EFFECTS OF HELICAL UNDULATORS ON BEAM DYNAMICS

L. Tosi and R. Nagaoka
Sincrotrone Trieste
Padriciano 99
34012 Trieste, Italy

Abstract

The presence of helical insertion devices in a storage ring introduces further non-linearities with respect to the ones generated by a traditional insertion device, which results in a further reduction of the dynamic aperture. The effects of several types of helical devices are considered including an undulator with variable polarization.

Introduction

There is an increasing interest in the utilization of circularly polarized light for some types of experiments. This may be generated by helical insertion devices, in which a charged particle oscillates in both horizontal and vertical planes. Already in the case of a traditional device, in which a particle oscillates only in one plane, there is a significant reduction in the dynamic aperture, due to the highly non-linear fields of the device and to the breaking of the symmetry of the optics. It is, therefore, of extreme importance to be able to predict in advance the effects of the helical devices on beam dynamics.

On the basis of a Hamiltonian[1] derived for a general helical structure, the main properties of the motion are deduced for the bifilar helix[2], the crossed undulator[3] and the planar circularly polarized undulator[4]. The linear and non-linear effects are compared, also by dynamic aperture computations, with those generated by a traditional device.

Furthermore, the effects on beam dynamics of the planar undulator with variable polarization[5] are investigated with particular attention to those arising when the polarization is being switched from circular right to left-handed.

Linear and Non-linear Effects of General Helical Structures

The magnetic field generated by an insertion device (ID) producing circularly polarized light may be written as[6]:

$$B_x = B_o \frac{k_x}{k_y} sh(k_x x) sh(k_y y) cos(kz) + B_o ch(k'_x x) ch(k'_y y) sin(kz)$$

$$B_y = B_o ch(k_x x) ch(k_y y) cos(kz) + B_o \frac{k'_y}{k'_x} sh(k'_x x) sh(k'_y y) sin(kz)$$

$$B_z = -B_o \frac{k}{k_y} ch(k_x x) sh(k_y y) sin(kz) + B_o \frac{k}{k'_x} sh(k'_x x) ch(k'_y y) cos(kz) \qquad (1)$$

where k_x, k_y, k_x', k_y' must satisfy the two divergence conditions $k_x^2+k_y^2 = k^2$ and $k_x'^2+k_y'^2 = k^2$ with $k = 2\pi/\lambda_o$, λ_o being the insertion device period, x, y are the horizontal and vertical Cartesian coordinates and z the longitudinal one coinciding with the device axis. According to the choice of k_x, k_y, k_x', k_y', different types of helical insertion devices may be achieved as follows:

a) Bifilar Helix[2] $k_x^2 = k_y'^2 = k^2/4$; $k_y^2 = k_x'^2 = 3k^2/4$
b) Crossed Undulator[3] $k_x = k_y'$; $k_y = k_x'$
c) Planar Circularly Polarized Undulator[4] $k_x = k_x'$; $k_y = k_y'$

In order to derive the main characteristics of motion for a particle in the central region of a long ID, the particle is assumed to oscillate around an equilibrium orbit generated only by the on-axis transverse components of the field. A Hamiltonian[1] for the motion expanded around the equilibrium orbit up to fourth order in the coordinates may be derived as:

