Studies on Linear and Dynamic Apertures in the LHC

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Abstract Installing the superconducting magnets of a large collider in ordered sequences can improve the stability of the transverse motion of the circulating particles, provided that an approximate compensation of the random magnetic imperfections along the ring can be obtained. In spite of the additional difficulties introduced by the two-in-one design of the magnets, solutions have been found for the LHC, which reduce the effects of the random sextupole components and allow an increase of the apertures in both the counter-rotating beams. To study this, computer simulations have been performed, for on- and off-momentum particles, on a realistic LHC lattice, including the finite closed orbit due to alignment errors, the systematic imperfections of the main magnets, and the lumped multipolar correctors in the regular cells.

Lattice with Imperfections

The LHC lattice used here is that of Ref. [1], retuned at $Q_x = 70.28$, $Q_y = 70.31$. It is made of eight arcs and eight insertions, each of them including two dispersion suppressors and one long straight section. An arc contains 49 regular half-cells with four dipoles each, and a dispersion suppressor contains four pseudo-half-cells with three dipoles each. Therefore there are 1760 dipoles in total, and all of them are 9.54 m long. There are 384 quadrupoles in the regular cells, split in two families, all of them 3.08 m long. The insertion quadrupoles, 160 in total, are of special design and lengths. Sextupoles, powered in two families, are included to correct chromaticity and are placed next to the main quadrupoles.

The injection optics considered consists of four insertions with β^* values fixed at $\beta^*_x = \beta^*_y = 6.5$ m, placed in the even straight sections, and four insertions with $\beta^*_x = \beta^*_y = 4$ m at the interaction point in the odd straight sections.

The collision optics, instead, is non-symmetric, with two different interaction points: one with $\beta^* = 0.25$ m in insertion No. 1, and the other with $\beta^* = 0.5$ m in insertion No. 5.

Crossing angles varying from 96 to 300 μrad are considered for both injection and collision optics; for this reason the particle trajectory in the inner triplets of quadrupoles is displaced off-axis.

The r.m.s. values of the random magnetic imperfections assumed in each dipole and quadrupole are given in Table 1. We disregarded the normal and skew quadrupole components, and we assumed the IR-triplets to be largely compensated by bore-tube windings up to the dodecapole term.

The values of systematic multipolar coefficients considered in the dipoles are reported in Table 2. To compensate their effect we used lumped correctors up to decapoles located in the regular cells close to the main quadrupoles and half-way between them: the setting of their strengths was fixed according to the so-called Simpson method of Ref. [2]. The systematic dodecapole imperfections in the quadrupoles are assumed to be corrected by bore-tube windings; the higher-order components are neglected.

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Order	Dipole		Quadrupole		IR–Triplet	
	b	a	b	a	b	a
1	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0
3	1.5	0.5	2.28	2.10	0.1	0.1
4	0.15	0.2	0.34	0.72	0.1	0.1
5	0.2	0.07	0.16	0.17	0.1	0.1
6	0.0	0.0	0.12	0.06	0.1	0.1
7	0.02	0.04	0.02	0.02	0.022	0.022
8	0.0	0.0	0.0	0.0	0.016	0.016
9	0.005	0.002	0.007	0.006	0.013	0.013
10	0.0	0.0	0.005	0.004	0.009	0.009

Table 1: R.M.S. Coefficients of Random Multipolar Errors (in units of 10^{-4} at $R_r = 1$ cm)

Systematic multipoles in the dipoles	b_3	b_4	b_5	b_7	b_9
	-4.05	± 0.05	0.56	0.13	0.02

Table 2: Multipolar Coefficients for Systematic Errors (in units of 10^{-4} at $R_r = 1$ cm) at injection ($B_o = 0.56$ T)

For the misalignment and the field errors we assumed the values listed in Table 3, based on the experience of LEP.

	Dipoles	Quadrupoles
Horizontal misalignment	0.14 mm	0.14 mm
Vertical misalignment	none	0.14 mm
Tilt (DPSI)	0.24 mrad	0.24 mrad
Relative field error	5×10^{-4}	5×10^{-4}

Table 3: R.M.S. of Misalignment and Field Errors

The average closed-orbit distortion and the related maximum excursion of the particle trajectory, after correction in both planes are:

$$X_{rms} = 0.26 \text{ mm}, \quad X_{peak} = 1.2 \text{ mm},$$
$$Y_{rms} = 0.25 \text{ mm}, \quad Y_{peak} = 1.1 \text{ mm}.$$

These values were obtained by averaging over 10 different distributions of the errors of Table 3.

