Effects and Tolerances of Injection Jitter in the SLC and Future Linear Colliders*

T. LIMBERG, J.T. SEEMAN AND W.L. SPENCE
Stanford Linear Accelerator Center, Stanford, California 94309, USA

1 Introduction

The bunch injected into the main linac of a linear collider may have offsets in transverse angle and position, may have a phase error (longitudinal position offset) and, furthermore, may be optically mismatched. Each of these injection errors reduces the luminosity and must be held within tolerances.

Transverse angle or position offsets increase the emittance throughout the linac due to filamentation caused by transverse wakefield and chromatic effects. Simulations[1,2] have been used to calculate the maximum injection jitter to keep the emittance blowup at the end of the linac below an acceptable level, say, 10 to 25%. In Section 2 tolerances are calculated using this maximum jitter for field stability and transverse vibrations for magnets of the Damping Ring and the Ring-To-Linac (RTL) transport line. No components upstream of the damping ring have to be considered, because after several damping times orbit and beam distributions reach an equilibrium entirely determined by the damping ring components themselves. Two examples show that the field stability tolerance, which determines the stability of the magnet power supply, may depend strongly on the interconnection of the magnets.

The effect of optical mismatches on the emittance at the end of the linac is calculated analytically. The tightest tolerances on magnetic elements stemming from these effects are listed.

The phase tolerance is determined by the energy acceptance of the final focus system. It imposes tolerances to the integrated field strength of the damping ring and RTL bending magnets and the bunch compressor rf-phase.

In Section 3, measurements of injection jitter and the effect of betatron oscillations caused by changes of the angle or position of the incoming beam are described. These measurements were taken with BNS damping which relaxes certain tolerances by an order of magnitude.

In Section 4 injection jitter tolerances for a linac of the next generation are given. As an example, parameters for the Next Linear Collider[3] (NLC) being designed at SLAC are used.

2 SLC Injection Tolerances

Tolerance for dipole-like field changes – For a bunch population of $5 \cdot 10^{10}$ particles the requirement that emittance growth not exceed 25% limits the amplitude $\|x\|$ of jitter in the injected beam position x or angle x', normalized by the beam size $\sigma = \sqrt{\beta\epsilon}$, to [1]

$$\frac{\|x\|}{\sigma} \equiv \frac{\left[x^2 + (\beta x' + \alpha x)^2\right]^{\frac{1}{2}}}{\sqrt{\beta\epsilon}} \leq 0.016 \tag{2.1}$$

where α and β are the Twiss parameters and the beam is round with a (non-invariant) emittance $\epsilon \cong 6.6 \cdot 10^{-3}$ mm mrad. A change in any dipole field seen by the beam on its way from the damping ring to the linac will excite a β-tron oscillation with normalized amplitude

$$\frac{\|x\|}{\sigma} = \sqrt{\frac{\beta_{\rm dipole}}{\epsilon}}\,\Delta\theta \tag{2.2}$$

$\Delta\theta$ being the orbit deflection due to the dipole variation.

BNS damping would loosen this tolerance by an order of magnitude (see Section 3). But the tolerance for every single element would have to be tightened a factor of $\sqrt{n}$ to take into account the cumulative effect of n injection elements. For the SLC, the single component tolerances are calculated without BNS damping which is then expected to compensate for the cumulative effects. Table 1 lists stability tolerances for dipole magnets calculated using eqns. (2.1), (2.2). It shows two examples how hardware design, *i.e.*, interconnections of magnets, can affect power supply tolerances:

- The extraction septum, being a very strong magnet, has an unattainable tolerance if not powered in series with a bending magnet of equal strength located approximately 180° in betatron phase advance away. The new tolerance is then relieved by nearly an order of magnitude, but is still the tightest of the whole injection system.
- The RTL (ring-To-Linac) is a transport line built around an embedded achromat[4] The last bending magnet before the linac '*HB0*' is shared by positrons and electrons and was powered seperately whilst all the other bending magnets of the RTL's were powered in series. Powering the two RTL bends and the *HB0* magnet in series loosened the power supply tolerance by nearly two orders of magnitude.

Finally, trajectory correction coils with the tightest tolerances because of their high strength and $\beta_{\rm dipole}$ are listed.

