FIELD CONSIDERATIONS FOR NON HOMOGENEOUS BENDING MAGNETS IN A RACETRACK MICROTRON.

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Two Racetrack microtrons being built at the Eindhoven University of Technology will be equipped with low-field sectors in the bending magnets for improving vertical focusing. Calculations performed use hard-edge field approximations and linear matrix theory. Results of these calculations will be given. More realistic fields include interacting fringe fields at all edges. The sectors will be constructed in such a way, that all orbits are both isochronous and closed. For the optimum shape of the sectors, we use a model for approximating the three-dimensional field from the two dimensional magnet-shape for arbitrary sector geometry.

Introduction

In the accelerator laboratory at the Eindhoven University of Technology, two Racetrack microtrons are designed [1] and will be built, suitable for a peak current of 100 A. At these high currents, space charge effects[2] can not be neglected anymore. To maintain a stable bunch, extra focusing forces have to be provided. Fröhlich[3] proposed a three-sector magnet configuration, with alternating high and low field regions. In this magnet configuration, edge focusing between the sectors is used to improve the focusing strengths. Figure 1 shows a schematic drawing of the proposed Ridge-Valley-Plateau (RVP) configuration. Using a hard-edge approximation, it is shown from geometrical considerations [4], that the focusing properties are determined by two parameters: the valley angle δ and the relative depth $a=B_v/B_0$ with B_v the magnetic induction in the valley and B_0 the induction in both plateau and ridge. Using this hard-edge description, it

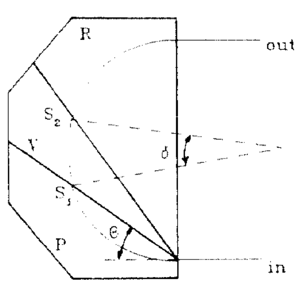

Fig.1 *Schematic drawing of the RVP-configuration.*

turns out to be possible to design a valley shape that satisfies two important requirements: isochronism and closed orbits. Besides these two, we have to meet the requirement of both horizontal and vertical stability. Because we deal with realistic magnetic fields, including fringe fields, we investigated the fringe fields, using 2-D magnetic field calculations. It turns out to be possible to describe the fringe fields with an analytical function. This description allows us to use the hard-edge theory for the realistic fields. As a check, the analytically calculated magnet configuration can be used in 3-D magnetic field calculations.

To check the requirement of stability, numerical orbit calculations have to be performed. It is to be expected that the three requirements will give a selected range of possible a, δ combinations. From this a valley shape will be chosen and the magnets will be manufactured.

Isochronism

In order to accelerate the electron bunches, the synchronous particle must have the same phase with respect to the cavity voltage. This condition is called isochronism. In the hard-edge approximation the orbit length s_n in the n^{th} orbit is given by:

$$s_n = 2(L + s_p + s_v + s_r) \tag{1}$$

with L the drift length (i.e. space between the two bending magnets), s_p the path length in the plateau, s_v the path length in the valley en s_r the length in the ridge. In this hard-edge approximation, the particle travels the angle 2θ in the plateau, δ in the valley and $\pi - 2\theta - \delta$ in the ridge. Setting $B\rho=p/e$ with B the magnetic field, ρ the curvature, e the electron charge and p the momentum, we get an isochronism relation for s_0, the path length of the first orbit and a relation that describes the path length difference $\Delta s=s_n-s_{n-1}$ between two successive orbits. For an arbitrary shaped valley with δ a function of the energy, we have:

$$s_0 = 2L + 2\gamma_0\frac{m_0 c}{e}\left[\frac{\pi-\delta_0}{B_0} + \frac{\delta_0}{B_v}\right] = n_0\lambda \tag{2}$$

$$\Delta s = 2\frac{m_0 c}{e}\left[\gamma_n\left[\frac{\pi-\delta_n}{B_0} + \frac{\delta_n}{B_0}\right] - \gamma_{n-1}\left[\frac{\pi-\delta_{n-1}}{B_0} - \frac{\delta_0}{B_{n-1}}\right]\right] = h\lambda \tag{3}$$

with n_0 and h integers. In these formulae γ is the usual relativistic factor, B_0 the magnetic induction in the plateau and ridge, and B_v the induction in the valley. From equation (2) and (3) we can derive a recurrence relation for δ_n for the n^{th} orbit as a function of the (fixed) values for B_0, a and λ. The solution of this relation is given by:

$$\delta_n\gamma_n = \delta_0\gamma_0 + \delta_c n\Delta\gamma \tag{4}$$

and

$$\delta_c = \left[\frac{eh\lambda B_0/\Delta\gamma}{2m_0 c} - \pi\right]\frac{a}{1-a} \tag{5}$$