Ordering Strategy

On the assumption that the random multipole components of the magnetic imperfections do not depend on the manufacturing sequence, we aim to order the LHC magnets in such a way that the magnetic errors are approximately self-compensated in the two rings.

The ordering rules are based on the following very demanding hypotheses:

1. all the main dipoles as well as the main quadrupoles are of a modular design and are thus interchangeable;

2. at least the sextupolar component of the magnetic field in the main dipoles and quadrupoles is known with high precision;

3. all the magnets of a full octant are available at a time in a large storage area, and can be installed in the LEP tunnel in any possible sequence.

Each octant will be separately ordered, thus 220 dipoles, 24 QF and 24 QD cell-quadrupoles will be concerned at a time. The magnets will be ordered according to the value of their sextupolar random error. The compensation is required to be as local as possible. The basic rules are:

1. two consecutive dipoles have approximately equal and opposite sextupolar content;

2. two dipoles 2π apart in phase have approximately equal and opposite sextupolar content;

3. dipoles without a partner 2π apart in phase have approximately zero sextupolar content;

4. QF (QD) quadrupoles 2π apart in phase have approximately equal and opposite sextupolar content.

To fulfil these rules the dipoles and the quadrupoles are classified in families depending on the value of the sextupole error in the two magnetic channels (two-dimensional binning). There are 16 families of dipoles, four of QF, and four of QD. The particular sequence in which the magnets are extracted has been optimized in Refs. [3] and [4] by a "trial and error" method. Considerations that support our choice of the ordering rules are presented in Ref. [5].

The insertion quadrupoles are assumed to be non-interchangeable, and are thus in random sequence. The separating dipoles near each crossing point are assumed to be perfect.

Computational Tools

The particle-tracking simulations were carried out along the test lattice with imperfections using the code FASTRAC [6] to identify the dynamic aperture of a round beam over 400 revolutions. The "linear aperture" based on a given maximum variation of the horizontal and the vertical Courant–Snyder invariant (smear) was also determined; in each plane the threshold was chosen at 10% r.m.s. relative variation.

No physical aperture limit was used around the beam axis. Appropriate routines to generate samples of random errors and to assign them to the dipoles and quadrupoles in a given ordered sequence, were coded and included in FASTRAC. The random components considered were set with random Gaussian distributions centered around zero and truncated at $\pm 3\sigma$. Random multipole components of different orders, or located in different rings, were supposed to be statistically uncorrelated. To improve the statistical meaning of our results we considered 20 different samples of the random multipoles components, generated with 20 different seeds in the random number generator routine. Only 10 seeds were considered when also closed-orbit distortion was included. Tracking for off-momentum particles was done with fixed $\Delta p/p$, excluding synchrotron oscillations. At injection we chose $\Delta p/p = \delta = 1.25 \times 10^{-3}$, equivalent to the bucket half-height. The magnetic imperfections in each dipole and quadrupole were represented by a set of thin-lens multipoles, all located in the middle of the magnetic length.

The dynamical quantities of interest were evaluated by the average value and the spread over the 20 (10) samples, expressed in mm and normalized at $\beta = 1$ m.

The values of the "ensured linear aperture", namely the lower edge of the spread of the linear aperture over the 20 (10) samples, is considered to be the best indicator of the detrimental effect of the non-linearities. Alternatively, the value of the "ensured dynamic aperture" can be used. These two quantities, indeed, are representative of the behaviour of the machine in the most pessimistic conditions that can be expected.

Results of the Simulations

The beneficial effects of ordering the LHC magnets are substantially impaired by the two-in-one design of the magnets, by the presence of higher-order random multipole imperfections, and to a smaller extent by the imprecise knowledge of the random sextupole imperfections. This has been shown in Refs. [3] and [4], by tracking on-momentum particles with perfect closed orbit, and by neglecting the systematic errors and the multipolar correctors. An additional reduction of the dynamic and linear aperture appears for off-momentum particles and to a smaller extent for finite closed orbit. The feed-down effects of the systematic imperfections and of the multipolar correctors in combination with the finite closed orbit are small.

On-Momentum, On-Axis Particle Tracking

In a single bore ring, with only sextupolar magnetic errors in the injection optics, the ensured dynamic aperture of on-momentum particles with perfect closed orbit is increased by a factor of two, from 0.9 to 1.8 mm, by applying the ordering rules. With higher-order multipolar imperfections and two-in-one magnets, the improvement is reduced to about a factor 1.25, from 0.8 to 1.1 mm, but it holds in both rings. Changing the tune working point can be beneficial; for instance with $Q_x = 70.15$, $Q_y = 70.18$, the ensured dynamic aperture increases by about a factor of 1.2, both for random and ordered installation of the magnets.