Dipole magnets	$\frac{\Delta\theta}{\theta}$
Kicker	$1.90 \cdot 10^{-4}$
Extraction Septa *B-80*	$3.95 \cdot 10^{-6}$ $4.75 \cdot 10^{-6}$
Septa & *B-80* in series	$2.14 \cdot 10^{-5}$
RTL bend string *HB0*(last Bend in RTL)	$4.75 \cdot 10^{-6}$ $4.75 \cdot 10^{-6}$
HB0 & RTL bend string in series	$3.21 \cdot 10^{-4}$
Septum Backleg	$5.0 \cdot 10^{-5}$
XC 310/430 (RTL)	$4.0 \cdot 10^{-5}$

Table 1: Steering tolerances for important dipoles.

Quadrupole Steering Tolerances – Static trajectory displacements x, y in quadrupoles make the launch sensitive to fluctuations $\Delta f/f$ in integrated quadrupole strengths. To fulfill the requirements of the linac injection tolerance, the product of the trajectory offset and relative field change in the quadrupole has the upper limit:

$$\left|x\frac{\Delta f}{f}\right| \leq 1.30 \cdot 10^{-3}\,\mathrm{mm}\,\frac{\mathrm{m}^{-\frac{1}{2}}}{\sqrt{\beta}/f} \tag{2.3}$$

The tightest tolerance stems from the large offsets the extraction kicker produces on the last turn in the damping ring quadrupoles near the septum:

$$\frac{\Delta f}{f} \leq 3.32 \cdot 10^{-5} \text{ for QFI } (x \cong 12.8\,\mathrm{mm}) \tag{2.4}$$

$$\frac{\Delta f}{f} \leq 1.07 \cdot 10^{-4} \text{ for QDI } (x \cong 6.9\,\mathrm{mm}) \tag{2.5}$$

Vibration of the magnetic center of a quadrupole deflects the beam enough to violate the injection tolerance unless the vibration amplitude

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$$|\Delta x, y| \leq 1.30 \cdot 10^{-3}\,\text{mm}\,\frac{\text{m}^{-\frac{1}{2}}}{\sqrt{\beta}/f} \tag{2.6}$$

The strong 'matching' quadrupoles at the beginning of the RTL where the β function reaches 100 m have the tightest vibration tolerance:

$$|\Delta x, y| \leq 0.14\,\mu\text{m} \tag{2.7}$$

Dispersion mismatch - Dispersion in the beam injected into the linac is manifested after acceleration as an emittance increase

$$\frac{\Delta\epsilon}{\epsilon} = \left(1 + \frac{\|\eta\|^2 \langle\delta^2\rangle}{\beta\epsilon}\right)^{\frac{1}{2}} - 1 \tag{2.8}$$

where $\langle\delta^2\rangle^{\frac{1}{2}}$ is the initial fractional energy spread and the dispersion amplitude

$$\|\eta\| \equiv \left[\eta^2 + (\beta\eta' + \alpha\eta)^2\right]^{\frac{1}{2}} \tag{2.9}$$

An emittance blow up of $\Delta\epsilon/\epsilon \leq 0.12$ requires $\|\eta\| \leq 8\,\text{mm}$ (for $\langle\delta^2\rangle^{\frac{1}{2}} \cong 0.01$ and $\beta = 3\,\text{m}$, the design beam β-function at the injection point).

A focusing fluctuation $\Delta(1/f)$ in the RTL produces an invariant amplitude

$$\frac{\|\eta\|}{\beta^{\frac{1}{2}}} = \Delta\left(\frac{1}{f}\right)[\beta^{\frac{1}{2}}\eta]_{\text{quad}} \tag{2.10}$$

β-mismatch - Deviations in the geometry of phase space may be characterized by the invariant fractional 'β-beat' amplitude

$$\left\|\frac{\Delta\beta}{\beta}\right\| = \tag{2.11}$$

$$\left\{\frac{1}{4}\left[1 + \frac{\Delta\beta}{\beta} - \frac{1 + (\Delta\alpha - \alpha\Delta\beta/\beta)^2}{1 + \Delta\beta/\beta}\right]^2 + \left(\Delta\alpha - \alpha\frac{\Delta\beta}{\beta}\right)^2\right\}^{\frac{1}{2}}$$

