DESIGN METHOD FOR HIGH ENERGY ACCELERATOR WITHOUT TRANSITION ENERGY

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Abstract:

During acceleration at relativistic energies, high energy accelerators are confronted with the problem of going through transition energy. At transition energy the bunch length becomes very small while the momentum spread becomes very large. A design method for a circular accelerator which avoids transition is presented. The method uses Floquet's coordinate transformation of the dispersion function. A detailed example of a 150 GeV accelerator ring without transition is also given.

1. Introduction

Transition is one of the major restrictions for high intensity beams (of the order of 10^{11} protons per bunch). At transition the bunch length becomes very small while the momentum spread becomes very large. Transition occurs during acceleration due to relativistic effects. The transition gamma is defined by:

$$1/(\gamma_t)^2 = (\Delta L/L)/(\Delta p/p), \qquad \ldots (1)$$

where "L" is the total length of the accelerator while ΔL is the difference in the path of off momentum particles (p+Δp). To avoid transition $1/(\gamma_t)^2$ should be less than zero. The sources of dispersion are dipoles where higher/lower momentum particles are bent less/more. The transverse displacement of an off momentum particle is related to the dispersion D_i by:

$$\Delta x_i = D_i * \Delta p/p. \qquad \ldots (2)$$

The total difference in the path around the ring ΔL is:

$$\Delta L \simeq \Sigma_i D_i \theta_i \qquad \ldots (3)$$

The quantity $1/(\gamma_t)^2$ will be negative if the sum of the dispersion function through the dipoles is a negative number. In these cases γ_t is an imaginary number.

2. Design Method

The horizontal dispersion vector $(D_x, dD_x/ds, 1)$ presents a general solution to the inhomogeneous Hill equation of motion [1] when the horizontal and vertical motions are treated separately. A new vector (χ,ξ) is defined [2] by Floquet's coordinate transformation as:

$$\chi \equiv D/\sqrt{\beta} \equiv A*\sin\Phi \text{ and } \xi \equiv D'\sqrt{\beta} + D\alpha/\sqrt{\beta} \equiv A*\cos\Phi \quad \ldots (4)$$

where β, α, and Φ are the Twiss parameters [1]. A change of the vector amplitude "A" occurs only at the corresponding dipole (along the ξ-axis), while propagation through all other elements is described with a change of the betatron phase Φ as presented in figure 1. To provide an average negative value of the horizontal dispersion through the dipoles most dipoles should be placed in a lattice within the third and fourth quadrant of the χ and ξ coordinate system.

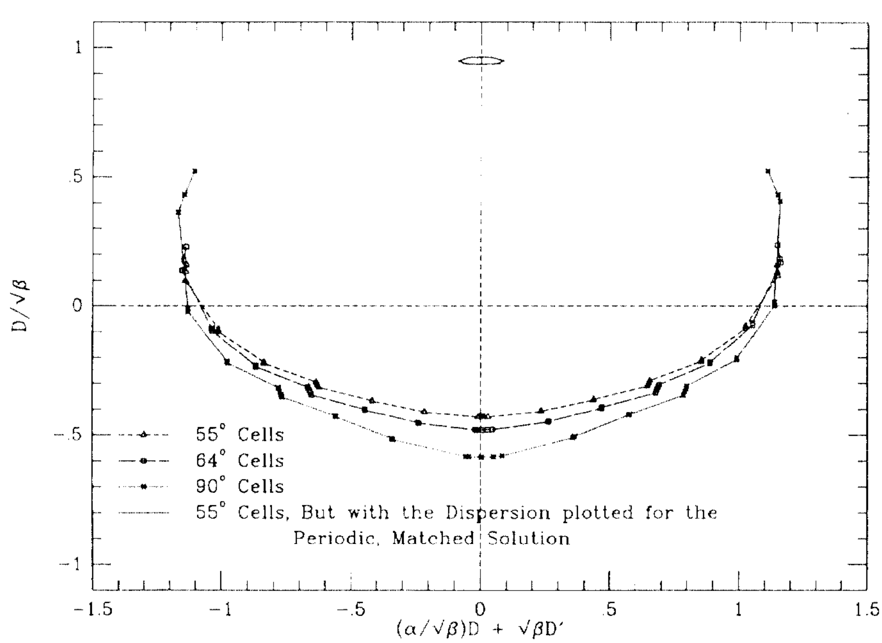

Fig. 1 Normalized dispersion function within two FODO cells: 90° (-*-), 64° (-°-), and 55° (-Δ-) degree phase advance per cell.

