THE DECAPOLE CORRECTION SCHEME FOR RHIC*

J. Claus, G.F. Dell, H. Hahn, G. Parzen, M.J. Rhoades-Brown and A.G. Ruggiero
Brookhaven National Laboratory
Upton, NY 11973 USA

Abstract

The systematic decapole error in the arc dipoles is expected to be $|b_4'| \leq 0.7$. However, the decapole tolerance has been determined to be $|b_4'| \leq 0.56$, indicating the need for a decapole corrector system in RHIC. A two-family corrector system situated next to the focusing and defocusing quadrupoles can reduce the systematic decapole error by a factor of about two and will be provided. If additional correctors are situated in the insertions, and optimally placed where β_x and β_y are equal, one can reduce the decapole error by a factor of ~ 20. A simpler yet adequate decapole corrector system is obtained by adding the insertion correctors while making the corrector strengths at the quadrupoles equal. This system requires only two independent power supplies but can reduce the decapole error by an order of magnitude.

Introduction

The systematic decapole error in the arc dipoles of the Relativistic Heavy Ion Collider (RHIC)[1] is expected to be $|b_4'| \leq 0.7$, where b_4' is the decapole field harmonic in primed units, i.e., at a distance from the magnet center of 2.5 cm, multiplied by 10^4: $b_4' = 10^4 \times b_4 \times (2.5\text{ cm})^4$. The design requirement of limiting the tune spread in the beam to $|\Delta\nu| < 3 \times 10^{-3}$ for the most severe operating conditions, i.e. gold beams at 30 GeV/u, established the decapole tolerance[2] to be $|b_4'| \leq 0.56$. Although not absolutely necessary, it would seem prudent to provide a decapole corrector system at RHIC. The planned corrector system will have two families of correctors situated next to the arc quadrupoles QF and QD. With this scheme a factor of about two reduction in the decapole error can be achieved.[3,4]

Recently, a novel decapole corrector system was discussed by Neuffer.[5,6] He proposed using three correctors, two situated next to the arc quadrupoles QF & QD, and one at the midpoint of the arc dipole, QC. With this three-corrector arrangement, a reduction in the decapole error by three orders of magnitude or more is claimed.[6] Because the dipoles at RHIC have an unbroken 9.45 m length in a half-cell, it would not be possible to implement directly this version of a three-corrector scheme.

In this paper, the decapole corrector scheme planned for RHIC is discussed and the effectiveness of a two-family scheme is compared with an alternative three-family scheme, which has the advantage that it can be added later on without disturbing the original magnet system configuration. The solution here proposed takes advantage of the fact that within the insertions, there is a region between quadrupoles Q8-Q9 where the lattice functions exhibit similar behavior as in the arc dipoles. Together with the original correctors at QF and QD, correctors in this region will be utilized to form a three-family corrector scheme for decapole errors. It will also be shown that by making the corrector strengths at QF and QD equal, an adequate corrector scheme for RHIC can be obtained that uses only two families of power supplies.

Tuneshifts Due to Decapole Errors

The first-order perturbative expression for the tune shift due to a decapole error in a magnet element is given by[7]

$$\Delta\nu_x^{(4)} = \Big\{2\delta^3\langle\beta_x X_p^3\rangle - \frac{3}{2}\delta\left(2\epsilon_y\langle\beta_x\beta_y X_p\rangle - \epsilon_x\langle\beta_x^2 X_p\rangle\right)\Big\} b_4 \quad (1)$$

$$\Delta\nu_y^{(4)} = \Big\{-2\delta^3\langle\beta_y X_p^3\rangle + \frac{3}{2}\delta\left(\epsilon_y\langle\beta_y^2 X_p\rangle - 2\epsilon_x\langle\beta_x\beta_y X_p\rangle\right)\Big\} b_4 \quad (2)$$

where b_4 is the systematic decapole error, δ is the momentum deviation, ϵ the beam emittance, and the average over the lattice functions $\langle\beta_x^l\beta_y^m X_p^n\rangle$ is defined by

$$\langle\beta_x^l\beta_y^m X_p^n\rangle = \frac{N}{2\pi\rho}\int \beta_x^l(s)\,\beta_y^m(s)\,X_p^n(s)\,ds \quad (3)$$

with the integral being evaluated over a single magnet element and N representing the total number of magnet elements of the same type.