$$\begin{aligned} H = {} & \frac{1}{2}(p_x^2+p_y^2) + \frac{1}{4k^2\rho^2}\{(k_x^2+k_x'^2)x^2+(k_y^2+k_y'^2)y^2+\left(\frac{k_x^4}{3}+\frac{k_x'^4}{3}\right)x^4 \\ & +\left(\frac{k_y^4}{3}+\frac{k_y'^4}{3}\right)y^4+k^2(k_x^2+k_y'^2)x^2y^2\} - \frac{\cos(ks)\sin(ks)}{k^2\rho^2}\{(k_y'^2+k_x^2)xy \\ & +\left(\frac{k^2k_y'^2}{2}+\frac{k_y'^4}{6}+\frac{k_x^2k_y^2}{6}\right)xy^3+\left(\frac{k^2k_x^2}{2}+\frac{k_x'^2k_y'^2}{6}+\frac{k_x^4}{6}\right)x^3y\} + \\ & - p_x\{\frac{\sin(ks)}{\rho k}\left[\frac{k_y^2}{2}y^2+\frac{k_x^2}{2}x^2+\frac{k_y^4}{24}y^4+\frac{k_x^4}{24}x^4+\frac{k_x^2k_y^2}{4}x^2y^2\right] + \\ & - \frac{\cos(ks)}{\rho k}\left[k_y'^2 xy+\frac{k_y'^4}{6}xy^3+\frac{k_x'^2k_y'^2}{6}x^3y\right]\} \\ & - p_y\{\frac{\cos(ks)}{\rho k}\left[\frac{k_y'^2}{2}y^2+\frac{k_x'^2}{2}x^2+\frac{k_y'^4}{24}y^4+\frac{k_x'^4}{24}x^4+\frac{k_x'^2k_y'^2}{4}x^2y^2\right] + \\ & - \frac{\sin(ks)}{\rho k}\left[k_x^2xy+\frac{k_x^2k_y^2}{6}xy^3+\frac{k_x^4}{6}x^3y\right]\} \end{aligned} \qquad (2)$$

where s, the arc length of the equilibrium orbit, is the independent variable.

The helical ID gives a focussing effect in both planes of strength:

$$K_H = \frac{k_x^2 + k_x'^2}{2\rho^2k^2} \qquad K_V = \frac{k_y^2 + k_y'^2}{2\rho^2k^2} \qquad (3)$$

Since the magnetic field is the sum of the fields produced by two traditional IDs (one with horizontal oscillation and the other with vertical), the two strengths are the superposition of the strengths produced by the single fields. Substituting k_x and k_x' by imaginary values, gives horizontal defocussing. Adding the two divergence conditions and dividing by $2\rho^2k^2$ yields the condition $K_H+K_V=1/\rho^2$ for the horizontal and vertical strengths (taken with the adequate signs according to (3)), showing that there is no way of choosing k_x, k_y, k_x', k_y' so as to make the device act linearly as a quadrupole or as a drift space.

Since the strengths produced by the helical devices are generally small with respect to the ones of the adjacent quadrupoles, the β function in the device varies approximately as $(\beta^* + s^2/\beta^*)$ where β^* is the value at the center of the device. Hence the horizontal linear tune shift produced is (the vertical one is given by substituting the indices x with y):

$$\Delta\nu_x = \frac{(k_x^2 + k_x'^2)}{8\pi\rho^2k^2} L\beta_x^* \left[1+\frac{1}{12}\left(\frac{L}{\beta_x^*}\right)^2\right] \qquad (4)$$

For the case of the planar circularly polarized undulator, since $k_x = k_x'$ and $k_y = k_y'$, the strengths and the tune shifts produced are doubled in value with respect to the corresponding traditional undulator, having the same characteristics and β^*. Instead for the crossed undulator and the bifilar helix, the strict relations between the values of k_x, k_x', k_y, k_y' and k always results in horizontal and vertical focussing of strengths of $1/(2\rho^2)$ and the linear tune shifts may be found by substituting the appropriate values in (4).

For the non-linear effects, there also is a superposition of the effects produced by the single fields. (This becomes even more evident if one considers the exchanging roles that k_x and k_y' play in the expressions for the two single fields.) It results in an increase of the strengths of the non-linear non-oscillating terms and in an introduction of further non-linear and coupling oscillating terms, with respect to a traditional device. For the planar circularly polarized device, the Hamiltonian yields a symmetric structure in x and y coordinates and the same non-linearities are present in both planes.

