NONLINEAR EFFECTS OCCURING DUE TO FRINGE FIELDS OF CYCLIC ACCELERATORS DIPOLES

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1. This paper presents the results of investigation on a beam dynamics in cyclic accelerators and storage rings with the regard to the dipole fringing fields. Here the Hamiltonian formalism is used, the disturbance occurs due to the fringing field. The Hamiltonian of disturbance written down in the beam-concomitant coordinate system has the form [1]:

$$H_1 = H - H_0 = \frac{R^2}{B} A_\vartheta(x'z'\vartheta') \frac{R}{B} [p_x A_x(x'z'\vartheta') + p_z A_z(x'z'\vartheta')] + \dots, \qquad (1)$$

where x', p_x, z', p_z are the transverse coordinates and momenta; $\vartheta = S/R$ is the azimuthal angle; R is the average machine radius; $B\rho$ is the magnetic rigidity of a particle; $A_{x,z,\vartheta}(x', z', \vartheta')$ are the components of magnetic vector potential of disturbance (fringing field in this case) in the above beam-concomitant coordinate system. The Hamiltonian (1) is written down on the assumption that $|x|, |z| \ll \rho$.

2. Let us consider as a model a flat magnetic dipole in which the vertical field component $B_z(z,s)$ has the properties:

$$\left.\frac{\partial B_z(z,s)}{\partial z}\right|_{s=0} = \left.\frac{\partial B_z(z,s)}{\partial x}\right|_{s=0} = 0; \qquad (2a)$$

$$B_z(z,s) = B_z(z,-s) = B_z(-z,s) = B_z(-z,-s). \qquad (2b)$$

Eqs. (2a) are related to the homogenity of the field inside the dipole and Eqs.(2b) to the existence of the symmetry plane (median plane) and the mirror symmetry of entrance ($s<0$) and exite ($s>0$) edges.

Using the conditions (2) and to separate the variables we expand B_z into even powers of z:

$$B_z = B_0 \sum_k b_{2k}(s) z^{2k}, \qquad (3)$$

where B_0 is the field in the dipole gap $B_0 = B_z(s=0)$.

It follows from conditions $\mathrm{rot}\vec{B}=0$, $\mathrm{div}\vec{B}=0$ that the coefficients $b_{2k}(s)$ can be expressed in terms of derivations of this field on the median plane $b_0(s)$ about the longitudinal coordinate s; thus the magnet potential components within the "magnet" frame (x, z, ϑ) obtain the form:

$$\begin{aligned} &A_z = A_\vartheta = 0; \\ &A_x = B \sum_k \frac{(-1)^{k+1}}{(2k)!} b_0^{(2k-1)}(\vartheta) z^{2k}, \end{aligned} \qquad (4)$$

where $b_0^{(2k-1)}(\vartheta) = d^{2k-1} b_0(\vartheta)/d\vartheta^{2k-1}$

To investigate the motion described by the Hamiltonian (1), the components of vector $\vec{A}$ are presented in the beam-concomitant coordinate system moving along the trayectory of the central particle (x', z', s' or ϑ') which is tilted by the angle $\alpha(s)$ to the "magnet" frame (x, z, s or ϑ): $\alpha(s) = B_0/B\rho \int_s^\infty b_0(s)\, ds + \alpha_0$, (here α_0 is the face angle of the magnet. $B = B_0(1 + \delta E/E)$, $\delta E/E$ is the energy dispersion, ρ is the radius of orbite curvature inside the dipole. At small $\alpha(s)$ the components of $\vec{A}$ in concomitant coordinate system has the form:

$$\begin{aligned} &A_{x'} = B_0 \sum_k \frac{(-1)^{k+1}}{(2k)!} b_0^{(2k-1)}(\vartheta) z^{2k}; \\ &A_{z'} = 0; \\ &A_{\vartheta'} = A_x \cdot \alpha(\vartheta). \end{aligned} \qquad (5)$$

Difference between the concomitant and "magnet" coordinates is shown in fig.1.

