TWO-PARAMETER SORTING OF DIPOLES IN LARGE SYNCHROTRONS*

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Abstract: A relatively simple procedure for finding the optimum arrangement of a given set of dipoles in a large synchrotron has been studied when normal and skew sextupole components in dipoles are the dominant factors in its aperture reduction. The analytical figure-of-merit (F.M.) used previously in the presence of normal sextupole field has been extended to include the contribution from skew sextupole field as well. This F.M. has been used to weed out several different sorting schemes which are not very promising. For a model lattice composed of 576 dipoles in six superperiods, the dynamic aperture for 2000 turns and the linear aperture corresponding to a 5% "smear" parameter have been found from numerical trackings for several unsorted arrangements and for a "one-parameter" and a "two-parameter" sorted arrangement. In general, results indicate a significant improvement in aperture, both dynamic and linear, when two parameters (normal and skew sextupole components) are taken into account.

Introduction

It is obvious that problems arising from systematic multipoles in superconducting dipoles such as the normal sextupole component at injection must be solved by means of special packages of correction magnets. The scheme proposed by D. Neuffer[1] is undoubtedly the most powerful one for this purpose. On the other hand, for reducing the adverse effects of multipole field components in dipoles when these show nontrivial amount of fluctuations from magnet to magnet, sorting (or shuffling) of dipoles in selecting their locations in a given lattice is surely the easiest way available in many cases; for one thing, it is practically cost-free. In real applications, however, many factors come into play in choosing the optimum arrangement: total number of magnets in the ring, number in each cell, natural partition of the ring such as cryoloops and power supply stations, magnet installation schedules, magnet storage capacity, type and scope of the planned correction systems, and magnet acceptance criteria.

The first reported sorting was performed for the Tevatron at Fermilab with a rather limited but well-defined goal.[2] The intention of the sorting was simply to reduce a few (six, to be precise) harmonic components of sextupoles and octopoles that can drive resonances. Subsequently, Gluckstern and Ohnuma proposed a scheme that can reduce a large number of harmonic components around the most troublesome ones when the normal (or skew, but not simultaneously) sextupole field is responsible for the aperture reduction.[3] Improvements in performance with this scheme or its refined versions have been investigated numerically[3-7] but the comparison with unsorted arrangements has been limited to the smear or some other equivalent quantities representing the deviation from linear beam behavior. Although a corresponding improvement in the dynamic aperture can be expected, it has not been demonstrated explicitly. Another weakness of the sorting schemes used in these studies is that only one multipole field is taken into account in the optimizing procedure ("one-parameter" sorting). A candidate for two-parameter shuffling has been suggested by J. Schonfeld at Fermilab[8] but it has not been tested for any realistic lattice. The only reported numerical study of a two-parameter sorting and its impact on the aperture of the proposed SSC lattice has been done by L. Schachinger.[9] Two parameters she considered for the sorting are the skew quadrupole and normal sextupole field errors expected to exist in the SSC dipoles. The present study is motivated by her work and is similar to it but the skew sextupole field errors (rather than the skew quadrupole) are taken into account together with the normal sextupole component. The underlying assumption here is that, with a suitable set of correction packages, the adverse effect of skew quadrupole errors can be reduced to such an extent that their contribution to the aperture reduction is much less than that from the sextupole field errors. Furthermore, in order to demonstrate the effectiveness of sorting in as simple a manner as practicable, it is assumed in this study that all other multipole components are absent in dipoles. This is a departure from Schachinger's work in which all multipoles up to the 20-pole components are included.

