AUTOMATIC INJECTION TUNING USING TWO SUCCESSIVE SINGLE TURN TRAJECTORY MEASUREMENTS

M. Martini, J.P. Potier and T. Risselada
CERN, PS Division, 1211 Geneva 23, Switzerland

Abstract

A new method is proposed for the correction of injection oscillations using single turn trajectories measured in the first few turns after injection. The coordinates of the closed orbit, the amplitude and phase of the coherent betatron oscillation at injection may be obtained from the measurement of the beam trajectory in two successive turns. If the betatron tune is not precisely known it may be derived from the trajectory difference between the two turns - which is a pure betatron oscillation - provided the nominal tune dependance of the phase advances is known.

For a given tune value the betatron phase and amplitude are found by fitting a sinewave to the normalized measured difference. A one - dimensional optimization technique finds the tune value yielding the lowest rms fit quality. The selected tune, phase and amplitude determine the value of two kicks which will cancel the oscillation. The method is used at the CERN PS for its proton, positron and electron injections.

Introduction

A method using the difference between the closed orbit and the first turn trajectory was proposed at the CERN ISR [1] and is presently used at LEP. At the PS, where the closed orbit cannot be measured directly, it has to be derived from the trajectory measurements in two successive turns. The main difficulty in this case is the estimation of the tune value. In an early version of the procedure a fixed nominal tune value was used, but the method does not allow tune variations of more than 0.05 whereas at the PS the tune values for different beams may vary by as much as 0.2. In a recent version the tune value is estimated from the trajectory measurements, thus improving the efficiency of the correction procedure. The three cases:

- known tune and closed orbit,
- known tune and unknown closed orbit,
- unknown tune and closed orbit,

will be discussed below.

Known tune and known closed orbit

If the closed orbit is known, it may be subtracted from the measured single turn trajectory in order to isolate the betatron oscillation. If the same hardware is used for orbit and for trajectory measurements the result of the subtraction does not contain systematic errors, like alignment or calibration errors. If the machine tune is equally known, the β values and phases of all position monitors and of the injection steering elements (septum, kicker etc.) may be assumed to be known precisely. The normalized difference is then a pure sinewave plus a small amount of random noise.

For a given set of two correctors the kicks to be applied may be directly evaluated as follows [2]. The trajectory modification at position monitor number i resulting from kicks K_1 and K_2 at the two injection correctors may be written:

$$\sum_{n=1,2} K_n \sqrt{\beta_i \cdot \beta_n}\, \sin(\mu_i - \mu_n) = K_1 A_i + K_2 B_i \qquad (1)$$

If dif_i is the measured difference (trajectory - closed orbit) at monitor number i the number to be minimized is

$$S = \sum_i \left(dif_i + K_1 A_i + K_2 B_i\right)^2 \qquad (2)$$

Putting $dS/dK_n = 0$ yields the kicks K_1 and K_2 to be applied at the two correction elements:

$$K_1 = \frac{\left(\Sigma B_i^2\right)\left(\Sigma A_i\, dif_i\right) - \left(\Sigma A_i B_i\right)\left(\Sigma B_i\, dif_i\right)}{\left(\Sigma A_i B_i\right)^2 - \left(\Sigma A_i^2\right)\left(\Sigma B_i^2\right)} \qquad (3)$$

$$K_2 = \frac{\left(\Sigma A_i^2\right)\left(\Sigma B_i\, dif_i\right) - \left(\Sigma A_i B_i\right)\left(\Sigma A_i\, dif_i\right)}{\left(\Sigma A_i B_i\right)^2 - \left(\Sigma A_i^2\right)\left(\Sigma B_i^2\right)} \qquad (4)$$

However, the choice of the correcting elements is not always possible *a priori*. Therefore in practical applications the oscillation is often analyzed first in terms of phase and amplitude, and an estimation is made of the trajectory position and slope in septum and kicker. A set of correctors is chosen according to the available aperture and the phase and amplitude values are transformed into corrector kicks.

The horizontal beam position in the injection septum must always be adjusted with great care in order to avoid beam loss. Two strategies are frequently used for the horizontal tuning:

- adjust the beam position inside the septum (if a position monitor is available) and cancel the oscillations with septum and injection kicker
- set the injection kicker to a precalculated value and cancel the oscillations using the septum and a transfer line dipole

In principle the two methods yield the same result. In the vertical plane two transfer line dipoles are usually used for the correction.

Known tune and unknown closed orbit

In cases where the closed orbit cannot be measured directly it may be derived from the trajectory measurements in two successive turns shortly after injection. The tune Q and the Twiss parameters of the machine are assumed to be known.

