

## COHERENT INSTABILITY LIMITATIONS OF FRMILAB'S UPGRADES

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This study is motivated by the proposed upgrades of a whole sequence of Fermilab's accelerators; Linac, Booster, Main Ring or Main Injector and the Tevatron. Two leading high-luminosity collider upgrade scenarios involve larger numbers of colliding bunches of higher intensity. This obviously raises a question of coherent instabilities<sup>1</sup>, which was already quite vital in the present fixed target and collider scenarios. Furthermore, multi-bunch instability limitations due to the resistive wall impedance are studied for the fixed target mode. The same question of coupled bunch instabilities is also addressed for new collider schemes involving large number of colliding bunches (up to 100 on 100 bunches), where the inter-bunch communication may become important.

Coupling Impedance

We tentatively identified four dominant sources of coupling impedance. These potentially offending vacuum structures are listed as follows:

- (a) bellows
- (b) kicker magnets
- (c) beam position monitors
- (d) resistive wall and Lambertson magnet laminations.

One can estimate both longitudinal and transverse impedances induced by the above elements, using simple quantitative models or numerical simulations. Our calculation<sup>2</sup> shows that the longitudinal impedance is virtually dominated by the broad-band contribution (bellows). Similarly, bellows contribute substantially to the broad-band part of the transverse impedance, together with the kicker magnets, which significantly raise the reactive component of the impedance spectrum. Finally, the low frequency region of the transverse impedance is dominated by the singular resistive wall contribution.

Microwave Instability

The classical picture of the microwave instability assumes that the wavelength of the perturbing field is much shorter than the bunch length. In a limit of fast blow-up one can use a modified Boussard criterion to define the instability threshold.

$$|Z_{||}/n| \leq \frac{2\pi |\eta| (E/e) \delta_p^2}{I_p} \quad (1)$$

where  $\delta_p \equiv \sigma_p/p$  represents the fractional rms longitudinal momentum spread and  $I_p$  is the peak current of a bunch.

The Tevatron's thresholds are not very restrictive therefore one should not expect any danger of the microwave instability; even for very short bunches ( $\sigma_L \sim 10^{-1}$  m) and small transverse emittances ( $\epsilon \sim 4$  mm mrad) as long as the bunch intensities do not significantly exceed  $10^{11}$  ppb.

As one might expect in the lower energy range (the Main Ring, Main Injector) the microwave instability is space-charge dominated. Characteristic threshold values (collected in the table below) show slightly lower thresholds for the Main Ring compared to the Main Injector, nevertheless both machines test quite well against the microwave instability.

$\langle\beta\rangle$ [m]	h	R [m]	E [GeV]	$N_{  }$ [ppb]
Tevatron (p-injection)				
56.7	1113	$1 \times 10^3$	150	$1.6 \times 10^{11}$
Main Ring (p-injection)				
56.7	1113	$1 \times 10^3$	8.9	$2.4 \times 10^{10}$
Main Injector (p-injection)				
30	588	519.4	8.9	$3.2 \times 10^{10}$

Mode Coupling Instability

The characteristic wavelength of the instability is of the order of the bunch length and its growth time is somewhat longer than the synchrotron period. At small currents coherent motion of a single bunch can be described in terms of its multipole modes. As the current increases, the coherent frequencies of these modes move and at some point two modes may cross. The resulting degeneracy of two eigensolutions is responsible for the following intensity thresholds

$$N_{\perp} = \frac{8(\sqrt{\pi})^3 (E/e) \sigma_L v_s}{e c \langle\beta\rangle \text{Im} \langle Z_{\perp} \rangle} \quad (2)$$

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Here the effective impedance is defined in terms of the following spectral average  $\langle \dots \rangle$

$$\langle Z_{\perp} \rangle = \int_{-\infty}^{\infty} d\omega \rho_0(\omega) Z_{\perp}(\omega) . \quad (3)$$

A summary of the intensity thresholds for the mode coupling instability is included in the table below.

$\langle \beta \rangle$ [m]	h	R [m]	E [GeV]	$N_{\perp}$ [ppb]
Tevatron (p-injection)				
56.7	1113	$1 \times 10^3$	150	$9.5 \times 10^{11}$
Main Ring (p-injection)				
56.7	1113	$1 \times 10^3$	8.9	$9.7 \times 10^{11}$
Main Injector (p-injection)				
30	578	519.4	8.9	$2.4 \times 10^{12}$

#### Resistive Wall Coupled Bunch Instability

Assuming  $M$  equally spaced bunches in a storage ring environment, characterized by the transverse coupling impedance,  $Z_{\perp}$ , there are  $M$  possible dipole modes of coherent transverse motion. Each mode, labeled by  $m = 1, \dots, M$ , is described by its characteristic growth-time,  $\tau_m$ , given by the following formula

$$\frac{1}{\tau_m} = - \frac{e c M I_b}{4\pi v_{\beta} E} \sum_{p=-\infty}^{\infty} \text{Re} Z_{\perp}((pM - m + v_{\beta}) \omega_0) \quad (4)$$

Coupling of adjacent bunches requires relatively long-range wake fields; this wake field contributions may be provided either by the high-Q parasitic resonances of the rf cavities, or by the resistive wall zero-frequency singularity.

