COMPUTER SIMULATION OF SYNCHRO-BETATRON RESONANCES INDUCED BY A NON-ZERO CROSSING ANGLE IN THE LHC

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Abstract:

The beam-beam interaction at a finite crossing angle can excite satellite resonances since it couples the longitudinal and transverse particle motions. A computer simulation has been used to show the existence of these synchrobetatron resonances for the LHC and to study the dependence on LHC parameters such as the betatron tune, the linear beam-beam tune shift and the crossing angle α. Possible constraints on the crossing angle are investigated and discussed.

INTRODUCTION

Synchro-betatron resonances or satellite resonances can be excited when the transverse motion of the particles is coupled to the longitudinal motion and the relation

$$n \cdot Q_x + k \cdot Q_z + m \cdot Q_s = p \qquad (1)$$

is satisfied by the betatron tunes Q_x, Q_z and the synchrotron tune Q_s where n, k, m and p are integer numbers. Different effects can drive synchro-betatron resonances and the effect investigated in this report is the beam-beam interaction at a non-zero crossing angle. Satellite resonances induced by the beam-beam effect have limited the luminosity of the DORIS e^+e^- storage ring [1]. In the original design of the ep storage ring HERA at DESY a crossing angle was forseen. Computer simulations have been used to investigate its effect and led to the decision to abandon the crossing angle [2]. For the LHC a crossing angle is foreseen and studies have been made to investigate the effect of this crossing angle on the short and long range beam-beam interactions [3]. It has been shown that the proposed crossing angle of $\alpha = 96\ \mu$rad is not sufficient to reduce the tune spread induced by long range forces in the very high luminosity intersection regions [3]. A larger crossing angle, however, may cause the excitation of satellite resonances and could cause the loss of the particles. The purpose of this report is to investigate the constraints on the crossing angle due to the excitation of synchro-betatron resonances and to determine whether the suggested crossing angles [3] can be used.

BEAM-BEAM EFFECT WITH NON-ZERO CROSSING ANGLE

When two bunches collide at an angle α the particles in the bunch centre experience a different beam-beam interaction than particles at a distance δs from the centre. Since the longitudinal position of the particle always changes as it performs synchrotron oscillations, the longitudinal motion is coupled into the transverse motion and causes the excitation of satellite resonances. The geometry of the collision is shown in Fig.1.

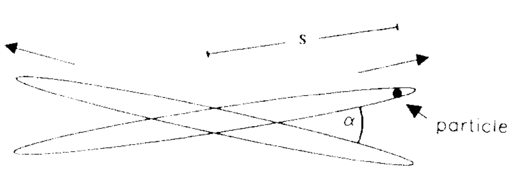

Fig.1: Beam-beam interaction at a non-zero crossing angle.

It shows two bunches crossing at an angle α. A particle at a distance δs from its own bunch centre and without transverse displacement passes the centre of the opposing bunch at a distance $d = \tan(\alpha/2) \cdot \delta s$ which is approximately $\approx \delta s \cdot \alpha/2$ for small α. The particle will receive a transverse kick which depends on d and therefore on the longitudinal position. For the head-on beam-beam interaction the kick is described by a function f(r) where r is the transverse displacement. For a round Gaussian beam the function f(r) becomes:

$$f(r) \propto \xi_{x,z} \cdot (1 - \exp(r^2/2\sigma^2))/r \qquad (2)$$

where $\xi_{x,z}$ is the tune shift that zero amplitude particles experience in a head-on beam-beam collision.

The parameter $\xi_{x,z}$ can be expressed as:

$$\xi_x = N r_p \cdot \beta_x / 2\pi k \gamma (\sigma_x + \sigma_z) \cdot \sigma_x \qquad (3)$$

where r_p is the classical proton radius, N the number of particles per bunch and σ_x and σ_z the transverse beam dimensions. The number of bunches in each beam is k, γ is the relativistic factor and β_x and β_z are the horizontal and vertical betatron amplitudes at the interaction point. For the expression for ξ_z the values for σ_x and σ_z have to be interchanged. For a head-on collision of two round Gaussian beams $\xi_x = \xi_z = \xi$.

