EFFECTS OF INSERTION DEVICES ON BEAM DYNAMICS IN THE PRESENCE OF CLOSED ORBIT DISTORTIONS

E. Karantzoulis and R. Nagaoka

Sincrotrone Trieste, Padriciano 99, 34012 Trieste, Italy

Introduction

ELETTRA is an electron/positron storage ring under construction in Trieste (Italy). It will be used as a light source for photon energies in the range of ultraviolet to hard X-rays (i.e a few to some tens of keV). Like all the new generation radiation sources its goal is high spectral brilliance/flux with good tunability. These design goals can be achieved by a storage ring in the energy range of 1.5 to 2.0 GeV with very low electron/positron beam emittance (i.e. $\varepsilon_x < 10$ nm·rad) and the use of wigglers and undulators.

In obtaining such a small emittance one must use strong quadrupoles for focusing and strong sextupoles to correct for the chromatic effects. On the other hand however, strong quadrupoles introduce large amplification factors (i.e between 50 and 100) in the case of closed orbit distortions whereas strong sextupoles introduce nonlinearities that limit the dynamic aperture. In a similar way the insertion devices (ID) may further limit the performance of the ring through a combination of linear and nonlinear effects resulting in a reduction of the dynamic aperture. This occurs mainly because the IDs excite additional nonlinear resonances as well as destroy the sextupole optimization in the lattice. These devices also limit the physical aperture because they require a small gap size (15-20 mm for ELETTRA). Since a large dynamic aperture (and momentum acceptance) is necessary to accomodate gas scattered and Touschek scattered electrons, it is evident that ELETTRA will be a very sensitive machine to operate and the examination of all these effects that reduce its dynamic aperture is therefore vital.

The effects of IDs on the beam in the ideal (i.e. no errors in the machine and no closed orbit distortions (COD)) situation has been already studied analyticaly [1] as well as numericaly [2]. With these studies we have obtained a rather good qualitative and quantitative understanding of the dynamics in the presence of an ideal ID. However, the very important aspect of the performance deterioration due to no ideal situations (i.e. random errors and closed orbit distortions) in the presence of ID, which are very likely to occur in a real machine, to the best of our knowledge has not yet been studied. Furthermore, analytic approaches such as given in reference [1] are inapplicable in our case - since they asume no COD or errors - and therefore cannot predict the important changes in tune shifts already reported from machine experiments [3].

The aim of our work is a systematic study - qualitative and quantitative - of ID effects on the beam dynamics in the presence of various possible errors. The study comprises the effects of IDs alone as well as its interaction with the ring. In such problems one already knows Hamiltonians/equations of motion, but the real difficulty lies on their integration. We do not use perturbative approaches since the ID magnetic fields are highly nonlinear. We rather integrate the most general equations of motion using RACETRACK [4] in a symplectic way [5]. Therefore one can study the effects quantitatively as accurately as desired and afterwards rely on analysis for their interpretation. What we shall explicitly study here is the effects of COD and errors on the linear tune and dynamic aperture in some realistic cases for ELETTRA with one typical plane wiggler and undulator.

General Description of the Numerical Approach

As already stated, in order to simulate particle motion in the presence of arbitrary closed orbit distortions inside an ID one can not use the L. Smith Hamiltonian [1], since it describes the betatron motion with respect to the ideal equilibrium orbit in the device. Since for our investigations the ideal orbit is no longer the reference trajectory we prefer to describe the particle's position in space using the same fixed Cartesian frame as the one where the magnetic field of the device is expressed. One may write the Hamiltonian describing the transverse motion in a Cartesian frame as:

$$H = \frac{1}{2}\left[p_x - \frac{e}{p_0}A_x\right]^2 + \frac{1}{2}\left[p_y - \frac{e}{p_0}A_y\right]^2, \qquad (1)$$

where $\mathbf{A} = (A_x, A_y, A_z)$ is the vector potential , A_z is assumed to be identically zero and the other symbols have their usual meaning. For simplicity, only the nominal energy particle is considered. The magnetic field expressions, correct to the first order, of a plane ID may be derived from the following vector potentials:

$$A_x = \frac{B_0}{k}\cdot\cosh(k_x x)\cdot\cosh(k_y y)\cdot\sin(kz), \qquad (2a)$$

$$A_y = -\frac{k_x}{k_y}\frac{B_0}{k}\cdot\sinh(k_x x)\cdot\sinh(k_y y)\cdot\sin(kz), \qquad (2b)$$

with

$$k_x^2 + k_y^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2, \quad \lambda\text{: period length of an ID}$$

and B_0 the peak field.