Relevant contributions to the total tuneshift are made by the decapole error in the 144 arc dipoles, which will be corrected by the 72 each decapole correctors at QF and QD and the 12 optional decapole magnets at the points midway between the Q8 and Q9 insertion quadrupoles.

It is advantageous to rewrite the tuneshift equations in terms of the total emittance

$$\epsilon_T = \epsilon_x + \epsilon_y = \epsilon_T F + \epsilon_T(1-F) \quad (4)$$

with $0 \leq F \leq 1$, leading to the expression for the total tuneshift due to all magnet elements in the ring,[8]

$$\frac{\Delta\nu_x^{(4)}}{b_{4D}\delta\epsilon_T} = \sum_i \Big\{2\langle\beta_x X_p^3\rangle_i\frac{\delta^2}{\epsilon_T} - \frac{3}{2}\left(2(1-F)\langle\beta_x\beta_y X_p\rangle_i - F\langle\beta_x^2 X_p\rangle_i\right)\Big\} S_i \quad (5)$$

$$\frac{\Delta\nu_y^{(4)}}{b_{4D}\delta\epsilon_T} = \sum_i \Big\{-2\langle\beta_y X_p^3\rangle_i\frac{\delta^2}{\epsilon_T} + \frac{3}{2}\left((1-F)\langle\beta_y^2 X_p\rangle_i - 2F\langle\beta_x\beta_y X_p\rangle_i\right)\Big\} S_i \quad (6)$$

with S_i the decapole strength of a magnet element normalized to the dipole error

$$S_i = \frac{b_{4i}l_i}{b_{4D}L}$$

where $b_{4i}l_i$ denotes the integrated strength of a single magnet unit and $b_{4D}L$ the decapole error due to a single dipole. Table I gives the averaged lattice functions for the various magnet elements relevant to the decapole correction system.

The tuneshift is evidently odd in δ and linear in δ^2/ϵ_T and F which therefore are convenient independent variables in optimizing the correction system.

Two-Corrector Scheme

The RHIC dipole has been redesigned by increasing the space between coil and iron yoke to reduce the saturation-induced decapole term.[1] The remaining decapole error is small enough that operation is expected to be possible without correction. The most severe operating conditions in RHIC are for gold beams at 30 GeV/u when intrabeam scattering results in a momentum spread $\delta = \pm 0.55\%$ and an emittance of $\epsilon_T = 2\times10^{-6}$ m·rad after 10 h storage. The design requirement of limiting the tune spread

* Work performed under the auspices of the U.S. Department of Energy.

Table I: Averaged lattice functions for RHIC.

	Dipole	Corrector @ QF	Corrector @ QD	Corrector @ QC	Corrector @ Q8/9
$\langle\beta_x X_p^3\rangle$, m^4	34.9	82.2	2.1	24.92	33.0/12
$\langle\beta_x^2 X_p\rangle$, m^3	671.8	1622	30.8	466.4	775.8/12
$\langle\beta_x\beta_y X_p\rangle$, m^3	490.7	314.7	158.7	466.4	774.8/12
$\langle\beta_y X_p^3\rangle$, m^4	26.2	16.0	10.5	24.92	32.9/12
$\langle\beta_y^2 X_p\rangle$, m^3	559.4	61.1	818.5	466.4	773.8/12

in the beam to $|\Delta\nu| < 3\times10^{-3}$ established the decapole tolerance to be $|b_4'| < 0.56$. The systematic decapole error is expected to be less than 0.7, suggesting that only a minimal correction system, such as a two- corrector scheme with a corrector near each of the arc quadrupoles, is required.

