TRANSVERSE MODE MEASUREMENTS ON SUPER-ACO*

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Abstract

To cure the head-tail-effect, a slightly positive chromaticity is required. Then, the transverse vertical mode coupling instability limits the intensity to 30 mA per bunch.

Simulations indicate that, with the Super-ACO bunch length, an increase of chromaticity pushes the limit away.

Indeed, new measurements have shown that with a high chromaticity the transverse modes do not merge any more and that *the intensity per bunch can be increased above 165 mA, only limited* by injection rate and beam lifetime.

Introduction

At the beginning of Super-ACO operation [1] we observed that an increase in chromaticity reduced the amplitude of the transverse instability, but the current limitation remained the same, limited by the dynamic aperture reduction due to high sextupole field values. Later, after a better sextupolar optimization [2] and the improvement of beam gas lifetime we can inject up to 165 mA. In this paper we analyze the behavior in terms of the mode coupling theory and compare it with experimental results.

Mode coupling theory

A single bunch can oscillate with different bunch shape modes which depend on the relative phase of the particle oscillation within the bunch.

The power spectrum of these modes consists of frequency lines $\omega_p = \omega_0 \; (p + Q + mQ_s)$ of magnitude $h_m \, (\omega_p)$ with $-\infty < p < +\infty$, $m = \pm 1, \pm 2, \ldots$ Q and Q_s being respectively the betatron and the synchrotron tunes and ω_0 the revolution frequency.

For positron bunches with Gaussian longitudinal distribution (rms length σ) the envelope $h_m(\omega)$ is expressed in terms of Hermitian modes

$$h_m(y) = \frac{1}{\Gamma\left(m+\frac{1}{2}\right)} \, y^{2m} \exp\left(-y^2\right) \quad ; \quad y = \frac{\omega\sigma}{c}$$

however for a finite chromaticity $\xi = \frac{dQ}{dP}\frac{P}{Q}$ the whole spectrum is shifted by the chromatic frequency

$$\omega_\xi = \frac{\xi Q}{\eta}\,\omega_0$$

where η *is the momentum compaction.*

It becomes

$$h_m\left(\omega - \omega_\xi\right) = \frac{1}{\Gamma\left(m+\frac{1}{2}\right)} \left(y - y_\xi\right)^{2m} \exp\left(y - y_\xi\right)^2$$

with $\quad y_\xi = \frac{\omega_\xi \sigma}{c}$

The interaction of the beam with its surroundings can be described by a frequency dependent impedance $Z_T\,(\omega_p)$. As a result, the oscillation frequency is shifted from its undisturbed value :

$$I_m(t) = \hat{I}_m \, e^{j\,\omega_p t} = \hat{I}_m \exp\left(j\left(\omega_{p0} + \Delta\omega_m\right)t\right)$$

The Real part of $\Delta\omega_m$ leads only to a change of the oscillation frequency while the Imaginary part if negative leads to an exponential growth of the oscillation amplitude at a rate of $1/\tau = j\,\Delta\omega_m$.

For rather low currents the different modes are uncoupled. In this case, the broad band impedance does not drive an instability if the chromaticity is ≥ 0. The real frequency shifts of the modes are given by

$$\Delta\omega = \frac{1}{m+1}\,\frac{c\,I\,R\,c}{8\sigma Q E}\sum_p \frac{\mathcal{I}\left[Z_T(\omega_p)\right] h_m\left(\omega_p - \omega_\xi\right)}{\sum\limits_p h_m\left(\omega_p - \omega_\xi\right)}$$

where I is the beam current and E the energy.

When the current increases, adjacent mode frequencies are shifted towards each other until they merge. They become complex and a very fast instability can develop.

Application to Super-ACO and Comparison with Experimental Results

The calculations are made using the BBI code [3], taking into account bunch lengthening and synchrotron frequency variation.

For bunch length we use the experimental variation fitted with the broad band resonator theory (Fig. 1), while the synchrotron frequency variation is calculated from the potential well model. The complex frequency of transverse modes is calculated by the program MOSES [4] inserted in BBI.

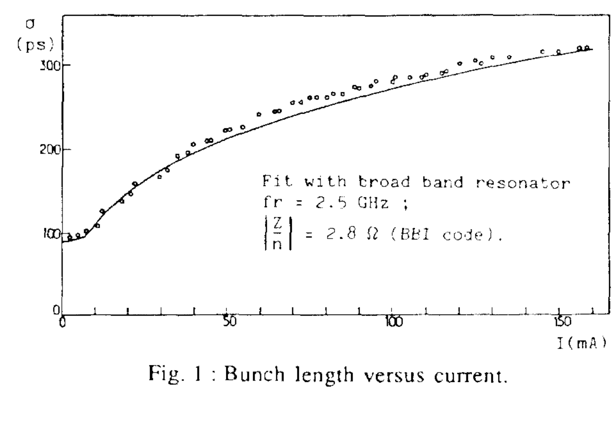

Fig. 1 : Bunch length versus current.

