

Theoretical Analysis of Transverse Stochastic Cooling in the Cooler-Synchrotron COSY

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Summary

The theory of transverse stochastic cooling is well established for high numbers of stored particles. For small particle numbers the equilibrium emittance will become too large for a reasonable open-loop response function. In this paper a consistent view of both aspects is presented. Explicite formulae are given for a $\lambda/4$ -directional coupler system. It is outlined how to find the minimal cooling time either for high or small particle numbers. The theory is applied for the cooler synchrotron COSY. It is also indicated that using a frequency dependent response function the necessary power can be decreased without affecting the cooling time.

Introduction

In chapter one we briefly present the theoretical background for our calculations. The theory is applied to the cooler synchrotron COSY [1] in chapter 2. The last part deals with frequency dependent transfer functions for high and small particle numbers.

Outline of the Theory

According to [2] the relative change in transverse emittance ϵ during cooling of a coasting beam containing N protons with nominal revolution frequency f_0 is rather well described by

$$\frac{1}{\epsilon} \frac{d\epsilon}{dt} = \frac{f_0}{N} \cdot \left\{ \sum_{j=m_+}^{m_-} -2g_j + \sum_{j=m_+}^{m_-} g_j^2 (M_j + \frac{U_j}{\epsilon}) \right\} \quad (1)$$

where the sum goes over all harmonics in the range $[m_+, m_-]$ covered by the bandwidth $W = f_0 \cdot (m_+ - m_-)$ of the cooling system. Including beam feedback the open-loop response g_j is replaced by $g_j \rightarrow g_j / (1 + S_j)$. The feedback factor S_j is related to the mixing factor M_j by the equations

$$S_j = g_j \frac{M_j}{2} \quad \text{and} \quad M_j = \frac{3}{4j\eta\delta} \quad (2)$$

M_j is here given for non-overlapping Schottky bands with a parabolic frequency distribution. In eq. (2) $\eta = |1/\gamma_{tr}^2 - 1/\gamma^2|$ and $\delta = \Delta p/p$ where Δp is the total width of the momentum distribution. The quantity U_j/ϵ gives the noise-to-signal ratio and a related expression is specified below. Eq. (1) implicitly assumes that the betatron phase advance from pickup to kicker is $\pi/2 \text{ mod. } \pi$. Furthermore, there is no phase error.

Eq. (1) is easily converted in the more convenient form

$$\frac{d\epsilon}{dt} = -\frac{1}{\tau} \cdot \epsilon + B \quad (3)$$

with the cooling rate

$$\frac{1}{\tau} = \frac{f_0}{N} \sum_{j=m_+}^{m_-} \frac{2g_j}{(1+S_j)^2} \quad (4)$$

and the emittance growth per time interval

$$B = \sum_{j=m_+}^{m_-} \frac{8g_j^2 u_j}{(1+S_j)^2} \quad (5)$$

In eq. (3) the emittance ϵ denotes the area of the ellipse divided by π which encircles 96% of all particles in phase space, $\epsilon = 4 \cdot \epsilon_{rms}$. Equation (3) has the simple solution

$$\epsilon(t) = \epsilon(\infty) + (\epsilon(0) - \epsilon(\infty)) e^{-t/\tau} \quad (3a)$$

with the equilibrium emittance $\epsilon(\infty) = B \cdot \tau$ and the initial emittance $\epsilon(0)$. It follows that the cooling time is

$$t(\epsilon) = \tau \cdot \ln \left(\frac{\epsilon(0) - \epsilon(\infty)}{\epsilon - \epsilon(\infty)} \right) \quad (3b)$$