In the two-corrector scheme, optimal settings for the correctors are obtained by requiring that the tuneshift be independent of F, i.e. $\partial_F(\Delta\nu_x) = 0$ and $\partial_F(\Delta\nu_y) = 0$ resulting in

$$\sum_{i=QF,QD}\left\{2\langle\beta_x\beta_y X_p\rangle_i + \langle\beta_x^2 X_p\rangle_i\right\}b_i l_i = -\left\{2\langle\beta_x\beta_y X_p\rangle_D + \langle\beta_x^2 X_p\rangle_D\right\}b_{4D}L \quad (9)$$

$$\sum_{i=QF,QD}\left\{2\langle\beta_x\beta_y X_p\rangle_i + \langle\beta_y^2 X_p\rangle_i\right\}b_i l_i = -\left\{2\langle\beta_x\beta_y X_p\rangle_D + \langle\beta_y^2 X_p\rangle_D\right\}b_{4D}L \quad (10)$$

Note that these conditions are independent of δ and ϵ_T. Using the numerical values in Table I leads to

$$b_{4QF}l = -0.58\times b_{4D}L$$
$$b_{4QD}l = -1.00\times b_{4D}l$$

In Fig. 1 the corrected tuneshift due to a decapole error in the arc dipoles and its reduction by a two-corrector system using these corrector settings is compared. The tuneshift is plotted in terms of a dimensionless variable η representing the ratio of momentum to betatron amplitude

$$\eta = \left(\frac{X_p\delta}{\sqrt{\beta\epsilon_T}}\right)_{QF} \quad (11)$$

evaluated at the focusing quadrupoles, where $X_p = 1.6$ m and $\beta = 50$ m. The maximum tuneshift is for Au-beams at 30 GeV/u which corresponds to $\eta\approx1$. From Fig. 1 follows that a two-corrector scheme will reduce the maximum tuneshift by a factor of two which in view of the above comments is considered adequate and it will be provided on day-one.

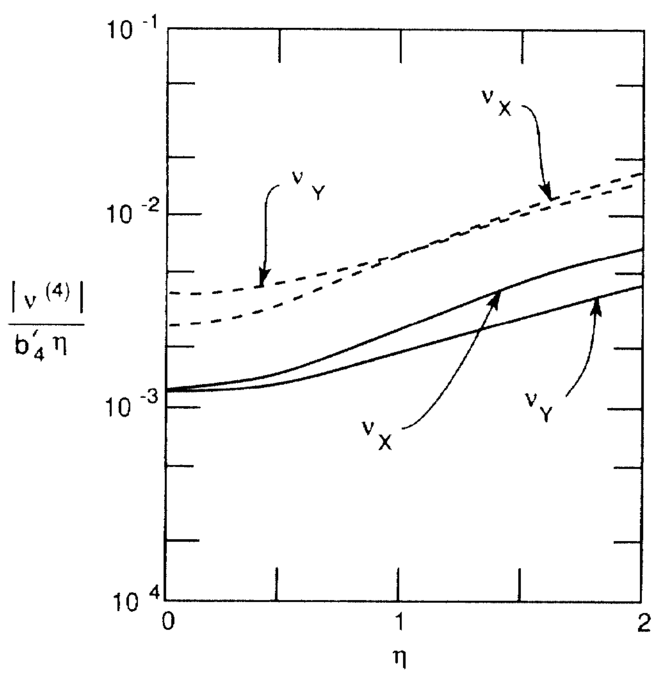

Fig. 1: Normalized horizontal and vertical tuneshifts $\Delta\nu^{(4)}/b_4'\eta$ as a function of the momentum/betatron amplitude ratio η. The corrected tuneshifts (solid curves) are shown for a two-corrector scheme and compared with uncorrected values (dashed curves).

