibunch one [3, 4]. Previously the coherent beam-beam instability in asymmetric rings has been studied in Ref. [5]. But in this paper the analytical results disagree with numerical calculations. A detailed calculation of coherent beam-beam oscillation spectra in asymmetric colliders as well as an analysis of possible ways for damping unstable modes can be also found in Ref. [1]. This report contains the main results of paper Ref. [1].

Let us consider a pair of asymmetric rings. The ring for e^+ (e^-) has q_1 (q_2) bunches accordingly, which are equally spaced and contain N_1 (N_2) bunches. T_1, T_2 are revolution periods in each ring (see Fig. 1). For the description of beam-beam effects we use the

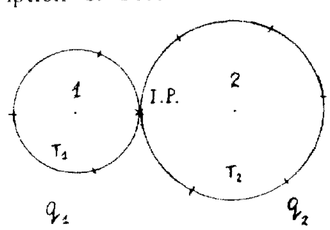

Fig. 1. Scheme of asymmetric ring-collider.

one-dimension rigid bunch model in linear approximation. In the simplest case when $q_1=1$, $q_2=q$ the coherent motion of beam baricenters z_1, z_2 can be described by a set of equations:

$$Z_1''+2\lambda_1 Z_1'+(q\nu_1)^2 Z_1=-4\pi q\nu_1\xi_1\sum_{a=0}^{q-1}\delta_T\left(\vartheta-\frac{2\pi a}{q}\right)[Z_1-Z_2^{(a)}],$$

$$Z_2^{(a)''}+2\lambda_2 Z_2^{(a)'}+\nu_2^2 Z_2^{(a)}=4\pi\nu_2\xi_2\delta_T\left(\vartheta-\frac{2\pi a}{q}\right)[Z_1-Z_2^{(a)}]. \quad (1)$$

Here λ_1, λ_2 are radiation damping decrements; ν_1, ν_2 are tunes; ξ_1, ξ_2 are beam-beam parameters; $\delta_T(\vartheta)$ is the periodical δ-function with a T_2-period; β is the value of β-function in the I.P.; ϑ is the azimuth.

The approximation of smoothed focussing permits one to take into account the radiation damping, but it is not of principle importance; calculations which take into account the Floquet modulation can be found in Ref. [1].

Simple calculations with set (1) yield eigenvectors of the problem χ_p:

$$Z_{1,2}(\vartheta)=e^{-i\nu\vartheta}\sum_{n=-\infty}^{\infty}Z_{1,2}(n)\,e^{-in\vartheta},$$

$$\chi_p=\sum_{a=0}^{q-1}\exp\left(\frac{2\pi iap}{q}\right)\sum_{n=-\infty}^{\infty}\exp\left(-\frac{2\pi ian}{q}\right)[Z_1(n)-Z_2^{(a)}(n)], \quad (2)$$

where

$$p=0,\ldots,q-1,$$

and the dispersion equation for eigenfrequencies:

$$1+\xi_2\sum_{n=-\infty}^{\infty}\frac{2\nu_2}{\nu_2^2-(\nu+n+i\lambda_2)^2}+$$

$$+\xi_1\sum_{n=-\infty}^{\infty}\frac{2q^2\nu_1}{(q\nu_1)^2-(\nu+nq+p+i\lambda_1)^2}=0. \quad (3)$$

The general form of this solution is typical for problems of the coherent stability of multibunch beams [3, 4]. Coherent dipole oscillations become unstable when ν_1, ν_2 are close to some resonance values which can be obtained from (3). The sum resonance is more specific for the examined problem. It is defined by equation:

$$q\nu_1+\nu_2=n, \quad n\in Z. \quad (4)$$

In this case (and neglecting the radiation damping) stability condition:

$$l+\frac{p-(\sqrt{q\xi_1}+\sqrt{\xi_2})^2}{q}\leqslant\nu_1+\frac{\nu_2}{q}\leqslant l+\frac{p-(\sqrt{q\xi_1}-\sqrt{\xi_2})^2}{q} \quad (5)$$