For a non-zero crossing angle, (3) has to be modified as:

$$\xi_x = N r_p \cdot \beta_x / 2\pi k \gamma (\sigma_{eff} + \sigma_z) \cdot \sigma_{eff} \qquad (4)$$

with an effective beam size σ_{eff}:

$$\sigma_{eff} = (\sigma^2{}_x + (\alpha/2)^2 \cdot \sigma^2{}_s)^{1/2} \qquad (5)$$

where σ_s is the longitudinal r.m.s. beam size. It is assumed that the dispersion is zero at the interaction point. For a vertical crossing , the parameters for σ_x and σ_z have to be interchanged in (4) and (5). Two more parameters are important for synchro-betatron resonances, the characteristic angle and the synchrotron tune. The characteristic angle is the crossing angle at which the ratio between the longitudinal and transverse dimensions is such that the ends of the bunches just separate when the bunch centres collide. The ratio $\alpha \cdot \sigma_s / 2\sigma_{x,z}$ should be smaller than one [5]. The longitudinal and transverse beam sizes are expressed as the r.m.s. values σ_s and $\sigma_{x,z}$. For the nominal LHC crossing angle of 96 μrad [4] this ratio is ≈ 0.3 but approaches 1.0 for a crossing angle of 280 μrad. For comparison, this ratio is 4 for the original HERA design and 0.5 for the SSC with a crossing angle of $\alpha = 75\ \mu$rad. The main difference between e^+e^- storage rings and pp colliders is the synchrotron tune which is much smaller for hadron colliders and therefore the satellites, even of higher order, are clustered around the betatron resonances. The synchrotron tune Q_s is 0.0016 for the LHC as compared to 0.03 for the e^+e^- storage ring DORIS I or 0.016 for HERA. The linear tune shift ξ is also usually much smaller in hadron colliders.

SIMULATION MODEL

The simulation model used here concentrates only on the beam-beam aspect of the particle motion. It can be divided into three separate parts, the particle transport between the interaction regions, the beam-beam interaction at a crossing angle and the synchrotron motion.

Transverse variables and particle transport

The variables used for the simulation are the horizontal and vertical displacement (x and z) and angles (x′ and z′). Between the interaction regions the transverse variables are transformed linearly. Neither linear coupling between the two planes nor higher multipole errors in the magnets were considered. The tune values are $Q_x = 69.28$ and $Q_z = 69.31$ with a beta value of $\beta_{x,z} = 0.25$ for the high luminosity and $\beta_{x,z} = 0.5$ for the medium luminosity interaction region.

Longitudinal motion

The longitudinal motion is described by two variables: $\delta E/E$ which is the relative energy deviation and the longitudinal displacement δs. The synchrotron motion is described by a set of coupled equations [6]:

$$(\delta E/E)_{i+1} = (\delta E/E)_i + (2 \cdot \pi \cdot Q_s{}^2 \cdot s_i / \alpha_p \cdot R) \quad (6)$$

where α_p is the momentum compaction factor, R is the mean radius of the ring and Q_s is the synchrotron tune. For computing speed considerations the nonlinearity of the synchrotron potential is not taken into account in equation (6) but its effect on the results has been checked and found negligible. The longitudinal position is updated as

$$\delta s_{i+1} = \delta s_i - \alpha_p \cdot C \cdot (\delta E/E)_i \quad (7)$$

The momentum compaction factor is α_p and C is the circumference of the machine. The other parameters used for the simulation can be found in [3].

Beam-beam interaction

At the intersection regions the transverse and longitudinal motion is changed by the beam-beam interaction. The beam-beam interaction is represented by a kick which is calculated from (2) and x′ and z′ are changed accordingly. The calculation is done by replacing x by $x + \alpha \cdot \delta s/2$ in the formulae used for the beam-beam kick. If only horizontal crossing is assumed we get:

$$\Delta x' = f(x + \alpha \cdot \delta s/2) \quad (8)$$

Since the beam-beam kick has a longitudinal component, the synchrotron motion is also influenced by the beam-beam interaction and can be written as

$$\Delta E/E = \alpha/2 \cdot f(x + \alpha \cdot \delta s/2) \quad (9)$$

The complete coupling is described by equations (8) and (9).