With co_i the closed orbit position at monitor number i, μ_i the phase advance of this monitor with respect to a reference location R in the machine, A and ϕ the amplitude and the phase of the oscillation at R, the horizontal or vertical trajectory position in turn n at monitor i may be written:

$$x_{i,n} = co_i + A\sqrt{\beta_i/\beta_R}\, \cos\left(\mu_i + \phi + 2\pi(n-1)Q\right) \qquad (5)$$

The difference between the positions at two consecutive turns contains only contributions from the betatron oscillation but none from the closed orbit or systematic errors. This can be written as:

$$x_{i,n} - x_{i,n+1} = C \sin\mu_i + D\cos\mu_i \qquad (6)$$

where C and D are functions of the turn number *n*, the tune and the initial oscillation conditions at the reference location. For an injection oscillation producing a measured trajectory difference dif_i the values C and D are estimated by minimization of

$$S = \sum_i \left(dif_i - \hat{C}\sin\mu_i - \hat{D}\cos\mu_i\right)^2 \qquad (7)$$

From the estimators $\hat{C}$ and $\hat{D}$ it is possible to derive *A* and ϕ, or equivalently the position and angle at the reference location and the injection closed orbit using (5). A 2 by 2 matrix M transforms position and angle into the corrector strengths K_1 and K_2, which will cancel the oscillation.

In this calculation the tune of the machine has a very large impact, both on the reconstruction of the phase ϕ at the reference point because of the term 2π (n-1) Q, and on the oscillation amplitude. To limit its impact on the phase of the correction we have chosen to measure the trajectories either in the 1st and 2nd turn or in the 2nd and 3d turn after injection. Under these conditions a tune error of 0.05 can be tolerated without going into divergence of the correction process.

The impact on the correction amplitude is shown in the following. Let us consider two points X_1 and X_2 in normalized transverse phase space, representing the trajectory positions with respect to the closed orbit in turns number 1 and 2 respectively, at a reference location R in the ring. The normalized trajectories are pure sinewaves, and thus fully determined by a vector (position, angle) at any point around the machine. The closed orbit CO is represented by the origin of this diagram. The angle X_1 - CO - X_2 is equal to the fractional tune value q times 2π.

In the case of an *unknown* closed orbit only the vector $X_2 - X_1$ is known, and the closed orbit has to be estimated by finding the position CO yielding an angle X_1 - CO - X_2 equal to $2\pi q$. Figure 1 shows that the length of the correction vector to be applied in order to inject onto the closed orbit is equal to

$$\|X_1 - CO\| = \|X_2 - CO\| = \frac{\|X_1 - X_2\|}{2 \sin \pi q} \qquad (8)$$

which shows the importance of the use of the correct tune value in the procedure.

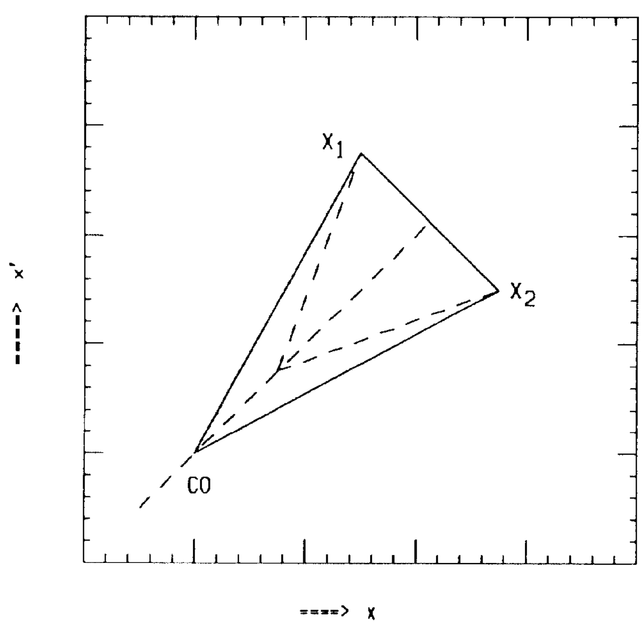

Fig. 1: Phase space diagram of trajectories and closed orbit

Unknown tune and unknown closed orbit

The method described above yields the betatron phase and amplitude by use of a linear regression model with two variables. The regression plane is fitted to the measured values by the method of least squares. As shown above the corrector strengths requested by the process are approximately inversely proportional to the sine of the fractional tune value used in the calculation, and hence it is vital to know the tune precisely.

The rms fit quality varies approximately as a quadratic function of the used tune around the true machine tune (figure 2). This property allows an estimation of the tune value by searching for the minimum rms value. The method can be applied even in case of bad monitor acquisitions. Elimination of faulty monitors is based on rms fit quality evaluation and is carried out only once at the beginning of the procedure. Afterwards, the same population of monitors is considered for subsequent calculations. Starting with an initial realistic tune, a single variable constrained optimization method is used to find the tune value which minimizes the rms fit quality, called objective function for convenience. The phase advance is assumed to be linear around the ring circumference. This calculated tune value will then be taken as the real tune of the machine at injection.