δ_0 is the valley angle in the first orbit at an relativistic energy γ_0. In (5) δ_c is called the critical valley angle. The value of angle δ for a triangular shaped valley, as proposed by Fröhlich, is obtained by setting $\delta_n=\delta_{n-1}$. From (5) it is seen, that the shape of the valley in this hard-edge approximation depends on the sign of δ_c. Choosing an accelerator frequency of 1.3 GHz and harmonic number h=1, δ_c changes sign for $B_0/\Delta\gamma=0.0458$. When the relative depth a changes from a<1 to a>1, the sign of δ_c changes also, so that the isochronism relations remain the same for both cases high-low-high and low-high-low field. From (5) we can distinguish three different types of valley shapes: Type I is: δ increases with increasing kinetic energy, type II is the type with δ being constant and type III is: δ decreases as a function of the kinetic energy. In figure 2 [a] these three types are shown.

Using the hard-edge approximation it is possible to derive a relationship between the valley angle δ and the relative depth of the valley from geometrical considerations[4], so that the particles are bent 180^0 in the RVP-magnets. Setting δ constant at every energy and using the geometrical relations we can calculate the position of the points S_1 and S_2 in figure 1. In figure 2^b the valley shapes are presented, belonging to the three shape types I, II

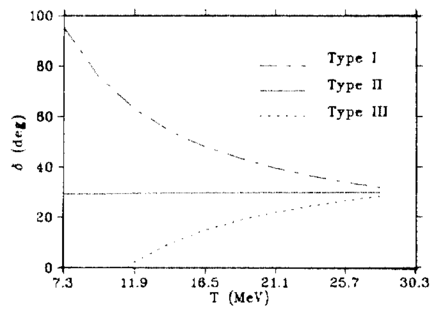

Fig.2[a] *Valley angle δ as a function of the kinetic energy T for the valley shape types I,II and III.*

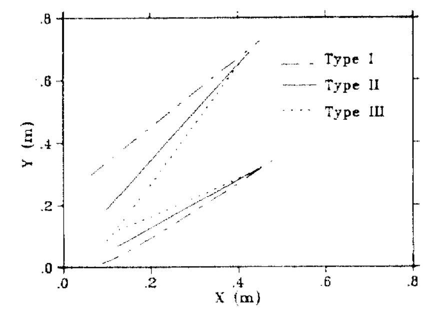

Fig.2[b] *Valley shapes corresponding to the δ-curves in figure 2[a].*

and III in figure 2[a]. Type I describes a valley with almost parallel boundaries. The shape of a type II valley is perfectly triangular, while the type III-shape is a crossed triangle. The disadvantage of a type III-valley is that edge-focusing is only available at higher energies, while type I doesn't provide sufficient focusing. So the only valley shape that will be investigated more in detail now is type II with a constant valley angle δ.

Stability

At this stage we calculated a valley shape, that fulfills the conditions of isochronism and closed orbits. In order to study the relative motion, with respect to the orbit of a central particle, a first order matrix theory for position and divergence deviations is used. In this theory the trace of the matrix M determines

