BETATRON PHASE ADVANCE MEASUREMENT IN LEP

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Abstract When switching on the machine, the beam position monitoring system is critical to check the correctness of the optics, but requires a reasonable optics to be itself commissioned. At this stage, the measurement of the betatron phase advance can be a convenient tool. It allows an accurate check of the focusing even if the beam does not circulate more than one turn and if the measurements are noisy. The method used consists in comparing a measured and modelled beam trajectory (or closed orbit) following a transverse kick. The comparison is achieved by a cross-correlation, for optimal noise rejection; it requires a regular sampling of the betatron oscillation, which is suitable for the LEP arcs. The method proved to be precise to 0.1° in cell phase advance and contributed to the identification of a spurious field gradient during the LEP injection tests. The present LEP phase advances are found to agree well with the model.

Introduction

The LEP arcs are made of regular FODO cells. Besides each horizontally *defocusing* quadrupole, a beam position monitor is installed. The sampling of the beam trajectory is thus made at equidistant points in betatron phase advance and at constant amplitude function β.

Under these conditions, the trajectory excited either by injection errors or deliberately by powering an orbit corrector appears as an exact sine wave in the ideal linear optics:

$$z_i = A\sin\left[(i-1)\mu + \psi_0\right] \tag{1}$$

μ is the betatron phase advance per cell and ψ_0 an arbitrary initial phase.

The measurement of μ allows deducing the exact integrated strength of the focusing magnetic fields, foreseen or unexpected.

Measurement of μ by cross-correlation

The phase advance μ, analogous to a frequency, can be found by Fourier transforming the observed beam oscillation. The small number of observations (14 for the injection test) and the large phase shift between them (60°), would have given a poor resolution.

We have rather used a technique based on the cross-correlation, which takes advantage of the a-priori knowledge of the signal (a pure sine wave in our case).

Principle of the method

The method is easier to present in the special case of a continuous sinusoidal signal observed over several complete periods.

Let $z(\theta)$ be the model of the ideal betatron oscillation; θ is the betatron phase angle. Let $\tilde{z}(\theta)$ be the observed oscillation, which may have a different frequency:

$$z(\theta) = \sin\theta \qquad \tilde{z}(\theta) = A\sin[\theta(1+2\epsilon)] \tag{2}$$

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A measures the amplitude, $2\epsilon\theta$ the relative difference between the model and measured betatron phase advances.

The cross-correlation is given by:

$$C(\tau) = \frac{A}{T}\int_0^T \sin(\theta+\tau)\sin(\theta+2\theta\epsilon)d\theta \tag{3}$$

We assume that the difference between the model and the measurement is reasonably small and develop the integral to first order in $2\theta\epsilon$:

$$C(\tau) \approx \frac{A}{T}\left\{\cos\tau\int_0^T \sin^2\theta d\theta + \sin\tau\int_0^T \sin\theta\cos\theta d\theta + \right.$$
$$\left. + 2\epsilon\left[\cos\tau\int_0^T \theta\sin\theta\cos\theta d\theta + \sin\tau\int_0^T \theta\cos^2\theta d\theta\right]\right\} \tag{4}$$

which can be readily integrated:

$$C(\tau) \approx \frac{A}{T}\left\{\frac{T}{2}\cos\tau + 2\epsilon\left[-\frac{T}{4}\cos\tau + \frac{T^2}{4}\sin\tau\right]\right\} \tag{5}$$

The relative phase shift 2ϵ and amplitude A are given by:

$$2\epsilon = \frac{1}{T\sin\tau}\left\{\frac{C(\tau)-C(-\tau)}{C(0)}\right\} \qquad A = \frac{2C(0)}{1-\epsilon} \tag{6}$$

The calculation of only three points of the cross-correlation function $C(0)$, $C(\tau)$, $C(-\tau)$, provides a way to access the actual betatron phase advance and oscillation amplitude, provided the optics is close enough to its model. An iterative use of the method makes it general.