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Let us now present some general observations on the mode measurements. The beam is excited transversally by a radio frequency signal (about 1,2 MHz) and we detect the signal on an electrode followed by a spectrum analyzer. For weak current and weak chromaticity we observe only one betatron frequency. When the chromaticity is larger, satellites appear on each side of the initial frequency, they are separated by approximately the synchrotron frequency. As the current increases, new satellites appear. When both chromaticity and current are large, the amplitude of the response at the first frequency decreases, some satellites may become more important than the initial frequency.

However the major point is that the transverse modes 0 and -1 which merge at low current for weak chromaticity, remain well separated for high chromaticity, making the instability disappear.

Fig. 2 shows the mode frequency plot for $\xi = 0$. At 30 mA the instability occurs. Below 15 mA only the m = 0 dipole mode is present. The fit of its tune variation versus current defines the value of the transverse impedance : $Z_T = 0{,}3\ M\Omega/m$. Simulation is in good agreement with the experimental instability threshold.

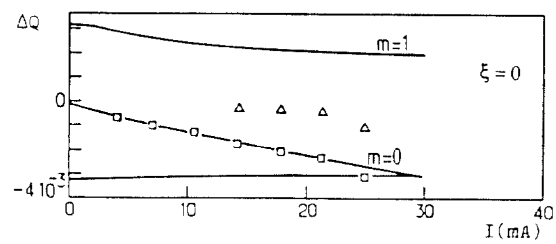

Fig. 2 : Transverse mode frequency variation with current for $\xi = 0$.

The curves show the fit by the MOSES code for m = 0.

Then we calculate the tune shift variations with current and instability threshold for different chromaticities.

In order to understand the results it is worthwhile to look at the plot of the transverse impedance and mode spectrum (Fig. 3)

We have drawn the spectrum for $\sigma = 100$ ps (weak current) and $\sigma = 270$ ps (100 mA) and each of them for $\xi = 0$ and $\xi = 6$.

First of all we can notice that the spectrum extension is much smaller at high current. The response to an external excitation being proportionnal to the amplitude of the mode at this frequency, this explains the amplitude variation observed versus intensity and chromaticity :

- At low current and $\xi = 0$ the response is zero for $m = \pm 1$. When $\xi = 6$ this response increases slightly.
- For I = 100 mA and $\xi = 6$ the response is very large for the $m = \pm 1$ mode and almost zero for m = 0.

Furthermore we see that for $\xi = 0$ the convolution of $\Im Z_T$ and h_0 is important so that the tune shift will be large while for $\xi = 6$ this quantity will be small leading to a small tune shift. For higher modes the tune shift will remain small for small chromaticity and

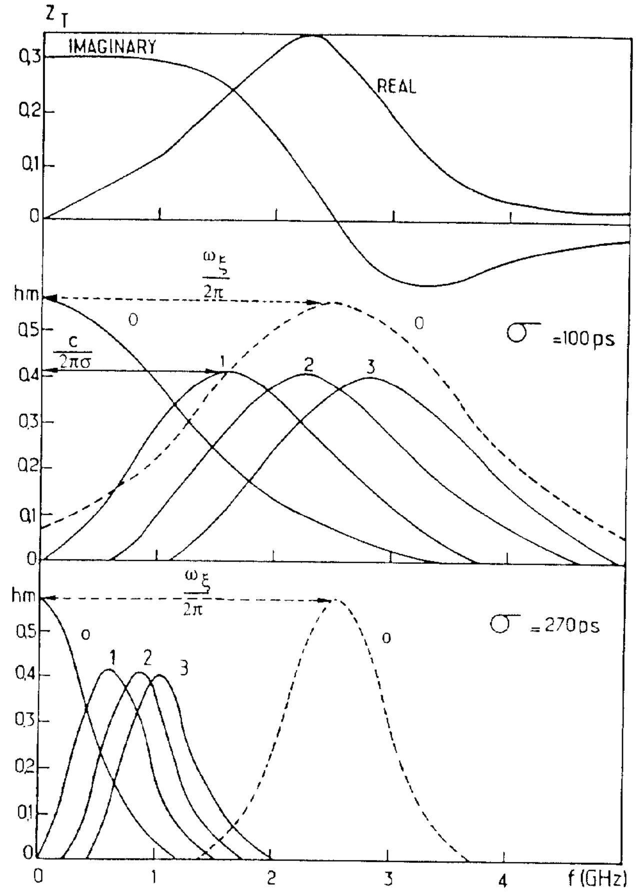

Fig. 3 : Impedance (broad band resonator model) and mode spectrum envelop for 2 different bunch currents.

zero for high chromaticity. So we expect that for a weak chromaticity an instability develops by mixing of modes 0 and -1 and that for high chromaticity their modes remain separate with no more instability.