So, the expression for the magnet potential in concomitant system are obtained. The appearence of $A_{\vartheta'} \neq 0$ is equivalent to the appearence of B_x at the magnet edges directed collinearly with equal z coordinate.

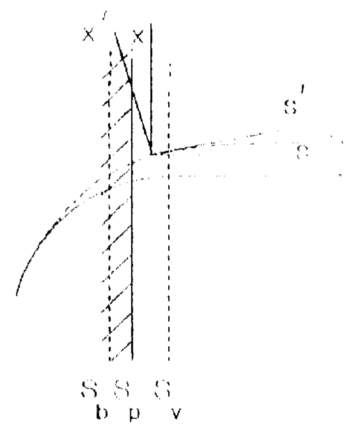

Fig.1. Orbits of equilibrium particle near the magnet exit angle. S_b: the calculate onset of fringing field. S_p: the pole edge boundary. S_v: the field virtual edge boundary.

3. After common used transfer to the new canonical variables a_y, a_y^* (see [1]),($y = x$ or z) and considering the vertical beam shift about the median plane by $z = z + z_0$, Fourier expansion of dependent on ϑ terms, summing over all $2M$ magnet edges under assumption that the Floquet function is constant along the edge field, separation of amplitudes and phases in a_y

$(a_y = r_y \exp(i\varphi_y);\ \dot{a}_y = r_y \exp(-i\varphi_y))$ we get the new Hamiltonian function (6). It consists of two parts, one of them (the stabilising term) does not depend on the azimuth , while another (the resonant one) depends on it:

$$U_{st} = \frac{R^2 B_0}{B\rho} \sum_{k=1}^{\infty} \sum_{j=0}^{2k} \sum_{m=1}^{2M} \frac{(-1)^{k+1}}{(2(k-j))!(j!)^2} z_0^{2(k-j)} |\Psi(\vartheta_m)|^{2j} r_z^{2j}$$
$$\frac{1}{2\pi}\int_0^{2\pi} \alpha(\vartheta) b_0^{(2k-1)}(\vartheta)\, d\vartheta; \tag{6a}$$

$$U_{res} = \frac{R^2 B_0}{B\rho} \sum_{k,m} \sum_{l=0}^{2k} \sum_{j=0}^{l} \sum_{p=-\infty}^{\infty} \frac{(-1)^{k+1}}{(2k-l)!(l-j)!j!} z_0^{2k-l} |\Psi(\vartheta_m)|^l r_z^l$$
$$\frac{1}{2\pi}\int_0^{2\pi} \alpha(\vartheta) b_0^{(2k-1)}(\vartheta)\, d\vartheta\, e^{i(((2j-l)\nu_z + p) + \Phi_{jlm})}, \tag{6b}$$

where $\Phi_{jlm} = (2j-l)(\arg\Psi(\vartheta_m) + \varphi_z - p\vartheta_m)$, $|\Psi(\vartheta_m)|$ is the module of the vertical Floquet function at m-th cut end, ν_z is the vertical tune. It may by shown that the terms responsed to $p_x A_x$ can excite the $2n\nu_z \pm \nu_x = p$-type resonances (n and p are integers), which usually are too weak becouse their dependence on the difference between the lattice functions at entry and exite ends of a dipole. We do not consider terms k=0 in (6) becouse these terms respound only for trajectory curvature in magnetic field.

Stabilising part of the Hamiltonian U_{st} determines the dependence of betatron tunes on amplitude [2,3]:

$$\nu_x = \nu_{x0};$$
$$\nu_z = \nu_{z0} + \frac{R^2 B_0}{B\rho} \sum_{k,j} \frac{(-1)^{k+1}}{(2(k-j))!(j!)^2} r_z^{2(j-1)} z_0^{2(k-j)} J_{kj}, \tag{7}$$

where $J_{kj} = \frac{1}{2\pi} \sum_{m=1}^{2M} \frac{\beta_z^j(\vartheta_m)}{(2R)^j} \int_0^{2\pi} b_0^{(2k-1)}(\vartheta)\alpha(\vartheta)\, d\vartheta$, $\beta_z(\vartheta_m)$ is the amplitude function value at m-th edge of dipole.