Cross-correlation of a sampled signal

In practice, the betatron oscillation is sampled at an arbitrary number of points n by the beam position monitors (PU's). Let z_i be the *model* of a betatron oscillation and $\Delta\theta$ the nominal phase advance between PU's:

$$z_i = \sin\left[(i-1)\Delta\theta + \psi_0\right] \tag{7}$$

Let the *measured* oscillation $\tilde{z}_i$ be different in relative phase advance by 2ϵ and in amplitude by A; the origins of the model and measured oscillations are the same, i.e. the position of the kicker magnet:

$$\tilde{z}_i = A\sin\left[(i-1)\Delta\theta(1+2\epsilon) + \psi_0(1+2\epsilon)\right] \tag{8}$$

The cross-correlation expression, analogous to (3), is written:

$$C(k) = \frac{A}{n-|k|}\sum_{i=a}^{b}\sin\left[(i-1)\Delta\theta + \psi_0 + k\Delta\theta\right].$$
$$.\sin\left[(i-1)\Delta\theta(1+2\epsilon) + \psi_0(1+2\epsilon)\right] \tag{9}$$

In order to retain the parity property of the cross-correlation function, the summation limits should be chosen:

$$\forall k \geq 0, \quad a = 1,\ b = n - k$$
$$\forall k \leq 0, \quad a = 1 - k,\ b = n$$

After converting the product of sines into a sum of cosines and summing the series, one finds equation 10. The terms within curly brackets $\{\ \dots\}$ are only to be considered for negative k.

As in the simple case, the zero shift cross-correlation provides a measure of the amplitude A perturbed by a non-vanishing ϵ: the left-right asymmetry is a measure of ϵ once A is known:

$$C(k) = \frac{A}{2(n-|k|)}\Big(\cos\left[k\Delta\theta - 2\epsilon\psi_0 - (n-|k|-1)\epsilon\Delta\theta + \{2k\epsilon\Delta\theta\}\right]\frac{\sin\left[(n-|k|)\epsilon\Delta\theta\right]}{\sin\epsilon\Delta\theta} - $$
$$- \cos\left[|k|\Delta\theta + 2\psi_0 + 2\epsilon\psi_0 + (n-|k|-1)\Delta\theta(1+\epsilon) - \{2k\epsilon\Delta\theta\}\right]\cdot\sin\left[(n-|k|)\Delta\theta(1+\epsilon)\right]\frac{1}{\sin\left[\Delta\theta(1+\epsilon)\right]}\Big) \tag{10}$$

$$A = 2nC(0)\Big(\cos\left[2\epsilon\psi_0 + (n-1)\epsilon\Delta\theta\right]\frac{sin[n\epsilon\Delta\theta]}{\sin\epsilon\Delta\theta} - \cos\left[2\psi_0 + 2\epsilon\psi_0 + (n-1)\Delta\theta(1+\epsilon)\right]\frac{\sin\left[n\Delta\theta(1+\epsilon)\right]}{\sin\left[\Delta\theta(1+\epsilon)\right]}\Big)^{-1} \tag{11}$$

$$C(1) - C(-1) = \frac{A}{n-1}\Big(\frac{\sin\left[(n-1)\epsilon\Delta\theta\right]}{\sin\left[\epsilon\Delta\theta\right]}\sin\left[\epsilon\psi_0 + (n-1)\epsilon\Delta\theta\right]\sin\left[\Delta\theta + \epsilon\Delta\theta\right] - $$
$$- \frac{\sin\left[(n-1)(\Delta\theta+\epsilon\Delta\theta)\right]}{\sin\left[\Delta\theta+\epsilon\Delta\theta\right]}\sin\left[2\psi_0 + 2\epsilon\psi_0 + (n-1)(\Delta\theta+\epsilon\Delta\theta)\right]\sin\left[\epsilon\Delta\theta\right]\Big) \tag{12}$$