Simulation strategy

To scan the satellite resonances, the particles at given amplitudes were tracked for different horizontal Q-values while the vertical tune and the synchrotron tune remained fixed. The amplitudes used were in the range of 0 σ to 4 σ and the particles were tracked between 50000 and $64 \cdot 10^6$ turns. The maximum and minimum amplitudes which occured during the tracking were recorded and the ratio is plotted against the horizontal tune value. The linear resonances as well as the satellite resonances should appear as enhancements in the diagram. Since this maximum amplitudes depend on the initial phase between the transverse and longitudinal motion, the inital phase of the transverse horizontal motion was varied. As an example the tune scan around the third order resonance with a crossing angle of $\alpha = 96$ μrad is shown in Fig.2.

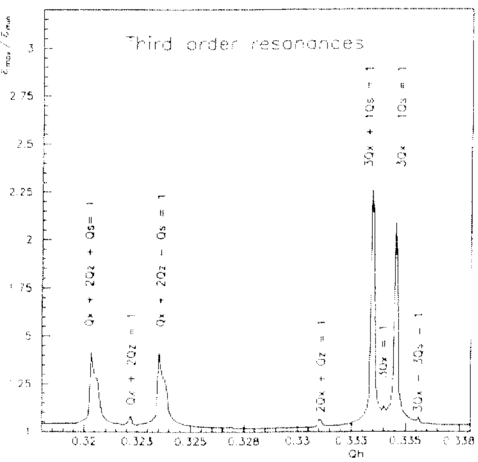

Fig.2: Third order resonance with $\alpha = 96$ μrad crossing angle.

The resonance and the synchro-betatron satellites are clearly visible. Normally an odd order resonance (like the third order) would not be observed as a beam-beam resonance since the beam-beam forces excite only even order resonances, but the symmetry of the potential is broken by a crossing angle. The resonances do not appear at their precise theoretical position since the tune is shifted due to the beam-beam effect. This shift is a function of the linear beam-beam tune shift ξ, the amplitude of the particle and the crossing angle.

HIGHER ORDER RESONANCES

To study the possible restrictions originating from beam-beam induced synchro-betatron resonances the main high order resonances in the vicinity of the LHC working point have to be investigated. The actual working point of the LHC, i.e. $Q_x = 69.28$ and $Q_z = 69.31$, is close to two resonances, namely the seventh order resonance $5Q_x + 2Q_z = 2$ and the twelfth order resonance $8Q_x - 4Q_z = 1$. The twelfth order resonance is expected to be excited even for a head-on collision and can therefore not be avoided, even with a zero crossing angle. The seventh order resonance is normally not excited by the beam-beam force in a head-on collision.

Twelfth order resonance

In Fig.3 the satellites of the 12th order resonance are shown for a crossing angle of $\alpha = 80$ μrad and in Fig.4 for $\alpha = 280$ μrad. Particles were tracked with amplitudes up to 4σ since high order beam-beam resonances are only excited for large amplitude particles.

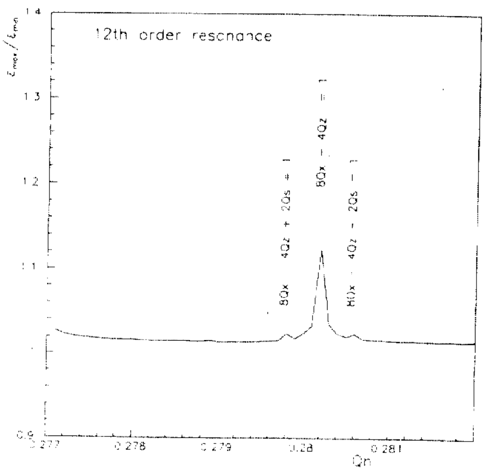

Fig.3: Twelfth order resonance with $\alpha = 80$ μrad crossing angle.