Once the Hamiltonian is specified, the particle motion may be followed by adopting the canonical integration technique which guarantees the symplecticity of the system. As in [5], a two-step integration is made for a given increment to achieve the accuracy up to the second order in the increment. Working in the fixed Cartesian frame one has also to transform the equilibrium orbit of the device, which in fact is nothing but one of the trajectories defined by the Hamiltonian with particular entrance coordinates. For the ideal plane sinusoidal device the equilibrium orbit is periodic and can be approximately derived by neglecting the field dependence on x and y :

$$x_e = \frac{\cos(kz)}{k^2\rho}, \qquad x_e' = -\frac{\sin(kz)}{k\rho},$$
$$y_e = 0, \qquad y_e' = 0. \qquad (3)$$

In reality, these devices have additional endpoles in order to shift the trajectory onto the equilibrium orbit. We have assumed, in order to avoid further complications, that the amount of shift is the same for any equilibrium orbit. This treatment is also consistent with the former approach [2] using the L.Smith Hamiltonian. To be accurate, the amount of shift is determined by searching an exact periodic orbit numerically, starting from the values at $z = 0$ in Eq. 3.

From the Hamiltonian (1) and the described treatment of the endpole shifts, the transformation over an ID is completely specified, closed orbits can be computed and what remains is the matrix calculations to derive the linear optics around it. To find the transfer matrices for the ID - which must not be simply approximated as a thin element like a multipole - four trajectories each differing infinitesimally in x, x', y and y' from the obtained closed orbit at the entrance are integrated through the ID. The exit values are then used to extract the linear part of the transformation which is represented by a 4×4 matrix. Thus, all the local changes of the linear components along the distorted equilibrium orbit in the device are effectively embedded in the resulting matrices.

We have implemented the described scheme in RACETRACK [4]. It should be stressed that the present approach is more general than the former [2] one. In the ideal case with no orbit distortion it should give identical results - apart from approximations made in the former - and in fact, reasonable agreement has been found in cases studied so far. Finally, we should also mention that the implemented routines are capable of treating helical IDs as well, although numerical studies with these devices are not presented in this work.

Numerical Results and Discussions

The numerical method described is applied to ELETTRA [6], including one ID in the ring. In the present study, two typical plane sinusoidal undulator and wiggler are examined whose parameters are shown in Table 1. Since our central interest lies in the effect of COD in the ID as well as in the machine performance with IDs in the presence of various magnetic errors, misalignment, field and tilt errors are assumed for dipoles, quadrupoles and sextupoles whose rms values are listed in Table 2. The errors are randomly generated according to the Gaussian distribution.

Table 1.
Insertion device parameters.

	Field (T)	Period (cm)	No. of period	k_x/k
wiggler	5.0	30	10	0.1
undulator	1.2	5.5	100	0.1

Table 2.
rms values of magnet error assumed in the simulation.

	Dipole	Quadrupole	Sextupole
field error	0.12 %	0.1 %	0.1 %
misalignment		0.2 mm	0.2 mm
tilt	0.5 mrad	0.2 mrad	0.2 mrad

Effect of COD in ID:

To be able to distinguish the effect of ID alone from that of rest of the ring, we first performed calculations with sextupoles turned off and errors consisting only of quadrupole misalignments that generate COD in the ID. The effect of ID is then studied as a function of the amplitude of distorted orbit by scaling the strength of dipole components in the quadrupoles.

The features of calculated COD in the ID itself is that, for one thing, the orbit passes through the device with a tiny wiggling motion on top of it, and for another, the amplitude is reduced compared to the case of drift space. The latter is a reflection of the fact that plane sinusoidal devices, with an assumed condition $k_x = 0.1k$, are linearly focusing in two transverse directions, whose strength are given respectively by $k_x^2/[2(k\rho)^2]$ in the horizontal, and $k_y^2/[2(k\rho)^2]$ in the vertical [1]. The reduction is therefore most enhanced with a wiggler in the vertical direction.