The following plots (Fig. 4) present the results of the simulation for three values of the chromaticity :

- For $\xi = 2$ though the mode 0 and -1 are very close to each other for 60 mA, $\Im \Delta\omega$ remains > 0 and no instability develops. For higher currents the two modes decouple again.
- For $\xi = 6$ the convergence occurs for 110 mA.
- For $\xi = 8$ the modes 0 and - 1 remain well separated up to at least 150 mA.

Fig. 5 shows the experimental results for a chromaticity slighty larger than 2. It is clear that there is no more instability but the modes are also well separated.

The calculations with MOSES are in good agreement for the threshold of the mode coupling instability while the variations of the modes with current are not very well described. The effect of synchrotron and betatron frequency spreads, not taken into account in the computation, may be responsible for this discrepancy.

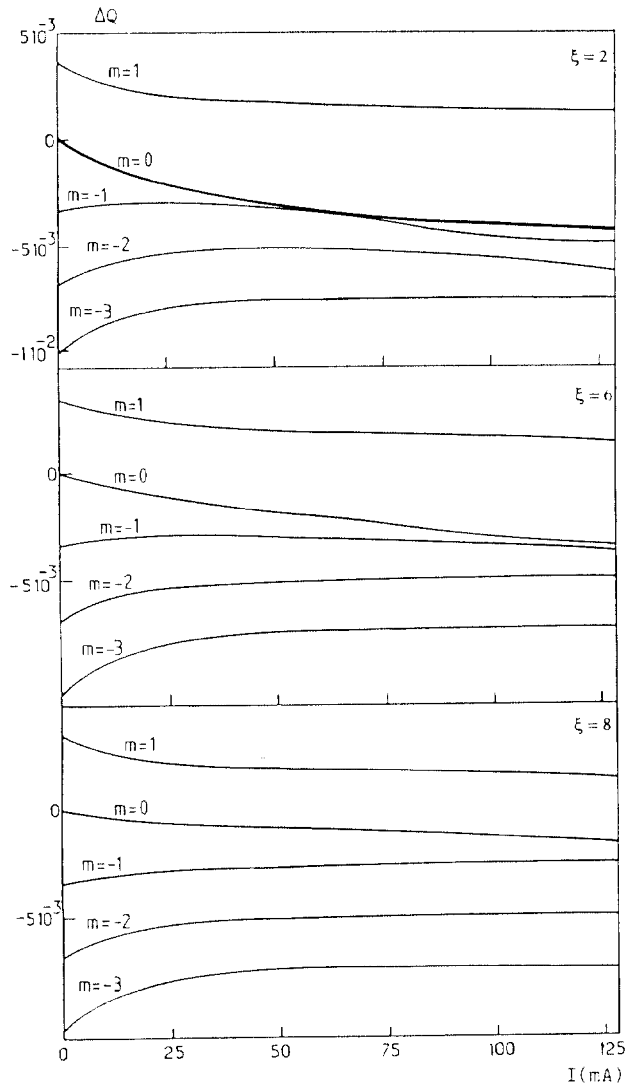

Fig. 4 : Theoretical mode frequency variation for 3 different chromaticities.

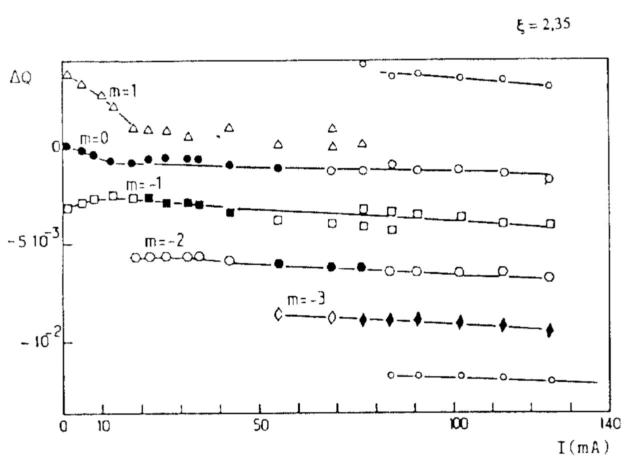

Fig. 5 : Experimental mode frequency variation for $\xi = 2.35$. Dark points show the modes whose measured amplitudes are the largest.

In conclusion, we can say that the theory of mode coupling explains rather well the beam behaviour and the transverse instability in Super-ACO and that the increase in chromaticity allows to get rid of it. It would be worthwhile to provide the future machines intended to be operated at high bunch current with a good deal of sextupole strength above the necessary chromaticity cancellation.

References

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