The $\|\Delta\beta/\beta\|$ tolerance is entailed by the fact that a β-beat is ('Landau') damped after a phase advance $\Delta\psi$ in the linac at the expense of emittance growth through incoherent chromatic filamentation

$$\frac{\Delta\epsilon}{\epsilon} = \left[\left(1 + \left\|\frac{\Delta\beta}{\beta}\right\|^2\right)\left(1 - \langle\cos(2\Delta\psi_\delta)\rangle_\delta^2\right) + \langle\cos(2\Delta\psi_\delta)\rangle_\delta^2\right]^{\frac{1}{2}} - 1 \tag{2.12}$$

$\Delta\psi_\delta \cong -(2\pi{\cdot}33)\,\delta$ for the full linac, so $\langle\cos(2\Delta\psi_\delta)\rangle_\delta \approx 0$. An emittance blow up of $\Delta\epsilon/\epsilon \leq 0.12$ thus requires $\|\Delta\beta/\beta\| \leq 0.50$.

A quadrupole strength fluctuation $\Delta(1/f)$ in the RTL produces a β-beat

$$\|\Delta\beta/\beta\| \cong |\Delta\alpha| = |\beta\Delta(1/f)| \tag{2.13}$$

The magnet field tolerances imposed by these optical perturbation tolerances are all looser than 10^{-3}.

Injection phase tolerance - A change in the bunch compressor bend strength $\Delta\theta$ shifts the phase $\Delta\phi$ of the bunch relative to the linac rf—shifting the energy after acceleration by

$$\frac{\Delta E}{E} = -\tan\phi\,\Delta\phi = -\tan(\phi)\,k_L R_{56}\frac{\Delta\theta}{\theta} \tag{2.14}$$

where

$$\tan\phi = \frac{\sum\limits_{\textbf{klystrons}} V_k \sin\phi_k}{\sum\limits_{\textbf{klystrons}} V_k\cos\phi_k - E_{\text{lost}} + E_{\text{initial}}} \tag{2.15}$$

k_L is the linac rf circular wave-number, and $R_{56} = \int\eta\,d\theta$ is the shift in longitudinal position per shift in fractional energy deviation. Since $R_{56} = 603\,\text{mm}$, $k_L = .05986\,\text{mm}^{-1}$ and $\phi = 15°$

$$\frac{\Delta\theta}{\theta} \leq 3.2 \cdot 10^{-4} \tag{2.16}$$

to maintain $\Delta E/E \leq 3 \cdot 10^{-3}$ or $\Delta\phi \leq 0.64°$ to remain within the acceptance of the final focus. A shift in beam energy in the bunch compressor has an equivalent effect—thus precisely the same tolerance applies to the damping ring bending magnets.

3 Injection Jitter Measurements

The transverse position of an electron bunch was measured at two position monitors 20 m from the SLC injection point and 90° apart in betatron phase. Due to the phase difference the data represent x and x' displacements. Data were taken for both monitors on a single pulse and 100 consecutive pulses were recorded. A scatter plot is shown in Fig. 1. The total observed injection error is on the order of 100 μm (gaussian sigma, $\beta \approx 10\,\text{m}$). The correlation of the two positions indicates that the injection jitter occurs at a particular betatron phase at which there are several upstream magnetic components. Subsequent to these measurements, efforts have identified and eliminated some jitter sources.

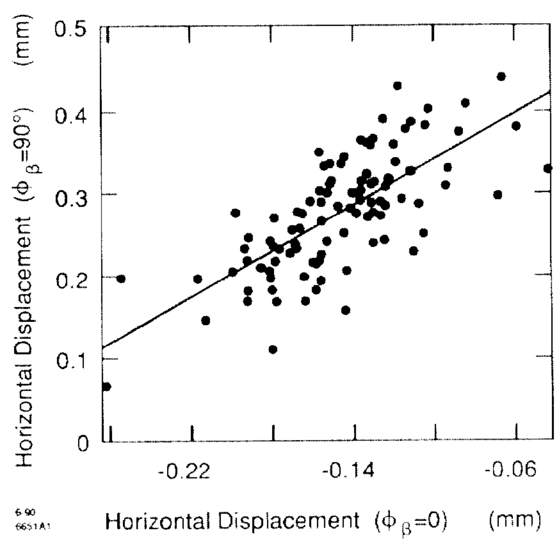

Figure 1: Position of 100 consecutive pulses in the x, x' space near the injection point in the linac.