We assume $\lambda/4$ -loop couplers with power combiners for the pickup and kicker arrays [3]. The open loop response function is then determined by

$$g_j = g(\omega) = F \cdot \frac{\omega_m}{\omega} g_A(\omega) \quad (6)$$

with the amplitude $g_A(\omega)$ of the amplifier gain at frequency $\omega = j 2\pi f_0$. $\omega_m = (\omega_+ + \omega_-)/2$. The factor F is given for $\beta \approx 1$ by

$$F = \frac{2}{\pi} N \sin^2(kL) \sqrt{n_p n_k \beta_p \beta_k} \frac{\sigma_p \sigma_k Z_p}{h_p h_k} \frac{L e^2 f_0}{\gamma m_0 c^2} \quad (6a)$$

where $k = \omega/c$. F contains the number $n_{P(K)}$ of loop pairs and the gap height $h_{P(K)}$ of the pickup (kicker). The beta-function at the pickup (kicker) is denoted by $\beta_{P(K)}$. The sensitivities $\sigma_{P(K)}$ follow from $\sigma_{P(K)} = 2 \cdot \tanh(\pi w / 2h_{P(K)})$. The common transverse width of a loop is w and the longitudinal length is L . The characteristic impedance of pickup and kicker is $Z_{P(K)}$. e is the elementary charge. Note that we use the mean of $\sin^2 kL$ (equal to 0.9 for one octave) which is a rather good approximation especially for bandwidths less or equal 1 octave. The quantity u_j

which is related to the noise-to-signal ratio now reads

$$u_j = u = \frac{1}{2} \left(\frac{f_0}{N} \right)^2 \frac{k_B (T_R + T_A)}{\langle \sin^2 kL \rangle n_p \beta_P Z_P \left(\frac{\sigma_P}{h_P} \right)^2 e^2 f_0^2} \quad (7)$$

and as indicated it does not depend on frequency. T_R and T_A denote the system temperature and the equivalent amplifier noise temperature, resp..

The response function $g(\omega)$ becomes independent of frequency if we choose a linearly increasing amplifier gain (eq. (6)). In this case eqs. (4) and (5) reduce to

$$\frac{1}{\tau} = \frac{2f_0}{N} \cdot g_0 \cdot J(g_0) \quad \text{and} \quad \epsilon(\infty) = \frac{4N}{f_0} \cdot g_0 \cdot u \quad (8)$$

$$\text{with } J(g_0) = \frac{W}{f_0} \cdot \left\{ 1 + 2x \ln \left(\frac{x+x_+}{x+x_-} \right) + \frac{x^2}{(x+x_-)(x+x_+)} \right\}$$

and

$$x = 3f_0 g_0 / 8\eta \delta W \quad \text{as well as } x_{\pm} = m_{\pm} / (m_+ - m_-).$$

Two important cases can be deduced from eq. (8): Since $g_0 \propto N \cdot \epsilon(\infty)$ the response g_0 will decrease with decreasing particle number for a given $\epsilon(\infty)$. Especially for a small final emittance we then have $J(g_0) \rightarrow W/f_0$ and the cooling rate converges to

$$\frac{1}{\tau} = \frac{2W}{N} \cdot g_0 \quad (9)$$

i.e., the cooling rate is independent of particle number for constant $\epsilon(\infty)$.

On the other hand a larger particle number makes it easier to achieve larger values of g_0 . It can be shown (for one octave: [4]) that there exists an *optimal* g_0 for which the cooling rate will be at maximum:

$$\frac{1}{\tau_{\text{opt}}} = \frac{4 \eta \cdot \delta \cdot W^2}{3 N \cdot f_0 \cdot \Lambda} \quad (10)$$

with $\Lambda = \ln(m_+/m_-)$. The optimal response g_{opt} is

$$g_{\text{opt}} = \frac{8\eta \delta W}{3f_0 \Lambda} \quad (11)$$

For τ_{opt} the same result holds neglecting beam feedback. However, the amplifier gain and $\epsilon(\infty)$ are then twice as large. It follows that eq. (9) becomes valid if $g_0 \leq 0.1 g_{\text{opt}}$.