Sorting Procedures and Test Lattice

The analytical figure-of-merit (F.M.) has been relied upon extensively in weeding out various sorting schemes which are not very promising. For a few cases, numerical tracking has been performed to insure that F.M. is a reliable semi-quantitative guidance for the resulting aperture. The procedure found to be most effective for one-parameter sorting is essentially the one proposed by Gluckstern and Ohnuma (G/O scheme).[3] It assumes that the sorting is to be done within cells covering the phase advance 2π: six cells and four cells, respectively, for 60^0/cell and 90^0/cell designs. Refinements similar to the one studied by R. Li and Gluckstern[4] have been tried but no substantial improvement has been obtained. The starting configuration of the optimum two-parameter sorting is G/O with the normal sextupole component as the parameter. Magnets are then grouped into pairs, two adjacent magnets making a pair, and the average skew sextupole component of two magnets is assigned to the corresponding pair. Pairs instead of individual magnets are now considered as elements for sorting with the average skew sextupole component as the sorting parameter. The sorting procedure is again G/O. Finally, inside every pair, two dipoles are switched if necessary so that the one with higher (larger positive) normal component is always on the left, for example. This is to avoid having two dipoles with high or low components next to each other when pairs are rearranged. A scheme in which the sorting order is reversed, that is, the skew first and then the normal component, has been tried also but the resulting performance is inferior. This is undoubtedly due to an assumption in this study that the fluctuation in the normal component is twice as large compared with the fluctuation in the skew component.

Lattice: The test lattice used for the tracking is composed of six superperiods, each superperiod containing twelve identical FODO cells with the phase advance 90^0 in both transverse directions. Four dipoles are placed symmetrically in each half-cell which is 30m long. The total number of dipoles is then 576. In addition, each superperiod has one matched insertion such that the tunes are 19.42 (horizontal) and 19.41 (vertical).

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Tracking: Sorting is performed for a group of 32 dipoles at a time and there are eighteen completely uncorrelated sortings to complete the dipole allocation in the entire ring. It is understood in G/O scheme that, if there are cells covering less than 2π phase advance at the downstream end of each superperiod, "good" dipoles will be installed in them so that the contribution from these extra dipoles will be insignificant. For tracking calculations, quadrupoles and dipoles are treated as a thin lens and the sextupole kicks

$$\Delta x' = -S_b(x^2 - y^2) + 2S_a xy \qquad (1)$$

and

$$\Delta y' = 2S_b xy + S_a(x^2 - y^2) \qquad (2)$$

are added at the center of each dipole. If the field in a dipole is expressed as

$$B_y(y=0) = B_0 b_2 x^2 \quad \text{and} \quad B_x(y=0) = B_0 a_2 x^2,$$

the sextupole kick strength parameters are

$$S_b = b_2 \times (\text{dipole bend angle})$$

and

$$S_a = a_2 \times (\text{dipole bend angle}).$$

A uniform distribution between $-S_0$ and $+S_0$ is assumed for S_b; for S_a, the fluctuation is taken to be one-half of this. The normalizing sextupole strenth S_0 is left unspecified since the aperture is also scaled with S_0. Although it is not possible to present results applicable to an arbitrary lattice with only one scaling factor β_0 to normalize the lattice parameters β_x and β_y, a dimensionless amplitude of oscillation in the horizontal direction is defined to be

$$A_x = (\beta_0^{3/2} S_0)\{x^2/\beta_x + (\frac{\alpha_x}{\sqrt{\beta_x}} x + \sqrt{\beta_x}\cdot x')^2\}^{\frac{1}{2}} \qquad (3)$$

and similarly for A_y; the scaling factor β_0 is taken to be 100m which is close to β_{max} = 102m of the test lattice. To find the dynamic aperture, the starting amplitude $A_x = A_y = A_0$ is gradually increased until the test particle is lost before 2000 turns. The definition of "smear" used in this study is

$$\text{"smear"} \equiv \sqrt{\langle R^2\rangle - \langle R\rangle^2} / \langle R\rangle \qquad (4)$$

where

$$R \equiv (A_x^2 + A_y^2)^{\frac{1}{2}} \qquad (5)$$

and < > indicates the average over 500 turns. For the evaluation of smear, it is in general not necessary to extend the tracking beyond this since its value changes little after a few hundred turns.

Results

Five seed numbers are used to generate five sets of random numbers, each set designating sextupole field components in 576 dipoles. For each set of dipoles, five random arrangements (R) and two sorted arrangements S_1 (one-parameter, b_2 only) and S_2 (two-parameter, b_2 and a_2) are used to calculate the smear as a function of the initial amplitude A_0 (=A_x and A_y) and the dynamic aperture for 2000 turns. Results are summarized in Tables 1 and 2; for Set No. 1, the dependence of smear on the initial amplitude A_0 is shown in Fig. 1. Curves for seven different arrangements are terminated at their dynamic apertures. Neither closed orbit deviations nor momentum dispersions are included in the tracking. For Set Nos. 1-5, tunes are fixed at 19.42 (horizontal) and 19.41 (vertical).