As the field inside the ID is highly nonlinear, a study of linearized field around a distorted equilibrium orbit may be of great importance since the resulting change of optics from the ideal case may be crucial to the machine operation. With the type of devices considered, the obtained 4×4 transfer matrix reveals that: (1) The effective focusing strength increases in both x and y direction with orbit distortions. (2) The focusing strength along the orbit is non uniform. (3) Coupling terms are negligible.

The first aspect can be clearly seen in Figs. 1, where the increase in the vertical tune is plotted against the vertical rms amplitude deviation from the ideal orbit, for undulator and wiggler, respectively. One notices a marked result that the tune shift with COD is more pronounced with an undulator, although in the absence of COD, it is larger with a wiggler. This may be plausible since the change of tune with COD should be more sensitive in a field with higher nonlinearity which is the case with a shorter period device [1, 2]. Similar behaviour was also seen in horizontal tune shifs, but much more reduced. The observed tune dependence on the COD, which seems to be quadratic, is yet to be understood. Analytical approaches have been investigated in this respect, one using a covariant formalism treating the equation of motion with respect to arbitrary orbit [7], and another in the Hamiltonian formalism with certain canonical transformations [8].

Effect on the nonlinear motion was investigated by calculating the tune shift with amplitude. With fixed initial betatron amplitudes particle tracking was made, again by scaling the dipoles to vary the COD in the ID. Changes in the tune shift were very small (less than ~ 0.001) in the range of COD considered, which had largest values in the vertical motion with an undulator. In this case, the tune shift has no definite direction, although on the average it seemed to increase with COD. Again, these aspects are to be confirmed by analytical approaches.

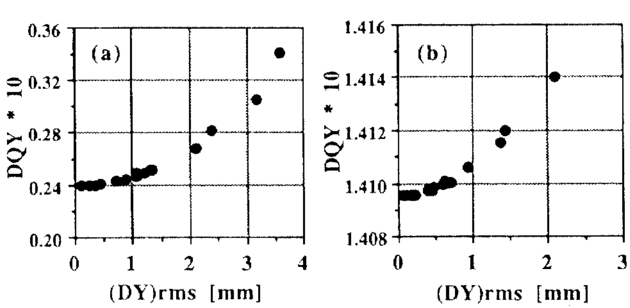

Fig.1. Vertical (linear) tune shift as a function of rms COD in the ID. (a) Undulator. (b) Wiggler.

Combined effect of ID and the ring in the presence of errors:

From the practical point of view, it would be very important to simulate the machine as closely as what would be in reality and to find the net effect on the particle motion. For this purpose, we generated 10 machines with errors according to Table 2, each for the case of undulator and wiggler. For every machine, correction of COD was made under the given corrector and monitor arrangement of ELETTRA [6] and using the two correction schemes provided by RACETRACK. The degree of correction achieved is summarized in Table 3. Linear tunes were then readjusted to the nominal values by

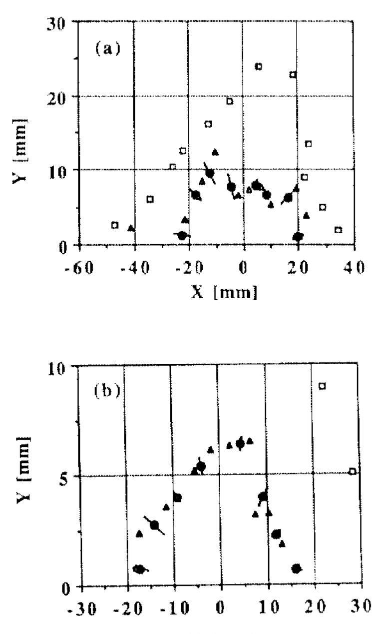

Fig. 2. Dynamic apertures with ID, in the presence of errors, calculated for 10 machines. Dark circles represent the averages, bars are the rms deviations. For comparison, those of the ideal ring with no ID (white squares), and with ID and no errors (triangles) are shown. (a) Undulator. (b) Wiggler.

Table 3.
rms values [mm] of residual COD in the calculation of Fig. 2.

	INSIDE ID		@ MONITORS	
	<Xrms>	<Yrms>	<Xrms>	<Yrms>
Undulator	0.218	0.025	0.391	0.070
Wiggler	0.175	0.012	0.394	0.070

varing two families of quadrupole triplet. Particle tracking was performed in the resulting rings to calculate the dynamic aperture.