3. Design of the Ring

The horizontal dispersion function within a planar accelerator ring usually follows the horizontal betatron function around the ring. The normalized dispersion function presented in equation (4) within a regular and matched FODO cell is shown in figure 1. The oscillations of the dispersion function along the cell are contained within a small trapezoid in the normalized dispersion space. The center of the trapezoid is located on the χ - axis above the origin and the oscillations are contained within the first and second quadrant of the ξ and χ space.

Lattice designs to date without transition have manifested high values of the dispersion function. The maximum value of the dispersion function through the regular FODO cell depends on the choice of a betatron phase advance through the cell. Higher values (90° per cell) of the phase advance provide smaller dispersion through the cell. The choice of a phase advance per FODO cell in the case of a transitionless ring is different. The horizontal dispersion function of two adjacent regular FODO cells can be presented in the normalized dispersion space with the starting point within the second quadrant. Fig. 1 presents two FODO cells for three different phase advances per cell: 55°, 64°, and 90° within the normalized χ and ξ space. Dipoles within the cells are presented with vectors parallel to the ξ axis and point toward the positive ξ. The dipoles are evenly distributed within both the third and fourth quadrant by a specific starting point in the second quadrant. Dispersion D has a positive value while the slope of the dispersion function D' is a negative number. The phase advance of 55° per cell is the best choice of the three due to the lowest dispersion values at both ends of the plot. In order to close and match the dispersion function as well as the other betatron functions (as presented in fig. 1) a low beta cell is used.

The highest value of horizontal dispersion in the low beta cell is at the focusing quadrupoles. The two 55° FODO cells provide the lowest starting value of the horizontal dispersion for the low beta cell. A combination of two regular adjacent FODO cells and the low beta cell, with all betatron functions matched, provides a basic repetitive unit for the lattice without transition.

This simple transitionless lattice, with a 55° phase advance per FODO cell, is presented in the normalized dispersion space in figure 2. The third and fourth quadrant in the χ and ξ space contain two FODO cells with six dipoles per cell. The low beta cell occupies most of the two upper quadrants. Two dipoles are placed in the middle of the cell. Because the betatron function β_x is very small through the cell, the effect of the two dipoles on the dispersion is very small.

4. Design of the Straight Sections

Ideal extraction and injection designs always require a 90° phase advance between the kickers and injection (fast extraction) and the magnetic septum or between the electrostatic septum and the magnetic septum (slow extraction). Transversely the beam should not be limited by the physical aperture of the downstream element. The straight sections are low beta insertions without dipoles. Two dipoles are taken out from the first half cell of the FODO cell to provide room for kickers for extraction or injection. Figure 3 presents the normalized dispersion space of the straight section. Two more dipoles are removed from the end of the last half FODO cell to allow the dispersion match to the rest of the ring.

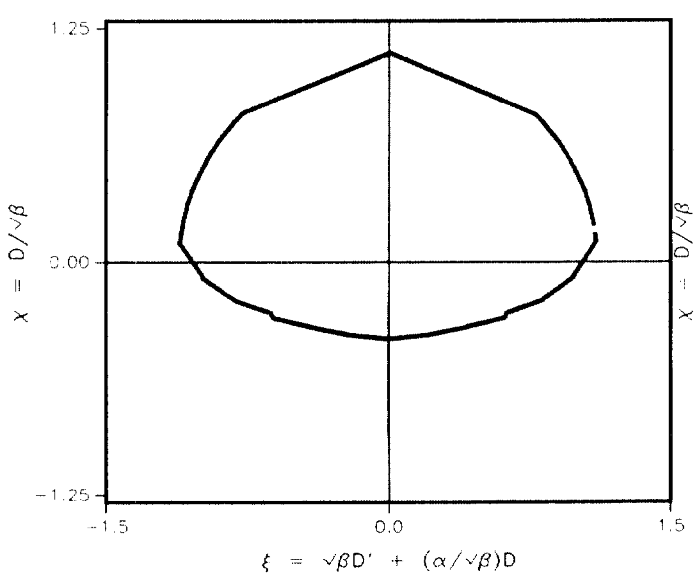

Fig. 2 Normalized dispersion function within the repetitive cell.