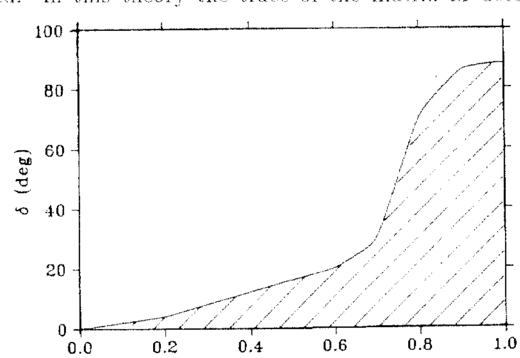

Fig.3 *Stability region as a function of the valley angle δ and relative depth a.*

whether the relative motion of a particle is stable or not. Stability only occurs for $|Tr(M)| < 2$, where Tr(M) is the trace of the matrix M that describes the relative motion. Therefore the trace for both horizontal and vertical movement is calculated. In figure 3 the resulting stability region for a particle at 5 MeV is dashed. It turns out that the stability region will be larger for higher energies, so the stability region is smallest at the lowest energy. To design the shape of the valley we have to choose the microtron parameters B_0, a and δ in such a way, that equation (5) is satisfied and that the combination δ, a is inside the stability region. Doing so yields the combination a≈ 0.40 and $\delta < 20^0$.

Magnetic field calculations

To perform orbit calculations in a real magnetic field with fringe fields, we first have to calculate the three dimensional field in the RVP-magnet. Since there is no symmetry to reduce the 3-D shape to a 2-D one, we have to use cross sections to calculate the magnetic field. This cross section of the RVP-magnet

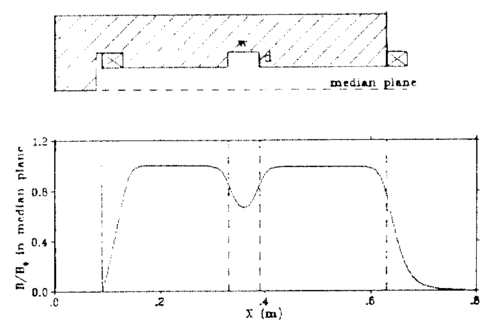

Fig.4 *Schematic drawing of the upper half of a cross section of the RVP-magnet and the magnetic field in the median plane.*

is shown in figure 4. We used the computer code POISSON[5] to perform these magnetic field calculations. From the results we will set up a field model. Measurements have to be performed to check the validity of the used model. Besides, the calculated valley can be used as an input for 3-D magnetic field calculations. In order to derive a model field we investigated the effect of a rectangular valley on the magnetic field in the median plane. This valley is characterized by its depth and width. The POISSON

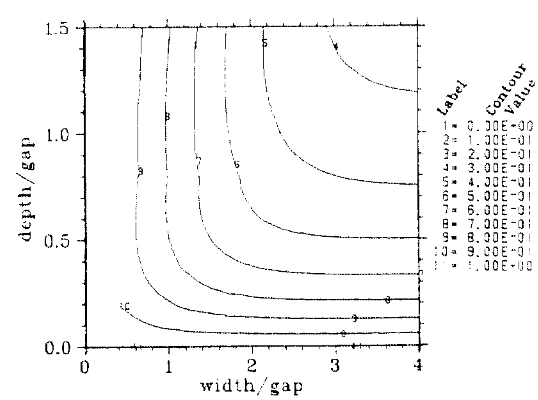

Fig.5 *Contours with constant relative depth a as a function of the width and the depth of the valley. Width and depth are scaled with respect to the gap.*

calculations give the minimum magnetic induction B_{min} and the magnetic induction B_0 in the homogeneous part of the field. The effect of the dimensions of the valley are visible in the relative depth $a = B_{min}/B_0$. Figure 5 shows contours with constant relative depth at different combinations of the depth and width of the valley. Now we postulate[6] that the magnetic field in the valley can be regarded as a mixture of two fringe fields. From curve fitting it appears that the exponential function $\exp[-\alpha(x+\Delta x)^2]$ where x is the coordinate in the fringe field, fits rather well with a root mean square of about 10^{-2}. The mechanical pole boundary is at x=0 and $\Delta x \approx 3$ cm. Because of its simplicity we use only one parameter α to describe the fringe field. The value of α depends on the dimensions and the position of the coil. Setting $f_1(x)$ the fringe field function:

$$f_1(x) = \frac{B(x) - B(\infty)}{B(-\Delta x)} \tag{6}$$

with $B(\infty)$ the induction far away from the pole boundary and b the width of the valley, the mix-function F reads:

$$F(x) = f_1(x) + f_2(x) - f_1(x) f_2(x) \tag{7}$$

Because of symmetry in $x=\frac{1}{2}b$ we write $f_1(x)=f_2(b-x)$. Fitting of the mix-function F to the calculated magnetic field profiles, yields a value for α with a root mean square of about 10^{-2}. From curve fitting it appears that α depends also on de depth of the valley. Inserting the depth dependence of α in the mix-function F and calculating the minimum magnetic induction in the median plane gives a very good agreement between the model field and the field calculated with POISSON.