The results are shown in Figs. 2 for undulator and wiggler, respectively. For comparison, the dynamic aperture of the ideal ring with and without ID are also plotted. It is seen that, even with errors, no drastic reduction of dynamic aperture is found in both cases, which is especially true with a wiggler where the reduction is roughly by only few percent.

The result may be interpreted as that: (1) The dynamic aperture is mostly determined by the ID itself (so that not by others such as accidental sextupole resonances), and that (2) The stability of motion is not very sensitive to the presence of small COD in the ID. To somehow confirm these points, we performed additional computation of dynamic aperture as a function of the magnitude of COD, by scaling the dipole strength (as was done previously) for the three cases: (i) No ID. (ii) With an undulator. (iii) With a wiggler. The results shown in Fig. 3 together with Table 4 seems to justify the above two conjectures.

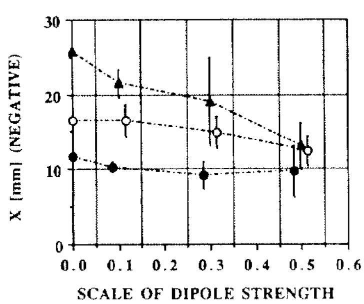

Fig. 3. Dynamic aperture as a function of the magnitude of COD, calculated for 10 machines (see also Table 4.). Triangles: Without ID. White circles: With an undulator. Dark circles: With a wiggler.

Table 4.
rms values [mm] of COD at the position of ID,† in correspondence to Fig. 3.

SCALE	0.1	0.3	0.5
< X rms >	0.588	1.732	2.824
< Yrms >	0.301	0.887	1.453

† Calculated without including an ID

Summary and Conclusions

In the present work, we have described a numerical approach to study the effect of ID on beam dynamics in the presence of arbitrary COD. The motion in the ID was solved in the Cartesian frame, using a canonical integration technique, and the (distorted) equilibrium orbit in the device was taken into account explicitly. From the integration of the orbit itself was simultaneously deduced the linear transformation of the ID to reconstruct the (distorted) optics of the ring.

Application to ELETTRA was made having a plane sinusoidal device in the ring with the COD. It was found that these devices do not contribute much to the distortion of the closed orbit, apart from small reduction in the magnitude when the focusing strength is large. An important findings was the increase in focusing strength with the increase in the COD, which was notable with a shorter period device. In the case studied, the additional tune change amounted as much as 40% of the ideal tune shift, with 4 mm rms orbit deviation. Changes in the nonlinear tune shift was also more stressed with a shorter period device, but was quite small in the range of COD considered (< 10 mm). Dynamic apertures calculated for realistic cases where the COD is corrected with steering magnets leaving small residual COD in the ID, showed no drastic reduction compared to those of the ideal system. It may be concluded that the stability of motion is not sensitive to the presence of COD in the ID to the extent as numerically verified.

It would be of great help to the further understanding of the issue if the features found from the numerical studies are confirmed by analytical approaches [7, 8]. It would also be important to extend the present work to other type of IDs investigated for future synchrotron light sources.

Acknowledgement

We would like to thank Dr. Albin Wrulich for suggesting the subject to us and for many helpful discussions.

References

[1] L. Smith, "Effects of Wigglers and Undulators on Beam Dynamics," ESG TECH. NOTE-24.
[2] L. Tosi, "Effects of Insertion Devices on Beam Dynamics," ST/M-88/25;
L. Tosi and A. Wrulich, "The Simulation of Insertion Device Effects on Single Particle Motion with RACETRACK," ST/M-88/12.
[3] Photon Factory Activity Report 1983/84, P. IV-34, National Laboratory for High Energy Physics KEK.
[4] A. Wrulich, RACETRACK: "A Computer Code for the Simulation of Nonlinear Particle Motion in Accelerators," DESY 84-026 (1984).
[5] R. Nagaoka and L. Tosi, "An Improved Scheme for Integrating the Particle Motion through the Insertion Device in RACETRACK," ST/M-90/6.
[6] Conceptual Design Report, "ELETTRA," Sincrotrone Trieste (1989).
[7] E. Karantzoulis, "Non-linear Covariant Equations of Transverse Motion in Arbitrary Magnetic Fields and with respect to Arbitrary Orbits," ST/M-89/21.
[8] L. Tosi, private communication (1990).