Iterative algorithm

The phase advance per cell results mainly from two quantities which are accurately measured, namely the quadrupole gradients and the cell length. The expected discrepancy ϵ between model and measurement cannot be large. It is thus legitimate to develop the equations (11,12). The small parameter is $n\Delta\theta\epsilon$. (in the case of the LEP injection test, its value was of the order of 0.01, i.e. small compared to 1). It is further legitimate to neglect in (11) the terms dependent on $\Delta\theta\epsilon$ which are n times smaller than the small parameter. One obtains in this way a decoupled system of linear equations:

$$A = 2\,C_0\left(\frac{1}{1-k_1}\right) \tag{13}$$

$$2\epsilon = \frac{1-k_1}{(n-1)\Delta\theta\sin\Delta\theta + k_2}\left\{\frac{C(1)-C(-1)}{C(0)}\right\} \tag{14}$$

with

$$k_1 = \frac{\cos\left[2\psi_0 + (n-1)\Delta\theta\right]\sin n\Delta\theta}{n\sin\Delta\theta}$$

and

$$k_2 = 2\psi_0\sin\Delta\theta - \frac{\Delta\theta\sin\left[(n-1)\Delta\theta\right]\sin\left[2\psi_0 + (n-1)\Delta\theta\right]}{(n-1)sin\Delta\theta}$$

A first evaluation of (14) allows a readjustment of the model. A second evaluation yields the accuracy. One is thus naturally led towards an iterative algorithm that is stopped when the required accuracy on ϵ is reached.

Estimate of the accuracy

The method is primarily limited by the PU's resolution. Indeed a pure oscillation may only be obtained by subtracting two measurements: some kick is excited to perturb the beam trajectory; the orbit measurements are made before and after the perturbation and subtracted. Any constant offset on the PU's is thus suppressed. The ultimate accuracy may be evaluated as follows: even if the model and the measurement have the same parameters, the cross-correlation function will not be symmetrical due to random measurement errors. This bias will be converted by the algorithm into a phase advance error.

Let $<\delta z_{PU}>$ be the rms PU reading error. The accuracy of each point of the pure betatron oscillation is thus

$$<\delta z> = \sqrt{2}\ <\delta z_{PU}>$$

The cross-correlation between the model and the noisy signal is obtained by adding the term $\delta z_i/A$ to the last term of equation 9. The mathematical expectancy of the cross-correlation function shows no bias. Its rms error is obtained after the usual approximations of statistics:

$$<\tilde{C}(k)> \approx \frac{<\delta z_{PU}>}{\sqrt{n-|k|}} \tag{15}$$

Neglecting small factors in (11) and (12), one gets the accuracies:

$$\frac{<A>}{A} \approx \frac{2}{\sqrt{n}}\frac{<\delta z_{PU}>}{A} \tag{16}$$

$$<2\epsilon> \approx \frac{1}{\Delta\theta\sin\Delta\theta}\frac{2\sqrt{2}}{(n-1)\sqrt{n-1}}\frac{<\delta z_{PU}>}{A} \tag{17}$$

The denominator of (17) shows that the accuracy of this method improves very quickly with n, as compared with other techniques.

Under the conditions of the LEP injection test, i.e.

$$A \approx 15\text{ mm} \qquad \Delta\theta = \pi/3$$
$$n \approx 11 \qquad <z_{PU}> \approx 0.1\text{ mm}$$

equation (17) yields:

$$<\mu> \approx 0.05^\circ$$

For the LEP commissioning, the full octants become available, i.e. n=30, which yields:

$$<\mu> \approx 0.01^\circ$$

If the gradient changes from cell to cell, it introduces a β-beating which limits the accuracy. A phase advance modulation of 1 % would produce a β-beating of less than 1 %. For a betatron oscillation amplitude of 15 mm, this effect would double the quoted uncertainties. There are yet other sources of inaccuracy,such as the random errors of the PU's calibrations,... that are likely to limit the accuracy of the method.

Results of the LEP injection tests

The LEP injection test was carried out in a special optical configuration. The sextupoles were not powered and both quadrupole chains QF and QD were connected to the same power supply. Ignoring the very small contribution of the dipole magnets to the focusing, the horizontal and vertical phase advance should have been be the same, even if the quadrupole integrated gradients would be different from design. The nominal phase advance per cell was 60°.

In order to verify it, pure betatron oscillations were measured by subtracting two trajectories excited by an orbit corrector magnet: between the two measurements, the sign of the kick was reversed so as to maximize the accuracy for a given kick strength. This procedure cancels all systematic effects, either due to the imperfect closed orbit or to dc PU offsets.