Three–Corrector Scheme

It is recognized that the introduction of a third corrector family will result in a significant improvement of the decapole correction scheme. The ideal location for the third corrector is the half-cell midpoint. At RHIC, the arc dipole is an unbroken 9.45 m long, so this scheme is not appropriate and it is necessary to place the third corrector family in the Q8–Q9 driftspace at each end of the arcs, where β_x equals β_y. As seen from the entries in Table I, the Q8/Q9 location is in first–order equivalent to the half-cell midpoint. However it is important to note that each corrector in the insertion must compensate for the decapole error in twelve dipoles thus requiring a correspondingly increased strength. A potential further disadvantage could be the excitation of 5th order resonances which would require compensating decapole magnets in the insertion at locations with zero dispersion.

The corrector settings can be optimized to satisfy different objectives. A unique solution is obtained by requiring the corrector strengths to be independent of the beam dimensions which in RHIC will vary with ion species and operating energy. The corrector settings are then determined by requiring the tuneshifts to be independent of F leading to a set of constraints as in the two–corrector scheme but now with i = QF, QD, Q89. In addition, the condition that $|\Delta\nu_x^{(4)}| = |\Delta\nu_y^{(4)}|$ for all δ is imposed leading to the additional constraint

$$\sum\left\{\langle\beta_x X_p^3\rangle_i - \langle\beta_y X_p^3\rangle_i\right\}S_i = -\left\{\langle\beta_x X_p^3\rangle_D - \langle\beta_y X_p^3\rangle_D\right\} \quad (12)$$

With the values in Table I, the following corrector settings are found

$$b_{4QF}l = -0.152\ b_{4D}L$$
$$b_{4QD}l = -0.159\ b_{4D}L$$
$$b_{4Q89}l = -12\times0.540\ b_{4D}L$$

Using these corrector settings a correction factor of ~ 20 is obtained. The optimum settings are close to the Simpson's Rule values (1/6, 2/3, 1/6), which would yield a correction factor of less than 10. A study is in progress to determine the source of this smaller than expected correction factor. However, the results emphasize the sensitivity of the three-corrector scheme and suggests the use of a more modest but simpler correction scheme.

Modified Three–Corrector Scheme

A simplified, yet adequate, corrector scheme may be readily found by equating the currents in the QF and QD correctors, which has the virtue of only requiring two families of power supplies. The required corrector strengths are again determined by requiring the tuneshifts to be independent of F. The resulting corrector strengths follow as

$$b_{4QF}l = b_{4QD}l = -0.144 \times b_{4D}L$$
$$b_{4Q89}l = -12 \times 0.550 \times b_{4D}L$$

A theoretical correction factor of about 16 is obtained. The corrected tuneshift is also less sensitive to variations in the corrector strengths than the full three-corrector scheme. The reduction in tuneshift calculated here would be more than adequate for any foreseeable error in RHIC. In fact, using the insertion correctors alone with a single power supply provides a theoretical reduction of the tuneshift by a factor of 5. The addition of warm corrector magnets at the Q8/9 location represents thus an attractive future option.

Comparison with Tracking Studies

The preceding discussion of a decapole correction scheme for RHIC is based on a first-order analytical approximation to beam-dynamics. A comparison of the analytical results with tracking studies is clearly indicated in order to determine the importance of second-order effects such as the momentum-dependence of the lattice functions.

The maximum tuneshift due to a decapole error in the dipole of $|b_4'| = 0.7$ obtained by tracking is shown in Fig. 2 for Au-beams at 30 GeV/u. The following comments on the tracking results can be made:

1. The tuneshift from tracking agrees with the analytical results in the absence of correctors.
2. The tuneshift reduction with correctors at QF and QD is at least a factor 2, again in agreement with theory.
3. The tuneshift reduction with correctors in the insertion and at QF/QD is smaller than the theoretical prediction by a factor of about 2. This difference is, at least in part, due to the limited precision in the determination of tune by tracking. A further contributing factor is the momentum dependence of the lattice functions: the corrector setting is not optimal for off-momentum particles and the effectiveness of the correction system is reduced.