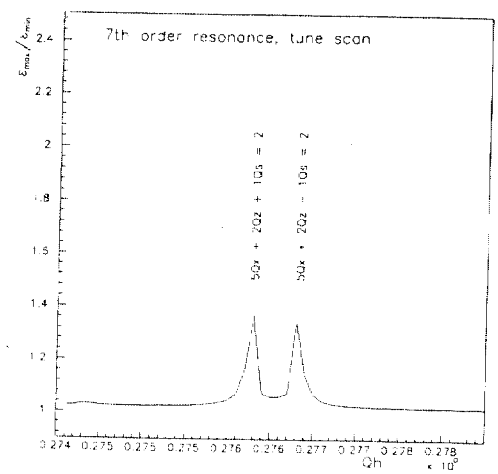

Fig.5: Seventh order resonance with $\alpha = 80$ μrad crossing angle.

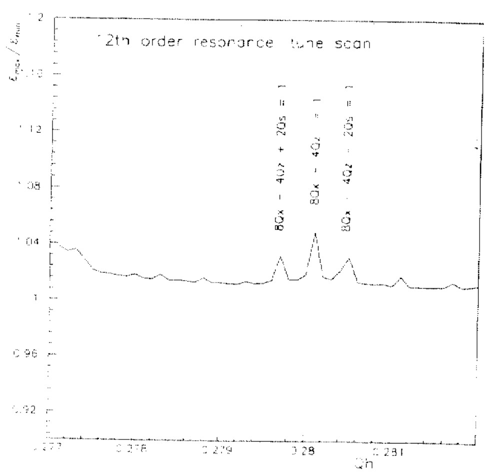

Fig.4: Twelfth order resonance with $\alpha = 280$ μrad crossing angle.

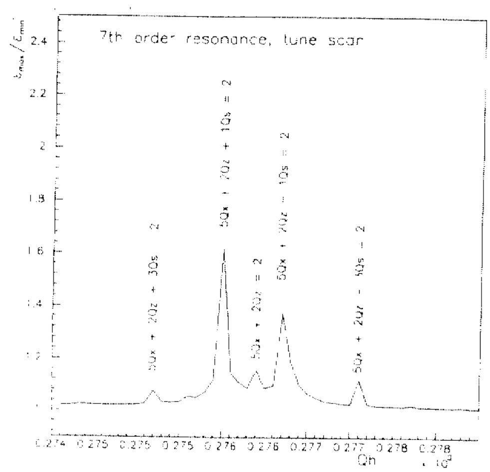

Fig.6: Seventh order resonance with $\alpha = 280$ μrad crossing angle.

This resonance is driven by the beam-beam effect and cannot be avoided even in case of a zero crossing angle (head-on). It is clearly visible that with an increased crossing angle more satellites are excited. i.e. satellites of higher orders, but the maximum amplitude excursion is not significantly increased with the beam-beam parameters investigated. The maximum amplitude increase was about 5%.

Seventh order resonance

The seventh order resonance is only excited for a non-zero crossing angle and in Fig.5 and Fig.6 it is shown for α = 80 μrad and 280 μrad.

The observation is again a larger number of satellites when the crossing angle is increased.

Stability of phase space

To test the stability of the phase space, the development of the particle amplitudes on the satellite resonances were investigated as a function of time. No dependence of the time has been found even after a very large number of turns. The width of the satellite resonances is very small (0.00001) and as the amplitude increases, the detuning is large enough to take the particles off the resonance and the amplitude increase is stopped.

If no other mechanisms, such as tune modulation, ripple on the power supply or other strong non-linearities are introduced, the phase space is stable with the parameters investigated. Computer simulations always suffer from the fact that it is difficult to prove the long term stability of particles and additional experiments have to be carried out. Such experiments are foreseen in the SPS/LEP complex to study the excitation of synchro-betatron resonances before final conclusions will be made.

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