The power in the band W applied to the kicker is the sum of the amplified *Schottky power*

$$P_S = \frac{\langle \sin^2 kL \rangle N}{2} n_p \beta_P \left(\frac{\sigma_P}{h_P} \right)^2 Z_P e^2 f_0 \epsilon_{\text{rms}} \langle g_A^2 \rangle W \quad (12)$$

and the *thermal noise power*

$$P_{\text{th}} = k_B (T_R + T_A) \langle g_A^2 \rangle W \quad (13)$$

Results for COSY

In *COSY* transverse cooling of protons in the range $N = 10^8$ ppp up to 10^{10} ppp and energies above $T = 1.5$ GeV will be done with a 2 GHz bandwidth splitted into two sub-systems consisting of band I and band II with a frequency range (1 - 1.8) GHz and (1.8 - 3) GHz, resp. The number of loop pairs of each system is $n_p = 30$ and $n_k = 15$. We use for pickup and kicker a width $w = 3$ cm and $Z_P = 50 \Omega$. Pickup and kicker plates are moveable up to maximal gap height of 15 cm. The cooling system will be installed in the telescopic section of the ring [1]. The results given below are calculated for a kinetic energy $T = 1.5$ GeV and a revolution frequency $f_0 = 1.5$ MHz. At this energy the relative momentum spread is $\delta = 2 \cdot 10^{-3}$. The lattice [5] is tuned so that $\gamma_{\text{tr}} = 2$ and $\beta_P = \beta_K = 20$ m. The initial emittance is $\epsilon(0) = 5$ mm mrad and the final emittance $\epsilon = 1$ mm mrad. We therefore use the gap height $h = 3$ cm for pickup and kicker.

In figure 1 the cooling time as well as the inverse of the cooling rate is plotted versus $\epsilon(\infty)$ for band I. For $N = 10^{10}$ ppp the minimum cooling time is found for $\epsilon(\infty) = 0.08$ mm mrad corresponding to τ_{opt} (eq. 10,11). On contrary, for $N = 10^8$ ppp the equilibrium emittance would be 8 mm mrad for τ_{opt} . Thus, for this particle number the amplifier gain must be reduced and τ_{opt} can not be attained. In this case eq. (9) provides a good approximation for the cooling rate. The figure clearly shows that there is a minimal cooling time at a gain indicated by the bold arrow.

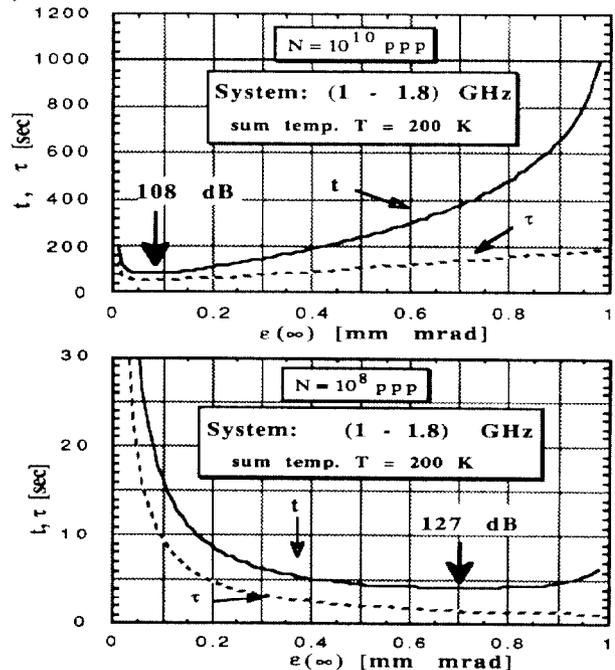


Figure 1: Cooling time t and τ versus $\epsilon(\infty)$

Note the broad minimum for 10^8 ppp. The amplifier gain can be reduced to 124 dB ($\epsilon(\infty) \approx 0.5$ mm mrad) without a significant change in cooling time.