One can see that for the high intensity fixed target scenario ( $N_b = 5 \times 10^{10}$ ) the injection to the Tevatron is limited by the transverse instability with the characteristic growth-time of about 300 msec., which can easily be suppressed by a feedback system. On the other hand, the same high intensity injection to the Main Ring faces very fast transverse instability with the order of magnitude shorter growth-time than the Tevatron's; this may pose a serious problem for an active damper.

The Main Injector design seems to be quite resistant to the transverse coupled bunch instability; even for high intensity injection the characteristic growth-time is about 12 msec. which may be handled by a fast feedback system. For the new collider mode with 96 on 96 high intensity bunches ( $N_b = 5 \times 10^{10}$ ) the communication between bunches is quite significant and it yields to the

growth-time of about 300 msec., which may call for a damper system for the collider mode as well.

m	M	$N_b$ [ppb]	E [GeV]	$\tau_m$ [sec]
Tevatron (collider)				
21	96	$6 \times 10^{10}$	150	$3.09 \times 10^{-1}$
Tevatron (fixed target)				
20	1008	$5 \times 10^{10}$	150	$3.24 \times 10^{-2}$
New Tevatron (fixed target)				
21	996	$5 \times 10^{10}$	150	$2.94 \times 10^{-2}$
Main Ring (p-storage)				
20	1008	$5 \times 10^{10}$	8.9	$2.01 \times 10^{-3}$
Main Injector (p-storage)				
23	498	$5 \times 10^{10}$	8.9	$1.35 \times 10^{-2}$

#### Slow Head-Tail Instability

Following the Sacherer's model<sup>3</sup> one assumes that the amplitude of the transverse beam oscillation is a superposition of a standing plane wave pattern (with the number of internal nodes defining the longitudinal mode index  $\ell$ ) and a propagating part describing the betatron phase lag/gain, governed by the characteristic chromatic frequency,  $\omega_{\xi} = \xi \omega_0 / \eta$ .

One can generalize a simple equation of motion describing a wake field driven coherent betatron motion of a coasting beam to model the head-tail instability of the bunched beam. The inverse growth-time and is expressed by the following formula<sup>4</sup>

$$\frac{1}{\tau} = - \frac{c e \beta I_0}{4\pi E v} \text{Re} Z_{\text{eff}}^{\ell} , \quad (5)$$

where the effective impedance defined as follows

$$Z_{\text{eff}}^{\ell} = \frac{2\pi}{\ell + 1} \frac{1}{2\omega_0 \hat{t}} \sum_{p=-\infty}^{\infty} Z_{\perp}(\omega_p) \rho^{\ell}(\omega_p - \omega_{\xi}) . \quad (6)$$

Assuming only one dominant contribution to the transverse coupling impedance (due to the kicker magnets), the inverse growth-times were calculated numerically. The resulting growth-rates as a function of chromaticity evaluated for different slow head-tail modes ( $\ell = 0, 1, 2, 3$ ) are summarized in the following table.

$\epsilon$ [eV-sec.]	$\mathcal{L}$	$\xi_{\max}$	$\tau^{\mathcal{L}}$ [sec]
Tevatron (p-injection) @ 150 GeV, $N_b = 6 \times 10^{10}$ ppb			
0.3	1	10	$19 \times 10^{-3}$
	2	18	$29 \times 10^{-3}$
	3	25	$42 \times 10^{-3}$
Main Ring (p-injection) @ 8.9 GeV, $N_b = 6 \times 10^{10}$ ppb			
0.3	1	18	$2.9 \times 10^{-3}$
	2	12	$6.7 \times 10^{-3}$
	3	30	$5.2 \times 10^{-3}$
Main Injector (p-injection) @ 8.9 GeV, $N_b = 6 \times 10^{10}$ ppb			
0.3	1	stable mode	
	2	16	$2.5 \times 10^{-3}$
	3	14	$5.1 \times 10^{-3}$

The Tevatron is dominated by the  $\mathcal{L} = 1$  mode with the characteristic growth-time of about  $20 \times 10^{-3}$  sec., with the higher modes also displaying significant instability at their critical chromaticities. Our study shows that careful adjustment of chromaticity (avoiding its critical values,  $\xi_{\max}$ ) may serve as an effective way of suppressing various modes of the coherent betatron instability in the Tevatron. On the other hand, low energy injection of short bunches to the Main Ring characterized by catastrophically unstable  $\mathcal{L} = 1$  mode ( $\tau^{\mathcal{L}} = 2 \times 10^{-3}$  sec.) can be significantly improved by an increase of the longitudinal emittance. In case of the Main Injector, which already performs much better for small emittances (the  $\mathcal{L} = 1$  mode stable and the growth-time of the dominant unstable mode of about  $10 \times 10^{-3}$  sec.) the above cure is even more miraculous – it simultaneously stabilizes  $\mathcal{L} = 2$  and 3 modes. This last superior feature of the Main Injector (compared to the Main Ring) can easily be explained by a larger chromatic frequency shift,  $\omega_{\xi}$ , which governs 'overlap' of the beam spectrum and the driving transverse impedance, for a smaller storage ring ( $\omega_{\xi} \sim R^{-1}$ ).

### References

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