Resonant part of Hamiltonian (6) U_{res} indicates on the possibility to excite the $n\nu_z = p$-type resonance. When k = 1, j = 1 the expression (7) describes the well-known fact of linear diminishing in ν_z [4].

Thus we have shown, that curvature of particles trajectories at the dipole edges give rise to the nonlinear component of B_x field in concomitant coordinate system and, as a result, to the resonant conditions (with pulsing beam dimensions) as well as to the shift of vertical betatron tune ν_z.

4. To use the results obtained in modelling codes the fringing field can be considered as a thin lens approximation and the motion equations from (1) after integration over the fringing field length have obtain got the form at $z_0 = 0$:

$$v_x = v_{x0};\quad v_z = v_{z0} + \frac{B_0}{B\rho} \sum_k \frac{(-1)^k}{(2k-1)!} z^{2k-1} \int_0^{\infty} b_0^{(2k-1)}(s)\alpha(s)\, ds, \tag{8}$$

where $v_y = dy/ds$.

This expression describes both the enter and exit edges of the magnet.

5. While choosing the function $b_0(s)$ it is of importance to take into account the following conditions:

-homogeneity and continuity of this function over the integration lenght;

-satisfaction to the boundary conditions $b_0(s)|_{s=0} = 1$; $b_0(s)|_{s\to\infty} \to 0$

- $\mathrm{sign}(b_0^{(2k)}(s)) = -\mathrm{sign}(b_0^{(2k+1)}(s))$.

The simplest function which satisfies these conditions and allows one to describe the effects considered is the exponent as follows:

$$b_0(s) = \begin{cases} 1, & \text{if } |s| < |S_b|; \\ e^{-\frac{|s-s_b|}{\sigma}}, & \text{if } |s| > |S_b|. \end{cases} \tag{9}$$

The parameter σ can be evaluated by using a MLS method taking into account the conditions of field integral (virtual border of field) invariance. We have carried out the experiments to measure the amplitude-freguency response of beam transverse oscillation in the N-100 storage ring. A comparison between theory and experiment shows a good agreement. The field of N-100 dipoles has been described by the function (9) with $S_b = 0$; $\sigma = 2.43$ cm. For this function $b_0(s)$ the tune shift ν_z is determined by expression:

$$\nu_z = \nu_{z0} + \frac{RB_0}{2\pi B\rho} \sum_k \frac{(-1)^{k+1}}{k!(k-1)!} r_z^{2(k-1)} \sum_m \left| \frac{\beta_z(\vartheta_m)}{2R} \right|^k \sigma^{2-2k} (\alpha_0 - \frac{B_0 \sigma}{2 B\rho}). \tag{10}$$

As it is obvious when taking the positive tilt angle $\alpha_0 = \sigma/2\rho$ for the monoenergetic beam one can compensate the effects considered. The physical meaning of it is that by obtaining the appropriate title angle to compensate the well-known linear defocusing [4], the nonlinear edge effects are also suppressed. For the expression (8) in the exponential fringing field model the v_z is:

$$v_z = v_{z0} + \frac{B_0}{2 B\rho} \sum_k \frac{(-1)^k}{(2k-1)!} \sigma^{2-2k} (\alpha_0 - \frac{B_0 \sigma}{2 B\rho}) z^{2k-1} \tag{11}$$

Linear decreasing of the vertical focusing force (k=1 in (10,11)) with good precision is equal to the quantity determined by well-known expressions [4] being applied in the codes [5].

6. The main results of the paper are as follows:

-The expression describing the magnet vector potential $\vec{A}$ of the dipole magnet fringing field is presented; it is shown that for the flat dipole this field is determined by the function describing the vertical field component on the median plane.

-The tipes of resonances excited by fringing fields are determined; the expression for nonlinear tune shifts as well as ones describing these effects in the thin lens approximation are presented;

-It is shown that edge effects are due to the tilt of particle trajectory with respect of the perpendicular to the dipole edge.

-It is shown that nonlinear effects can be compensated by small positive tilts of edges.

Theoretical predictions are proved by experimental measurements.

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