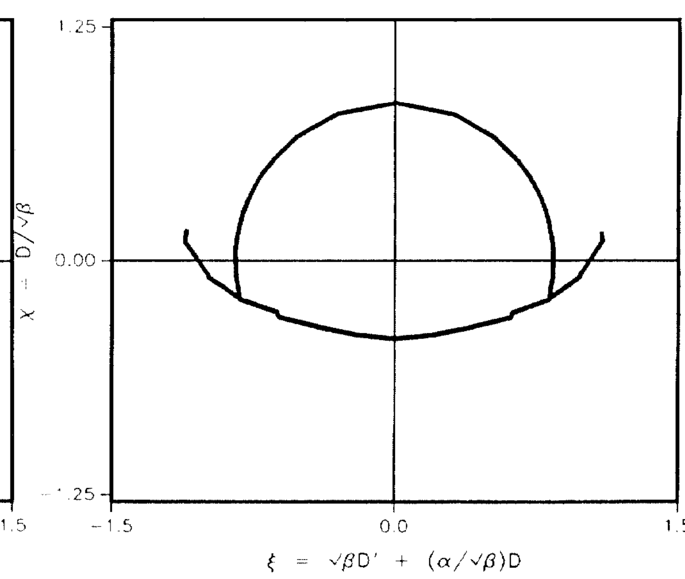

Fig. 3 Normalized dispersion function within the straight section together with the two FODO cells.

Figure 4 represents the β_x and β_y betatron functions and dispersion through the repetitive cell with the straight section included. The transition gamma in this example is equal to:

$$\gamma_t = i\ 18.27\ ,$$

and it is an imaginary number. The size of the ring presented in this example is 3071 meters with a radius of 488.8 meters. The horizontal and vertical tunes are slightly higher than 18, and the maxima of the dispersion function are between 2.7 and -2.7 meters, while the natural chromaticities are Q_x = -35.5 and Q_y = -26.8. The straight section has a dispersion of less than 2 meters. The maxima of the betatron function are: β_x = 75 meters and β_y = 78 meters. Other examples of transitionless rings were designed as well. A ring was designed with a 600 meters circumference and with only one dipole per half cell with a dispersion maxima of 1 meter.

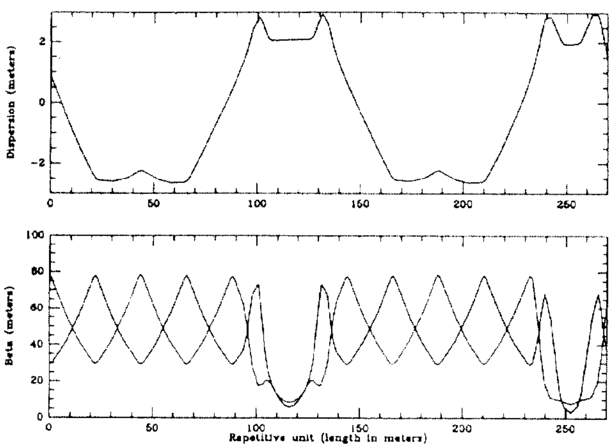

Fig. 4 Betatron functions through the repetitive cell together with the straight section

Conclusion

A simple design method for a high energy circular accelerator without transition is provided. It is applicable for any ring, regardless of size. The absolute values of the maximum of the dispersion function do not exceed 2.7 meters which is competitive with traditional lattice designs. The stability of the betatron and dispersion functions was examined by introducing an error of the quadrupole gradient. No noticeable change in all the functions mentioned was observed.

(1) E.D. Courant and H.S. Snyder, "Theory of the Alternating Gradient Synchrotron," Ann. Phys 3, 1(1958).

(2) D. Trbojevic and R. Gerig, "Design and Commissioning of the D0 Vertical Nondispersive Overpass in the Fermilab Main Ring", Proceedings of the 1989 IEEE Particle Accelerator Conference, March 20-23, 1989, Chicago, Illinois, pp.1891-1833.