In table 1 we summarize results for cooling with the two combined bands. Both sub-systems are operated independently with a total bandwidth of 2 GHz. The numbers listed in table 1 are calculated for an equivalent broadband system with a bandwidth of 2 GHz and $n_p = 30$ as well as $n_k = 15$. For the two sub-systems their independently optimized amplifier gains are nearly the values of the equivalent broadband system. Operating both bands simultaneously the expected cooling times are nearly those of the equivalent broadband system. The total power is calculated using eqs. (12) and (13) including 12 dB losses and reserve. The pickup plates will be cooled down to nitrogen whereas the pre-amplifiers will be operated outside the vacuum at room temperature. $T_R + T_A = 200$ K. For COSY the cooling time becomes independent of particle number for $N < 10^8$ ppp. It follows that for $N > 10^{10}$ ppp the cooling time scales with particle number. With respect to band I the cooling time is decreased by a factor of 4. Compared to other cooling systems the required power is rather low. This is due to the small final emittance which requires a corresponding small equilibrium emittance.

N	10^8 ppp	10^9 ppp	10^{10} ppp
cooling time: t [sec]	2	4	25
g_A [dB]	124	119	113
total power including losses:			
P_{tot} [W]	240	120	106
	2 dB loss in combiner+splitter 8 dB loss and reserve		

Table 1: Cooling time, gain and power for COSY

Frequency Dependent Gain Optimization

So far we have used a constant transfer function for the feedback system. In this chapter we try to optimize the cooling rate and/or the power by using a frequency dependent open loop response. Two different cases can be distinguished: a.) for high particle number the equilibrium emittance plays no important role. Here, the optimized response function is given by $g(j) = 2/M_j [2]$. It can be shown that the change in growth rate is small for one octave bandwidth. b.) for small particle numbers the equilibrium emittance becomes important. Here we use eq. (9).

Introducing the ansatz $g_\alpha(j) = g_0 \cdot \left(\frac{j}{j_m}\right)^\alpha$ with $\alpha = 1, 0, -1$ and $j_m = (m_+ + m_-)/2$ both, cooling rate and power are modified by a universal factor. These factors only depend on α and the ratio $x = m_+/m_-$. They are independent from lattice parameters. It turns out that up to $x = 4$ (2 octave bandwidth) the change in cooling rate is less than 10%. Figure 2 presents the power normalized to that of constant response.

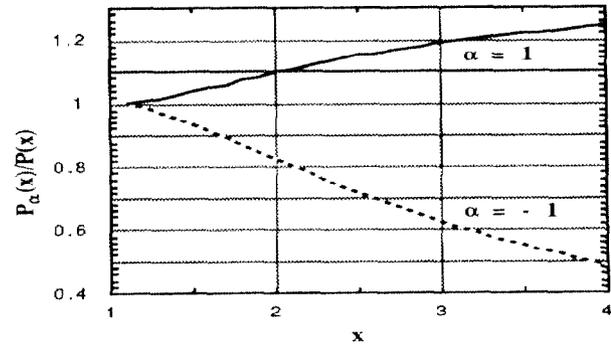


Figure 2: Power ratios for $\alpha = 1$ and -1 vers. x .

We observe that for one octave the power is slightly reduced whereas at $x = 4$ the power is decreased to 50% using a constant amplifier gain g_A ($\alpha = -1$).

Conclusion

A consistent view of transverse stochastic cooling is given for high and low particle numbers. The theory is applied for the cooler synchrotron COSY. It is shown that for all particle numbers the cooling time can be minimized by choosing the appropriate amplifier gain. The minimum in cooling time not necessarily coincides with the maximum cooling rate. This effect is especially pronounced at smaller particle numbers. It is demonstrated that for small particle numbers a frequency dependent response function will decrease the necessary power without affecting the cooling time. The decrease in power is described by a universal function which is independent of lattice properties.

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