Since helical devices introduce linear and non-linear effects in both planes, it may be expected that the differences in dynamic aperture with respect to a traditional ID will be manifest mainly in the plane with the larger β function. In order to investigate these effects, a comparison of the relative dynamic apertures has been made[1] for the following insertion devices; in all cases, except where indicated otherwise, the main parameters were taken to be E = 2.0 GeV, λ_o = 0.06 m, N_p = 75 and B_o = 0.35 T:

a) U1 : Traditional plane device ($k_x = 0$) with vertical focussing of strength $1/(2\rho^2)$;
b) U2 : Bifilar helix with horizontal and vertical focussing of strengths $1/(2\rho^2)$;
c) U3 : Crossed undulator (with $k_x = k_y' = 0$) having the same linear properties as U2 ;
d) U4 : Planar circularly polarized undulator having an imaginary $k_x = k_x' = j138.0$ giving a horizontal defocussing of strength $3.5/(2\rho^2)$ and a vertical focussing of strength $5.47/(2\rho^2)$;
e) U5 : as U4, but with an on-axis field value (B_o = 0.15 T) such that the vertical focussing strength results the same as for the first three cases.
f) U6 : as U3, but with λ_o = 0.074 m, B_o = 0.15 T and N_p = 60.

After performing an alfa-matching of the devices and a chromaticity correction with the sextupoles in the dispersive regions, four particles were tracked over 100 turns, yielding the dynamic apertures reported in figure 1. The case for sextupoles only, without an insertion device, is also shown.

While for the plane device U1, the main cause of dynamic aperture limitation are the sextupoles, for the helical devices there is a general decrease in the maximum horizontal stable amplitude, as expected since the horizontal β (8.2 m) is larger than the vertical (2.6 m) at the location of the devices. For the bifilar U2 and the crossed U7, the reduction in the maximum vertical aperture seems also caused by the sextupoles, but for increasing horizontal amplitudes there is an increasing reduction due to the non-linearities in the horizontal plane, showing that the two motions are strongly coupled.

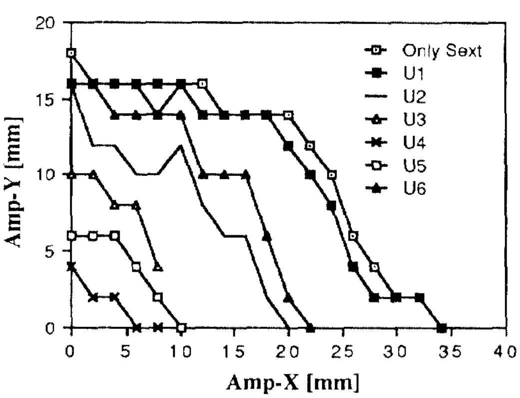

Fig.1. Dynamic Apertures for General Helical Insertion Devices

The aperture decreases strongly for the planar circularly polarized devices U4 and U5, linearly defocussing in the horizontal plane, with a significant decrease also in the maximum vertical aperture. A similar behaviour has already been found[7] during an investigation on how much horizontal defocussing could be tolerated in a traditional plane device. The main causes for this reduction is still under investigation.

The strong reduction in both horizontal and vertical dynamic apertures in the case of the crossed device U3, compared to U2 having the same linear effects and to U7 being the same type of device, might suggest that this particular device is driving a strong resonance. For this purpose, the tune shift with amplitude has been calculated along the line Amp-Y/Amp-X = 0.54 at maximum coupling. This resulted in a strong variation of the fractional part of the horizontal tune, which might confirm this hypothesis.

Effects of the Undulator with Variable Polarization

A new type of undulator consisting of two double arrays of permanent magnets has been proposed[5], which enables the switching of the polarization of the generated light from circular right to left-handed passing through linear by shifting the bottom jaw with respect to the top in the z direction. An analytical expression for the field has been found[4] to be:

$$
\begin{aligned}
B_x &= B_o\left(1+2b_1y-a_2x^2+b_2y^2\right)\cos(kz) + B_o\left(2a_1x-2a_2xy\right)\sin(kz+\varphi) \\
B_y &= B_o\left(2b_1x+2b_2xy\right)\cos(kz) + B_o\left(1-2b_1y-a_2x^2+b_2y^2\right)\sin(kz+\varphi) \\
B_z &= -kB_o\left(x+2b_1xy\right)\sin(kz) - kB_o\left(a_o-y-a_1x^2+b_1y^2\right)\cos(kz+\varphi)
\end{aligned} \tag{5}
$$

where the relations $2b_2-2a_2=k^2$ and $2b_1-2a_1=a_ok^2$ must be satisfied and the polynomial coefficients have been found for various period lengths in the above reference. The parameter φ has been introduced in order to describe the relative position of the two jaws. By varying φ from 0° to 180° passing through 90°, the changing of the helicity of the field from right-handed to left passing through plane may be simulated.