Table 1. Dimensionless Dynamic Aperture
See Eq.(3) for the definition.
"R" = five random arrangements,
S_1 = one-parameter (b_2 only) sorting,
S_2 = two-parameter (b_2 & a_2) sorting.

	"R"	S_1	S_2
Set No. 1	.049 ∿ .075	.11	.23
2	.054 ∿ .10	.11	.15
3	.054 ∿ .10	.11	.25
4	.055 ∿ .092	.14	.18
5	.054 ∿ .088	.16	.24

Table 2. Linear Aperture ("smear" = 5%)
See Eq.(4) for the definition of "smear".

	"R"	S_1	S_2
Set No. 1	.026 ∿ .048	.056	.10
2	.028 ∿ .062	.061	.078
3	.027 ∿ .055	.060	.13
4	.026 ∿ .052	.069	.095
5	.033 ∿ .061	.080	.12

With the one-parameter sorting S_1, the improvement in either the dynamic or linear aperture is not impressive. The most one can say is that it will avoid particularly unlucky combinations. When two parameters are included in the sorting, the apertures are increased by a factor two or more in all but one set, Set No.2. To improve the performance for this set, one may have to try some modified procedures rather than the relatively simple scheme outlined in this report. So far, however, this point has not been pursued.

It is interesting to see the change in performance when the tunes are very close to one of the third-integer resonances. Set No. 3 has been studied for this purpose with 1) ν_x, ν_y = 19.45&19.264, $\nu_x + 2\nu_y$ = 58 -0.022 (resonance driven by b_2), and 2) ν_x, ν_y = 19.404&19.208, $2\nu_x + \nu_y$ = 58 +.016 (resonance driven by a_2). Results are shown in Table 3.

Table 3. Apertures near Resonance (Set No. 3)

(ν_x, ν_y) = (19.45, 19.264), $\nu_x + 2\nu_y$ = 58 - 0.022

	"R"	S_1	S_2
dynamic	< .03	.10	.125
linear	< .01	.050	.055

(ν_x, ν_y) = (19.404, 19.208), $2\nu_x + \nu_y$ = 58 + .016

	"R"	S_1	S_2
dynamic	< .02	.015	.15
linear	< .013	.007	.05

The dynamic aperture is reduced significantly for all five random arrangements and for S_2 but no so much for S_1 when the resonance is driven by normal sextupole component. This is understandable since two-parameter

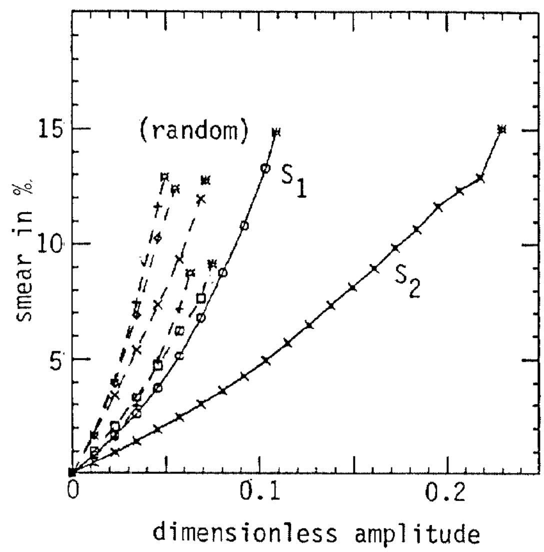

Fig. 1 Smear as a function of the dimensionless initial amplitude A_0 ($=A_x=A_y$) and dynamic aperture. Set No. 1 of Tables 1 and 2.

sortings cannot be as effective as one-parameter sortings. In the process of reducing the adverse effect of skew component, which is irrelevant as far as the resonance $\nu_x + 2\nu_y$ is concerned, one cannot avoid a disturbance to the "best" arrangement which was found by ignoring the skew component. Near $2\nu_x + \nu_y$, which is driven by the skew sextupole component, S_1 should be no different from random arrangements. This also is seen in Table 3.

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