When the collision optics is considered, the dynamic aperture is determined by the imperfections of the inner triplets, where the value of the β-functions is maximum. With perfect quadrupoles the ensured aperture is 0.7 mm and becomes 0.9 mm by applying the ordering rules. With realistic quadrupoles the ensured aperture drops dramatically down to 0.1 mm, and the ordering rules can only increase it to 0.2 mm.

Changing the crossing angle α can also have a strong effect, both with injection and collision optics. The previous results, obtained with $\alpha = 96$ μrad, are practically the same as with head-on crossing; larger values of α, instead, enhance the effect of the errors in the inner triplets and reduce the dynamic aperture: at 200 and 300 μrad the reduction factors are 0.9 and 0.8 respectively for both optics. To reduce the long-range beam–beam tune shift a crossing angle of 300 μrad is deemed necessary [7]; therefore, this value has been used for the simulations of the next section.

Methodical Particle Tracking

Methodical tracking simulations have been performed with injection optics, two-in-one magnets and crossing angles of $\alpha = 300$ μrad, the results of which are summarized in Table 4 (particles circulate clockwise and anticlockwise in ring No. 1 and ring No. 2 respectively). An ideal bare machine, chromaticity corrected, has a very large value (3.9 mm) of the dynamic aperture, well outside the vacuum chamber, whose radius of 20 mm corresponds to a normalized amplitude of 1.54 mm at $\beta = \beta_{max} = 169$ m, and a linear aperture of 1 mm. By including the systematic and the random multipole components in all the