In order to overcome the end pole effects, the particle is assumed to oscillate around the orbit generated by the transverse on-axis fields. The Hamiltonian yielding the linear equations of motion in the independent variable z may be found as:

$$
\begin{aligned}
H = {} & \frac{1}{2}\left(p_x^2+p_y^2\right) + \frac{1}{4\rho^2k^2}\left\{\left(4b_1^2-4a_2+4a_1^2\right)x^2+\left(8b_1^2+4b_2\right)y^2\right\} + \\
& - \frac{\cos(kz+\varphi)\sin(kz)}{\rho^2k^2}\left\{\left(2a_1+2b_1\right)x+k^2\left(1-2b_1a_o\right)xy\right\} + \\
& - p_x\left\{\frac{\cos(kz+\varphi)}{\rho k}2b_1y+\frac{\sin(kz)}{\rho k}2b_1x\right\} - p_y\left\{\frac{\cos(kz+\varphi)}{\rho k}2a_1x-\frac{\sin(kz)}{\rho k}2b_1y\right\}
\end{aligned} \tag{6}
$$

The resulting equations of motion are:

$$
\begin{aligned}
x'' = {} & \left[\frac{2a_2}{\rho^2k^2} - \frac{2b_1}{\rho}\cos(kz)\right] x + \frac{\cos(kz+\varphi)\sin(kz)}{\rho^2k^2}\left(2a_1+2b_1\right) + \\
& + \left[\frac{\cos(kz+\varphi)\sin(kz)}{\rho^2} + \frac{2b_1\sin(kz+\varphi)}{\rho}\right] y - \frac{ka_o\cos(kz+\varphi)}{\rho} y' \\
y'' = {} & \left[- \frac{2b_2}{\rho^2k^2} + \frac{2b_1}{\rho}\cos(kz)\right] y + \frac{ka_o\cos(kz+\varphi)}{\rho} x' + \\
& + \left[\frac{\cos(kz+\varphi)\sin(kz)}{\rho^2} + \frac{2a_1\sin(kz+\varphi)}{\rho}\right] x
\end{aligned} \tag{7}
$$

The "quadrupole-like" terms in the two equations are Mathieu's equations. Since the quantities involved are very small, it can be proved[8] that for the tabulated coefficients[4] the vertical motion will perform a stable non-periodic oscillatory motion around the reference orbit. Whereas the horizontal one will pass from a stable to an unstable one as soon as $a_2 \geq b_1^2$, which happens for $\lambda_o \geq 0.60$ m, in the present case.

It should be noted that there is a z-dependent dipole term in the horizontal equation, indicating that the reference orbit chosen is not

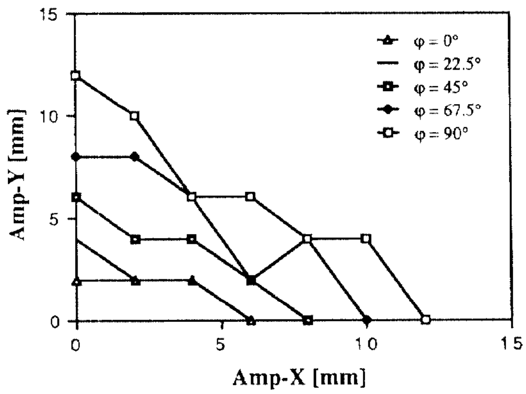

Fig.2. Dynamic Apertures for the Planar Undulator with Variable Polarization

the equilibrium one, for a particle entering the device on the assumed orbit will not remain on it. This dipole field taken by itself contributes to a horizontal angle with respect to the predefined machine's equilibrium orbit at the device exit by the quantity:

$$\Delta x' = -\frac{2a_1 + 2b_1}{2\rho^2 k^2} L \sin(\varphi) \qquad (8)$$

where L is the length of the device. For a particular device it will have a maximum when the undulator is passing through the plane mode. It should be underlined, however, that there is a deflection from the assumed orbit even for the helical mode. Even though in this case the dipole taken by itself does not contribute to a final angle, it influences the motion along the device interacting with the other terms and the particle exits on an orbit different from the machine's equilibrium one. Thus, expression (8) should not be considered as the effective angle of the particle at the device exit for a given φ, as it has been confirmed by simulating the motion in the field (5) with a particle entering the device on the assumed orbit.