About 12 measurements were made in each plane, to check the reproducibility and accuracy of the calculation. On each data set, the algorithm iterated typically four times. Averaging all the results obtained yields:

$$\mu_x \approx 58.48° \quad < \mu_x > \approx 0.12°$$

$$\mu_y \approx 61.79° \quad < \mu_x > \approx 0.20°$$

In fact, if one discards a few suspicious readings, the rms phase errors become respectively 0.07° and 0.13°, i.e. two to three times the theoretical prediction. The discrepancy may arise from a too tight estimate of the PU accuracy, a small non-reproducibility of the injection coordinates or from some ripple of the β function which has assumed to be the same at each PU.

The betatron phase shifts with opposite signs in the two planes were later attributed to a thin layer of magnetic material (Ni) used to bond the lead shield on the aluminium vacuum chamber.

Results of the LEP commissioning

An on-line program had been prepared to face commissioning difficulties, with some extensions to measure as well the betatron tunes (phase advance per turn) if the beam circulates for at least two turns. In fact, the commissioning of the LEP optics went smoothly. The betatron phase advance measurement revealed this time only small deviations from the design values (fig. 1).

The phase advance is found to vary from arc to arc by a small amount. This is not unlikely, as the parasitic gradient is expected to have some distribution due to the manufacturing tolerances of the vacuum chamber. Due to the operational tunes being slightly different from the theoretical ones, the average phase advances should have been:

$$\mu_x = 59.90° \qquad \mu_y = 59.87°$$

while the average measurements gave:

$$\mu_x = 59.92° \qquad \mu_y = 59.65°$$

The agreement in the horizontal plane is very good. The small discrepancy in the vertical plane (0.2°), if significant, shows that our optical model can be improved.

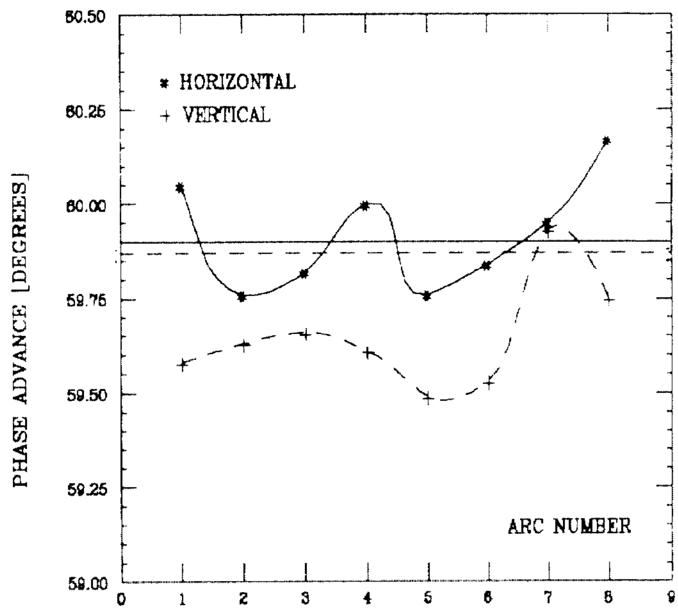

Figure 1: Measured and expected phase advance per cell in the LEP arcs

These measurements were carried out using the closed orbit instead of the betatron oscillation. The formula should then be slightly modified by a constant phase term: the pure closed orbit oscillation is indeed shifted in phase with respect to the orbit corrector used to produce the perturbation. Missing pick-up readings were merely assumed to give zero readings. A more elaborate treatment showed no significant change on the results.

Conclusion

The cross-correlation technique allowed to measure the phase advance per cell with an unexpected accuracy, given the small amount of data collected during the injection test.

Compared to other techniques like fitting or FFT followed by deconvolution, this method appears more economic in terms of computation, which may be of interest for control systems. It further directly yields an estimate of the accuracy. It however requires a regular sampling of the betatron oscillation, which is typically the case of the accelerator arcs.

The method can be extended to yield more insight into the optics:

- Cross-correlating a betatron oscillation observed over two successive LEP arcs allows the measurement of the betatron phase advance between the arcs, i.e. in the straight sections.
- Cross-correlating a betatron oscillation observed over two successive turns allows a fast measurement of the betatron tunes.
- The measurement can be repeated with an off-momentum beam, beyond the damping aperture. This should allow a more accurate measurement of the chromaticity.

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