MACHINE DEFINITION	LINEAR APERTURE based on SMEAR Normalized Amplitudes in mm ±0.02 mm									Short-Term DYNAMIC APERTURE Normalized Amplitudes in mm ±0.02 mm								
	$\Delta p/p = -1.25 \times 10^{-3}$			$\Delta p/p = 0$			$\Delta p/p = -1.25 \times 10^{-3}$			$\Delta p/p = -1.25 \times 10^{-3}$			$\Delta p/p = 0$			$\Delta p/p = +1.25 \times 10^{-3}$		
	A_{Min}	A_{Max}	A_{Ave}	A_{Min}	A_{Max}	A_{Ave}	A_{Min}	A_{Max}	A_{Ave}	A_{Min}	A_{Max}	A_{Ave}	A_{Min}	A_{Max}	A_{Ave}	A_{Min}	A_{Max}	A_{Ave}
IDEAL BARE MACHINE			1.00			1.00			**1.01**			**3.62**			**3.90**			**3.97**
All Multipole Errors Rand. <u>not</u> Sorted, Syst. <u>not</u> Comp.																		
Ring 1	<0.17	0.30	**0.24**	0.32	0.61	**0.47**	<0.15	0.21	**0.17**	0.34	0.59	**0.48**	0.63	0.76	**0.70**	0.38	0.71	**0.50**
Ring 2	<0.20	0.29	**0.25**	0.34	0.68	**0.52**	0.15	0.22	**0.17**	0.40	0.59	**0.51**	0.61	0.78	**0.69**	0.36	0.78	**0.50**
Random Multipole Errors in Dipoles and Quadrupoles																		
Not Sorted — Ring 1	0.32	0.63	**0.47**	0.33	0.61	**0.48**	0.34	0.62	**0.47**	0.48	0.76	**0.67**	0.63	0.72	**0.68**	0.49	0.79	**0.65**
Not Sorted — Ring 2	0.38	0.62	**0.52**	0.41	0.64	**0.52**	0.44	0.64	**0.51**	0.59	0.82	**0.68**	0.55	0.74	**0.66**	0.51	0.74	**0.63**
Sorted — Ring 1	0.52	0.69	**0.62**	0.53	0.71	**0.62**	0.53	0.71	**0.60**	0.63	0.79	**0.70**	0.63	0.76	**0.70**	0.59	0.76	**0.68**
Sorted — Ring 2	0.53	0.69	**0.60**	0.49	0.71	**0.62**	0.48	0.67	**0.59**	0.55	0.82	**0.69**	0.63	0.78	**0.69**	0.59	0.76	**0.68**
+ Systematic Mult. Errors in Dipoles - (Random Sorted)																		
Not Compensated — Ring 1	<0.24	0.36	**0.28**	0.58	0.70	**0.63**	<0.15	0.35	**0.23**	0.49	0.68	**0.58**	0.63	0.85	**0.72**	0.46	0.75	**0.62**
Not Compensated — Ring 2	<0.25	0.35	**0.28**	0.56	0.75	**0.64**	<0.15	0.35	**0.22**	0.50	0.68	**0.58**	0.63	0.82	**0.72**	0.44	0.74	**0.60**
Compensated — Ring 1	0.44	0.64	**0.57**	0.49	0.70	**0.61**	0.48	0.65	**0.57**	0.55	0.68	**0.63**	0.63	0.82	**0.72**	0.59	0.79	**0.68**
Compensated — Ring 2	0.52	0.63	**0.57**	0.43	0.72	**0.61**	0.51	0.66	**0.59**	0.59	0.68	**0.64**	0.59	0.82	**0.69**	0.59	0.74	**0.67**
+ Closed Orbit Residue (Rand. Sort., Syst. Comp.)																		
Ring 1	0.37	0.59	**0.50**	0.39	0.64	**0.55**	0.38	0.61	**0.51**	0.46	0.66	**0.58**	0.49	0.74	**0.63**	0.49	0.74	**0.61**
Ring 2	0.36	0.60	**0.50**	0.37	0.67	**0.56**	0.38	0.62	**0.51**	0.44	0.74	**0.58**	0.47	0.74	**0.65**	0.48	0.69	**0.61**
10 MACHINES SET Closed Orbit Residue +																		
Rand. <u>not</u> Sort., Syst. <u>not</u> Comp.																		
Ring 1	0.20	0.30	**0.25**	0.30	0.54	**0.42**	0.16	0.21	**0.18**	0.29	0.52	**0.42**	0.46	0.79	**0.62**	0.31	0.69	**0.43**
Ring 2	0.20	0.31	**0.25**	0.31	0.53	**0.44**	0.15	0.25	**0.18**	0.40	0.59	**0.49**	0.43	0.76	**0.62**	0.30	0.66	**0.43**
Rand. <u>not</u> Sorted, Syst. Comp.																		
Ring 1	0.32	0.46	**0.39**	0.31	0.61	**0.45**	0.32	0.57	**0.45**	0.51	0.59	**0.55**	0.58	0.72	**0.65**	0.51	0.74	**0.62**
Ring 2	0.31	0.55	**0.44**	0.31	0.60	**0.47**	0.32	0.53	**0.44**	0.42	0.68	**0.58**	0.43	0.69	**0.62**	0.44	0.76	**0.60**
Rand. Sorted, Syst. Comp.																		
Ring 1	0.37	0.59	**0.51**	0.39	0.64	**0.55**	0.38	0.59	**0.52**	0.46	0.67	**0.58**	0.49	0.74	**0.64**	0.49	0.74	**0.62**
Ring 2	0.36	0.60	**0.50**	0.37	0.67	**0.56**	0.38	0.60	**0.53**	0.44	0.67	**0.57**	0.47	0.74	**0.66**	0.48	0.68	**0.61**

Table 4: Linear and Dynamic Apertures in Different Configurations of the LHC

dipoles and quadrupoles, the detrimental effect on the particle motion is dramatic, especially off-momentum. When the systematic errors are neglected, the situation is slightly improved and the effect of the ordering rules is significant. Similar results are obtained when the systematic errors are compensated by the lumped Simpson correctors. A finite closed orbit provokes a further reduction of the ensured linear and dynamic aperture.

The size of the reduction seems to be significantly related to the size of the peak orbit distortion. For instance the ensured linear aperture changes by less than 0.10 mm and less than 0.16 mm for on- and off-momentum particles respectively, for a peak orbit distortion of $1.3/\sqrt{\beta_{max}} = 0.1$ mm and a peak dispersive distortion $D_{max}\delta/\sqrt{\beta_{max}} = 0.18$ mm. The effect of the ordering rules in the presence of a finite closed orbit is still visible in all the considered situations, summarized in the third part of Table 4. In the more realistic case, with random and systematic imperfections in all the dipoles and quadrupoles, and with lumped multipolar compensation, ordering the magnets may improve the ensured linear aperture by at least a factor of 1.16 for both on- and off-momentum particles, from 0.31 mm to 0.38 mm. Such an aperture corresponds to 4.3σ of a beam with $15\pi\times 10^{-6}$ m emittance. It remains to evaluate whether this is sufficient to ensure a safe operation of the LHC, and, if not, how the errors can be diminished.

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