From the beam dynamics point of view, the above effect is equivalent to errors in a perfect machine, generating thus a distorted closed orbit which varies with φ and presenting a maximum when φ is 90°. The device U7 with $\lambda_0 = 0.1$ m, $B_0 = 0.275$ T, $a_0 = 11.759$ mm, $a_1 = 0.05$ mm^{-1}, $b_1 = 0.054$ mm^{-1}, $a_2 = 0.006$ mm^{-2}, $b_2 = 0.007$ mm^{-2}, which generates a large final angle[4] (249 μrad) in the plane mode, has been introduced in the lattice. The new closed orbits have been computed as a function of φ and the maximum horizontal values were found to vary increasingly from 0.002 mm for $\varphi = 0°$ to 1.847 mm for $\varphi = 90°$. For each closed orbit, the relative effective transfer matrix of the device with respect to the new equilibrium orbit was obtained. A strong coupling between the two motions resulted for the helical mode which decreased when passing through $\varphi = 90°$.

By simulating the particle motion in the field (5) for the device U7 tracking calculations have been carried out as a function of φ, after resetting the initial tunes of the machine for each case. The resulting dynamic apertures, which include the effect of sextupoles, are reported in figure (2). The marked reduction in dynamic aperture of the helical mode ($\varphi = 0°$) compared to the linear mode ($\varphi = 90°$), despite the larger closed orbit distortion in the latter case, was observed also when matching was not performed.

Conclusions

It may be concluded that general helical devices introduce extra linear, non-linear and coupling terms in both horizontal and vertical planes with respect to a traditional ID. Dynamic aperture computations show that the particle's behaviour may be more strongly affected in the plane where the β function is large, which for ELETTRA is the horizontal one. For the bifilar and crossed device there is no obvious way of choosing the parameters so as to reduce the non-linearities in the horizontal plane (as may be done with a traditional ID by setting $k_x = 0$). For the planar device, a choice $k_x = k_x' = 0$ may reduce the horizontal non-linear effects, however this choice from a pratical point of view would result in a significant reduction in the maximum achievable on axis field[9]. Thus, if one plans to introduce one of these devices in a ring, the best solution may be to change the optics locally so as to decrease the horizontal β.

As for the planar undulator with variable polarization, the dynamic apertures computed show that there is an enlargement of the apertures when the device passes from the helical to the plane mode. On the other hand, the increasing closed orbit distortion generated would imply complicated beam steering corrections if the switching is intended to be done at high frequency. However, since this effect scales inversely with the square of the energy, a machine with a larger energy than ELETTRA may accept the closed orbit distortion without correction.

References

[1] L. Tosi, Sincrotrone Trieste Internal Report, ST/M-90/9

[2] J.P. Blewett and R. Chasman, "Orbit and Fields in the Helical Wiggler", J. Appl. Phys., vol. 48, pp. 2692-2698, July 1977.

[3] H. Onuki, "Elliptically Polarized Synchrotron Radiation Source with Crossed and Retarded Magnetic Fields", Nucl. Instr. Meth., vol. A246, pp. 94-98, 1986.

[4] B. Diviacco and R.P. Walker, "Fields and Trajectories in Some New Types of Permanent Magnet Helical Undulator", to be published on Nucl.Instr. Meth.

[5] P. Elleaume, ESRF Internal Report, ESRF-SR/ID-88-23, submitted to Nucl.Instr. Meth.

[6] R.P. Walker, Sincrotrone Trieste Tech. Note, ST/M-TN-89/41

[7] L. Tosi, Sincrotrone Trieste Tech. Note, ST/M-TN-89/39

[8] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions. New York: Dover Publications, Inc., 1970, ch. 20, pp. 721-730.

[9] R.P. Walker, Private communication