(l is an integer) determines q stopbands on the tune diagram (ν_1, ν_2), see Fig. 2. Note, that the widths of stopbands and the maximum value of the increment $\delta_m=\sqrt{q\xi_1\xi_2}$ increase like $\sqrt{q}$ with increasing the number of bunches in the large ring. As the distance between stopbands is

$$h=1-4\sqrt{q\xi_1\xi_2},$$

the stopbands overlap when $q\cong(16\xi_1\xi_2)^{-1}$. That evaluates the maximum permitted number of bunches for the given value of beam-beam parameters. Thus, for $\xi_1=\xi_2=.05$ $q_{max}\cong 25$. Under these conditions the limitation of the stopband width Δ_0 limits the available luminosity by the value:

$$L\leqslant\Delta_0^2\frac{I_1\gamma_1\gamma_2}{r_0^2\beta^2}\,\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}.$$

For coherent oscillations affected by the radiation damping the roots can be obtained from (3). This yields the stability condition in the form: $q\xi_1\xi_2\leqslant\lambda_1\lambda_2$. Therefore, in this region the luminosity will be limited by the maximum available RF-power:

$$L<\frac{\pi P}{r_0\beta m_e c^2} \quad \text{or} \quad L[\text{cm}^{-2}\cdot\text{s}^{-1}]<1.4\cdot10^{32}\,\frac{P(MW)}{\beta(\text{cm})}.$$

The above-mentioned sources of instability of coherent beam-beam oscillations in an asymmetric collider generally coincide with those found in Ref. [5] by computer simulation. But analytical calculations of the spectra in the present paper disagree in general with those in Ref. [5]. Thus, the eigenfrequencies determined

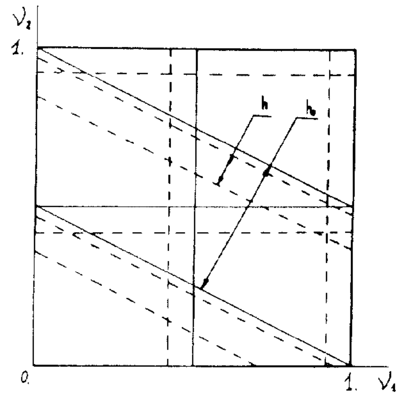

Fig. 2. Resonance stopbands of coherent dipole oscillations: h — stopband width, h_0 — distance between bands.

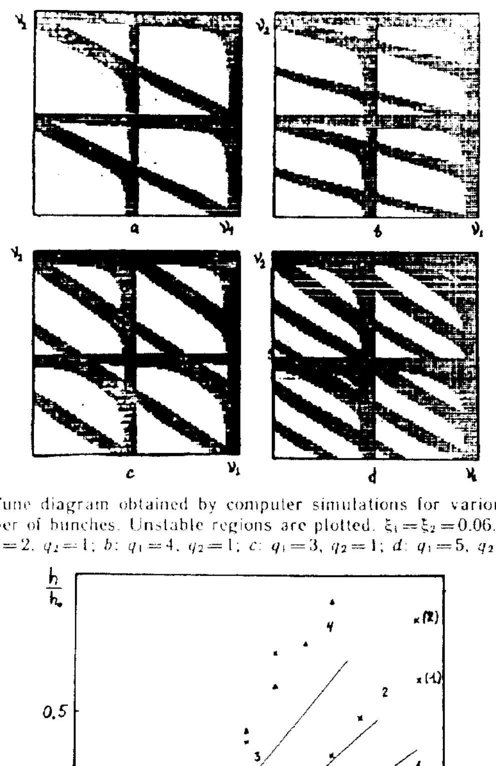

Fig. 3. Tune diagram obtained by computer simulations for various number of bunches. Unstable regions are plotted. $\xi_1=\xi_2=0.06$. a: $q_1=2$, $q_2=1$; b: $q_1=4$, $q_2=1$; c: $q_1=3$, $q_2=1$; d: $q_1=5$, $q_2=3$.