The effects of injection jitter can also be seen in trajectory measurements. A single electron bunch was injected into the linac at 1.15 GeV and accelerated to 47 GeV. A dipole magnet early in the linac was varied to produce a betatron oscillation. The difference between the before and after trajectories is shown in Fig. 2. At a low bunch intensity($5 \cdot 10^9 e^-$) the oscillation damps down due to acceleration. At a higher intensity($3.5 \cdot 10^{10} e^-$) transverse wakefields drive the core and the tail of the bunch to larger offsets. The resulting oscillations at the end of the linac are larger even though the injection error is identical. The measurements shown here were taken using full BNS damping[5] to contain the transverse wakefields[6] Clearly, the allowed injection error depends on the beam intensity.

The injection displacement which causes the average beam position at the end of the linac to be equal to the beam size ($\sigma_x = 120\mu\text{m}$) can be calculated from the data in Fig. 2. Including data at other intensities these displacements are shown in Fig 3. On a log-log plot the data fall on a line very accurately and can be extrapolated to higher currents. For example, the displacement at $7.5{\cdot}10^{10} e^-$ must be less than 40μm, as compared to the injection beam size of 365 μm. These values are overestimates. The monitors measure the average beam position only. It is likely that the tail of the beam, from the wakefields, extends significantly beyond the core. Thus, the effective emittance has

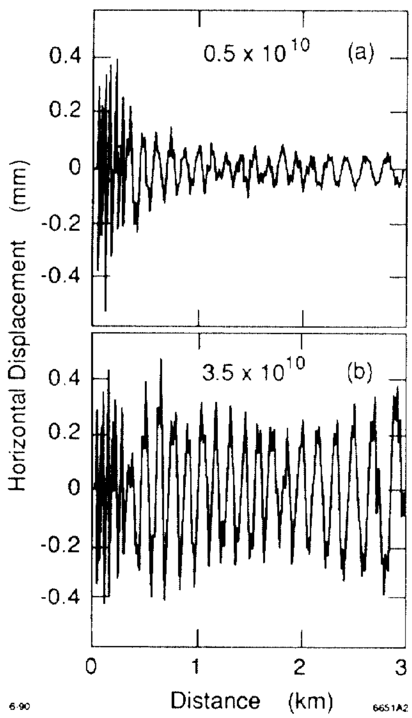

Figure 2: Difference orbit before and after changing a dipole in the beginning of the linac for two different bunch populations.

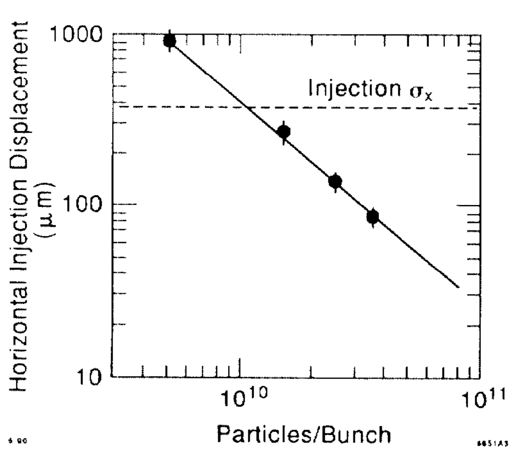

Figure 3: Injection tolerance vs. bunch population as measured in the SLC.

also grown and a smaller allowed displacement is required. A similar plot using measured emittances will soon be made.

4 Extrapolating for the Next Linear Collider

There are two main differences between the NLC and the SLC injection system as far as tolerances are concerned:

- Two bunch compression systems are needed to reach the required bunch length of 70μm. The first compression section is located right after the damping ring extraction. The bunch is then accelerated in a first linac (PL) to an energy of 17 GeV. It then enters the second bunch compressor and thereafter the main linac (ML). The second compression affects the phase stability requirements.
- The SLC operates with round beams, whilst the NLC demands low vertical emittances—tightening some optical tolerances in the vertical plane.