In conclusion, it can be stated that the tracking results confirm the viability of the planned RHIC decapole correction system with correctors at QF and QD and optional correctors in the insertions.

References

1. *RHIC Conceptual Design Report*, May 1989. BNL 52195,
2. G.F. Dell, H. Hahn, G. Parzen, M. Rhoades-Brown, A.G. Ruggiero, *Tolerances on Systematic Field Errors in RHIC Magnets*, RHIC Technical Note AD/RHIC-58.
3. H. Hahn, Report AD/RHIC-22 (1987).
4. G.F. Dell, H. Hahn, G. Parzen, A.G. Ruggiero, *RHIC Decapole Correction Magnet Requirements*, RHIC Technical Note AD/RHIC-28.
5. D. Neuffer, *Multipole Correction in Synchrotrons*, Proc. 1989 IEEE Accelerator Conference, Chicago, p. 1432.
6. D. Neuffer, *A Solution to Nonlinearity Problems*, Particle Accelerators, **27**, 209 (1990).
7. A. Jackson, *Tuneshifts and Compensation from Systematic Field Components*, Report SSC-107 (1987).
8. M.J. Rhoades-Brown, H. Hahn, *Analytic Study of Decapole Correction Scheme*, RHIC Technical Note, RHIC-66.

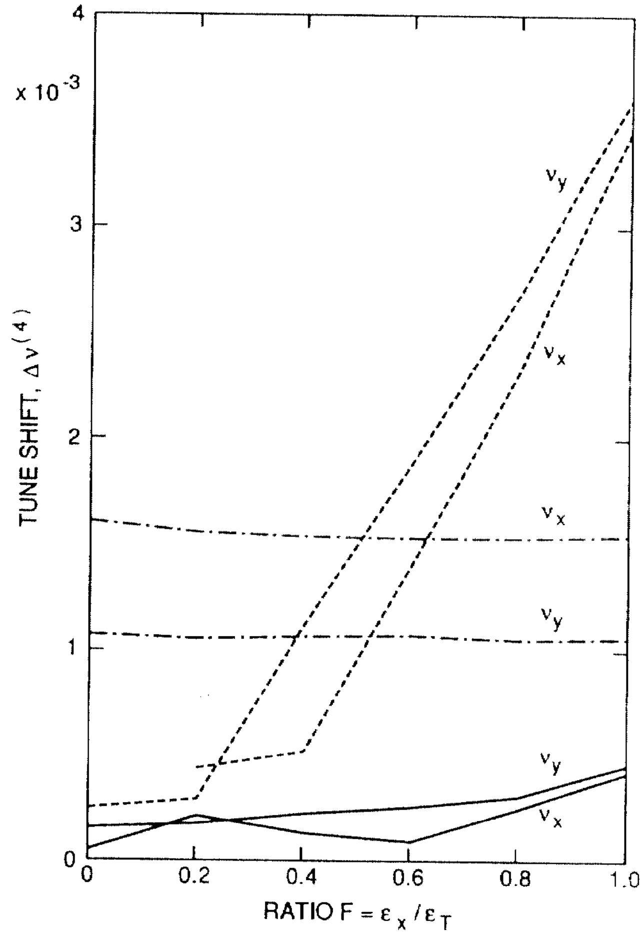

Fig. 2: The maximum tuneshift due to a decapole error in the arc dipole of $|b_4'| = 0.7$ for Au-beams at 30 GeV/u. The results were obtained with the *PATRICIA* tracking code for the cases of 1. no decapole correction (dashed curves), 2. a two-family system with correctors at QF and QD (dot-dashed curves), 3. a two-family system with correctors in the insertion Q89 and at QF & QD (solid curves).