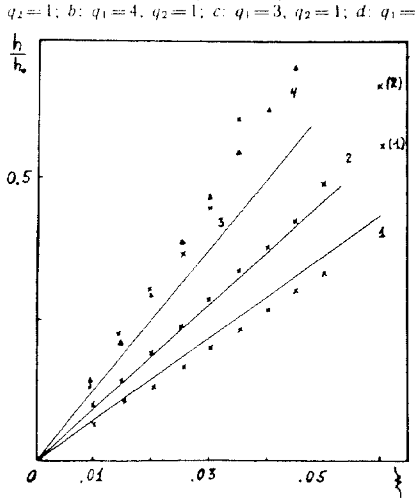

Fig. 4. Stopband width dependence on ξ for various number of bunches. $\times$ — computer simulation, —— — theoretical result (4). 1: $q_1=3$, $q_2=1$; 2: $q_1=5$, $q_2=1$; 3: $q_1=10$, $q_2=1$; 4: $q_1=5$, $q_2=3$.

by formula (4) in Ref. [5] always correspond to stable oscillations ($Q>0$). The difference in the interaction parameter seems to be more essential. The interaction parameter in Ref. [5] is obtained by averaging equations of motion before linearization and thus contains the contribution from the nonlinear part of the beam-beam force [6]. We use a parameter equal to that appearing in the calculations based on Vlasov equation

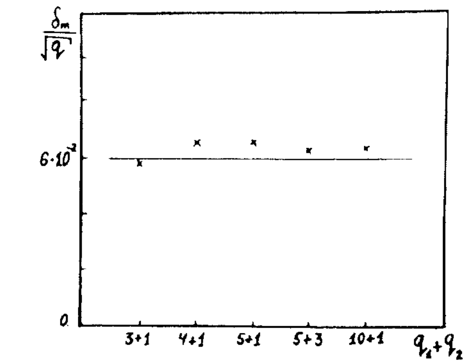

Fig. 5. Maximal increment δ_m dependence on number of bunches. $\xi_1=\xi_2=0.06$. $\times$ — computer simulation, —— — theoretical result.

[4]. As we suppose, this way is more adequate to linear approximation.

Let us also note that in accordance with the general results from Ref. [4], the multipole oscillation near the coupling resonance:

$$q m \nu_1 + n \nu_2 = p$$

may turn out unstable too. The widths of stopbands near these resonances decrease only as the power of a multipole number. This circumstance can impede applying the feedback damping system.

For a more detailed investigation of stability regions in the whole working cell with arbitrary ξ_1, ξ_2, q_1, q_2 we use computer tracking. Fig. 3 shows the results for several values of q_1, q_2. External and parametric resonances and also sum coupling resonances:

$$q_2\nu_1 + q_1\nu_2 \leqslant n \qquad (6)$$

are clearly seen. Condition (6) is a generalization of (4) for arbitrary q_1, q_2. Fig. 4 shows the width of stopbands h versus ξ for different q_1, q_2. When $q_1=1$ the plot agrees with (5). Fig. 5 shows the increment versus ξ for different q_1, q_2. A good agreement with analytical results is observed when $q_1=1$.

The presented results certainly paint out at the difficulties in the operation of high luminosity colliders with different circumferences for e^+ and e^- beams. The main difficulty is caused by the net of stopbands which covers the tune diagram (ν_1, ν_2). The size of the cell in this net, which is proportional to $(q_1+q_2)^{-1}$, limits the permitted value of ξ and, therefore, the maximum available luminosity. Increments of instability depend on the work point but in general are large which makes the instability damping by conventional feedback systems difficult.

References

[1] *A. Aleksandrov, D. Pestrikov.* Stability of Coherent Beam-Beam Oscillations in Asymmetric Colliders. INP-90-8, Novosibirsk.

[2] Proc. of the 5th Meeting of the Southern California *BB*-Factory Consortium. UCSD March 24, 1989.

[3] *C. Pellegrini.* Coherent Instabilities in Electron-Positron Storage Rings.— Physics with Intersecting Storage Rings. New York, Acad. press, 1971, p.221—243.

[4] *N. Dikansky, D. Pestrikov.* Physics of Intense Beams in Storage Rings. Novosibirsk: Nauka, 1989, p.333.

[5] *K. Hirata, E. Keil.* Coherent Beam-Beam Interaction Limit in Asymmetric Ring Colliders.— CERN/LEP-TH/89-54, GENEVA 1989.

[6] *N. Dikansky, D. Pestrikov.* Collective Beam-Beam Phenomena.— Proc. of the 3d ICFA Beam Dynamics Workshop Beam-Beam Effects in Circular Colliders. Novosibirsk, 29 May—3 June, 1989.