Injection Orbit – Given the choice of the multibunch mode ($N \lesssim 2.5 \cdot 10^{10}$ per bunch) the expected weakness of the intra-bunch transverse wakefield fixes the injection orbit tolerance at or near the limiting value (again for $\Delta\epsilon/\epsilon \leq .25$)

$$\frac{\|x\|}{\sigma} \leq \frac{1}{\sqrt{2}} \cong 0.71 \qquad (4.1)$$

determined by coherent chromatic filamentation.[7] The NLC emittances ($\epsilon_x = 850\,\mu$m μrad and $\epsilon_y = 8.5\,\mu$m μrad at 1.8 GeV) then entail $\|y\| \lesssim 3\,\mu$m (PL) and $\|y\| \lesssim 1\,\mu$m (ML) at $\beta \approx 2$ m.

Optical Perturbations – At the beginnings of both NLC linacs we expect $\langle\delta^2\rangle^{\frac{1}{2}} \approx .01$ and β to be a few meters. Then $\Delta\epsilon/\epsilon \leq .12$ requires

$\|\eta\|_x \leq 3$ mm, $\|\eta\|_y \leq 0.3$ mm at the pre-linac, and

$\|\eta\|_x \leq 1$ mm, $\|\eta\|_y \leq 0.1$ mm at the main linac

Phase Tolerances—Bunch Compressors – The acceptance of the final focus dictates that the energy error at the end of the linac $\Delta E/E \leq 10^{-3}$, or in terms of injection phase

$$\Delta\phi \leq (\tan\phi)^{-1}\, 0.057° \rightarrow 0.21° \qquad (4.2)$$

for $\phi = 15°$. We must choose $R_{56} = (\langle\tau^2\rangle/\langle\delta_0^2\rangle)^{\frac{1}{2}}$ in order to obtain a bunch length $\langle\tau^2\rangle^{\frac{1}{2}}$ given an initial energy spread $\langle\delta_0^2\rangle^{\frac{1}{2}}$ and so expect for the 2nd stage[8] $R_{56} \cong 50\,\mu\text{m}/10^{-3} = 50$ mm. Since $k_{ML} = 2\pi\,(11.4\,\text{GHz}) = 0.2394\,\text{mm}^{-1}$

$$\frac{\Delta\theta}{\theta} \leq (\tan\phi)^{-1}\, 0.835 \cdot 10^{-4} \rightarrow 3.1 \cdot 10^{-4} \qquad (4.3)$$

Precisely the same restriction must be placed on the fractional energy deviation at the start of the 2nd bunch compressor. Since the associated accelerating section must have a voltage obeying $eV/E = (k_C R_{56})^{-1} = 902\,\text{MeV}/16.2\,\text{GeV}$ for $k_C = 2\pi\,(17.13\,\text{GHz})$, its power source must have a stable rf phase up to

$$\Delta\Phi \leq (\tan\phi_{ML})^{-1}\, 0.086° \rightarrow 0.32° \qquad (4.4)$$

In addition the 1st stage initial linac phase must then satisfy

$$\Delta\phi \leq (\tan\phi_{PL})^{-1}(\tan\phi_{ML})^{-1}\, 0.0048° \rightarrow 0.07° \qquad (4.5)$$

The 1st stage bunch compressor has $R_{56} \cong 0.50\,\text{mm}/10^{-3} = 500$ mm and the linac has $k_{PL} = .05986\,\text{mm}^{-1}$, so its bending magnets, as well as those of the damping ring require

$$\frac{\Delta\theta}{\theta} \leq (\tan\phi_{PL})^{-1}(\tan\phi_{ML})^{-1}\, 2.79 \cdot 10^{-6} \rightarrow 3.8 \cdot 10^{-5} \qquad (4.6)$$

If its accelerating section rf source has $k_C = k_{PL}$, $\Delta\Phi \leq (\tan\phi_{PL})^{-1}(\tan\phi_{ML})^{-1}\, 0.0048°$ as for the linac.

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