At each iteration of the process the complete regression analysis is performed, yielding the sinewave parameters and the objective function. To overcome the lengthy computational task of a step-by-step linear search, a so-called Fibonacci optimization is used. It is an optimal search technique in the sense that it yields the minimax interval in which the optimum tune is known to exist. The procedure is a sequential elimination method, which successively reduces the interval in which the optimum value of the function lies. The magnitude of the final interval depends on the required accuracy. A specification of the accuracy determines the number of iterations. The location of tune values for objective function evaluation, assumed unimodal, is based on the use of Fibonacci numbers. No derivative evaluations are required. The algorithm proceeds as follows:

- Define the original tune search interval of uncertainty and the desired tune resolution, yielding the number of iterations to be run.
- Place the first two points within the original tune interval, symmetric about the center of the interval, and compute the objective function at both points. Then, shrink the search interval such that the new subinterval contains the optimum tune value obtained so far.
- For the subsequent iterations, place a new point within the remaining search tune subinterval, at the same distance to the center of the interval as the point remaining in the subinterval, but on the other side, and compute the objective function at that point. Then, narrow the search subinterval to surround the optimum tune value further. At the last iteration the optimum tune value is located in the final subinterval, of size equal to the required resolution.

A specified accuracy of 0.001, as is used at the PS, requires 13 iterations, to be compared to the number of 500 steps which would be required in a linear search scanning the tune range 6.0 to 6.5.

The determination of the tune value works well as long as the signal to noise ratio is large. Therefore the program has the option to perform the Q-search only in the first few measurement and correction cycles when the amplitude is still large, whereas the final cycles are done without fitting new Q values.

Results

The procedure "unknown tune and closed orbit" has been implemented at the PS machine for its proton, electron and positron injections and is currently used. The performance of the correction procedure may be expressed in terms of the residual oscillation amplitudes. These are related to the quality of the trajectory measurement, and to the knowledge of the β values and phases.

At the PS the final rms fit quality is of the order of 1 mm in the horizontal plane, and slightly less in the vertical plane. Typical residual amplitude values are below 1.0 mm (H) and 0.5 mm (V).

The fit quality depends on the accuracy of the optical parameters at the monitors, but is rather insensitive to phase or β value errors of the correctors. The accuracy of the corrector parameters is beneficial for the speed of the convergence, but is less critical as far as the final result is concerned, as usually several cycles (measurement and correction) are made.

If the injection channel optics is perturbed by stray fields (for example PS proton injection), it may be useful to use empirical corrector parameters. To this end the elements of the M matrix, relating A and ϕ to the corrector strengths, may be directly evaluated by changing each of the corrector settings successively by a given increment, and calculating A and ϕ from the trajectory differences in each case.

The convergence of the process has recently been enhanced by the introduction of new hardware allowing two successive trajectory measurements in the same magnetic cycle.

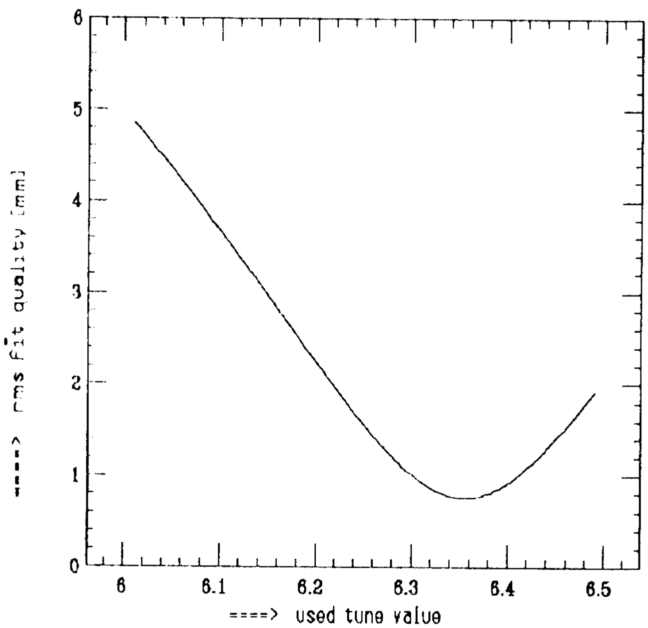

Fig. 2: Measured fit quality as a function of the used tune value

Acknowledgements

The authors wish to thank D. Gueugnon for the implementation of the correction procedure in the PS control system.

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