Fringe Fields

To describe the effect of a fringe field, Enge[7] introduced a SCOFF-field (Sharp CutOff Fringing Field) and an EFF-field (Extended Fringing Field). In our case, the EFF-field is described by the above mentioned exponential function. The position of the effective field boundary EFB in figure 6 is given by $\int B_{SCOFF} ds = \int B_{EFF} ds$. Using the above mentioned exponential function that describes the fringe field, we calculated the EFB:

$$\mathrm{EFB} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} - \Delta x \tag{8}$$

For our configuration with a gap of 0.05 m, a typical value for α is 300 m^{-2}. This yields an EFB ≈ 0.025 m. Enge showed that in the horizontal plane there is no change in focusing strength, although the path length in the SCOFF-field differs from the path length in the EFF-field. The order of magnitude is about 10^{-4} m for $\alpha=300$ m^{-2}. After one revolution, the path difference between EFF and SCOFF-orbit has become about 10^{-3} m. The phase shift with respect to the cavity voltage is about 5^0. In order to keep the orbits isochronous in the SCOFF-field, the drift length must be shortened.

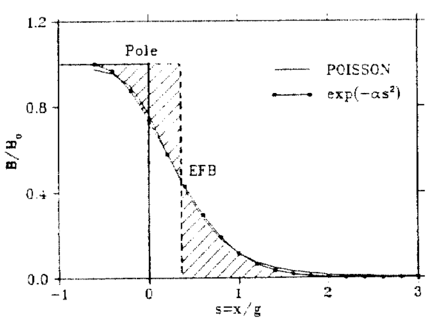

Fig.6 *Calculated and fitted fringe field function. s is the coordinate x scaled with respect to the gap.*

Summarizing, we can describe the EFF-field by a SCOFF-field, introducing the effective field boundary EFB given by (8), so that the isochronism relations (2) and (3) can still be used, although the drift length must change. Besides, the geometrical relations, which describe the shape of the valley, can also be used. Contrary to the horizontal plane, in the vertical plane there is a decrease of focusing strength. Although the bending angle and the path length are almost the same, the stability region showed in figure 3 will change.

Due to the fringe field, the particle is bent more in the plateau over a distance EFB. This implies the bending angle in the valley is smaller. To describe this correctly in the relations (2) and (3), the calculated valley boundaries must be moved over a distance EFB to the outside of the magnet.

Orbit calculations

At this stage we calculated the shape of the valley to provide both isochronism and closed orbits. To check the SCOFF approximation, orbit calculations were performed, using the computer code HIATT[8]. The required input is the magnetic field. To generate the model field we made the following assumptions: the magnetic field at a given position in the valley is described by the mix-function (7), and the distances x and b-x in (7) are the shortest distances to the valley boundaries.

At the neighborhood of the entrance point of the RVP magnet, there are three fringe fields. In order to reduce the effect of this mixing field on the particles, it is suggested to cut off the valley about 0.1 m from the pole boundary.

From orbit calculations, it turns out that the orbits at an energy 7.3-30.3 are closed, using the equations (2) and (3) to calculate the shape of the valley and using the above mentioned assumptions to generate the magnetic field. This proves that the model field provides both isochronism and closed orbits.

Conclusion

In this paper, we described isochronism relations for a three sector magnet, using a hard-edge approximation. From POISSON calculation we fitted a fringe field function, which enables us to describe the realistic magnetic field by a hard-edge approximation. Using this description, a possible valley shape was derived dependant on important microtron parameters, such as magnetic field B_0 and energy gain $\Delta\gamma$ in the cavity. Closed orbits as well as sufficient vertical stability were obtained with the following set of parameters: relative depth $a \approx 0.4$ and $\delta < 20^0$.

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