IMPEDANCE CALCULATION OF RF CAVITIES FILLED WITH GYROTROPIC LOSSY MATERIAL

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ABSTRACT

Usually ferrites are used in tunable cavities of particle accelerators hardware. Properties of ferrites (especially permeability) strongly depend on frequency. Attempts to use usual codes (like SUPERFISH [1]) to evaluate RF cavity characteristics in wide frequency range give unreliable results. Using finite elements method, special code was developed to calculate fields, exited by beam current, in cavities with either rotational or translational symmetry and arbitrary cross-section, below cut-off. In general case nonhermitian tensor with frequency dependent components describes the properties of material. Some examples of application are given.

INTRODUCTION

One of most serious problem in the development of high intensity cyclic accelerators is one to construct chamber with possible low impedance. In frequency range below cut-off are essential narrow-band impedances due to RF cavities of the accelerating system. As usual, to change operating frequency during accelerating cycle, RF cavities partially filled with ferrite are used. The permeability of the ferrite essentially depends on frequency and RF losses in ferrite may exceed RF losses in cavity walls in several orders. For these reasons problem of simulation of the electromagnetic field distribution (especially for high order modes) can not come to linear eigenvalue problem, as it is done in usual codes for RF cavities calculations.

FORMULATION OF THE PROBLEM

Consider the problem of excitation of the cavity with driving beam current given by

$$j = j_0 \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{r^2}{2\sigma^2}\right) \qquad (1)$$

where σ is "effective radius" of the beam. The factor $\exp(-i\omega t)$ has been omitted from all fields and currents.

The distribution of the fields depends on ferrite state. In difference of isotropic dielectrics, tensor describes permeability of the ferrite. The components of this tensor depend on frequency, ferrite properties and external conditions (value and direction of external magnetic field).

We assume cavity to be divided in several subregions, filled with material. Within one subregion components of the tensor do not depend on coordinates. Consider ferrite as a material with given properties, we don't take into account dependence of properties on electromagnetic field value (low level regime). Permittivity of all materials assumed to be complex constant. Let's consider typical (and interesting for practice) variants in cavities with rotational symmetry.

MONOPOLE MODES IN GYROTROPIC LOSSY MEDIUM

In the case of isotropic dielectric only one independent variable (H_φ or E_φ) is needed to describe full field distribution for monopole modes. When ferrite is in gyrotropic state and permeability tensor has nondiagonal components, subdivision into TM and TE modes is illegitimate and two independent variables are needed. After analysis of all couple possible H_φ together with E_φ were chosen.

Assuming permeability tensor given [2]:

$$\hat{\mu} = \begin{vmatrix} \mu & i\mu_\alpha & 0 \\ -i\mu_\alpha & \mu & 0 \\ 0 & 0 & \mu_z \end{vmatrix} \qquad (2)$$

After transformation Maxwell equations are driven equations:

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial rH_\varphi}{\partial r}+\frac{\partial}{\partial z}\frac{1}{r}\frac{\partial rH_\varphi}{\partial z}-\omega^2\left[\frac{\mu_\alpha \varepsilon \partial rE_\varphi}{r\omega\mu\partial z}+\frac{\varepsilon(\mu_\alpha^2-\mu^2)}{\mu}\right]H_\varphi=\frac{\partial j}{\partial r}$$

$$\frac{\partial^2 rE_\varphi}{\partial z^2}+\omega\mu_\alpha\frac{\partial H_\varphi}{\partial z}+\frac{\mu}{\mu_z}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial rE_\varphi}{\partial r}+\omega^2\mu\varepsilon E_\varphi = 0 \qquad (3)$$

At the boundaries ferrite-isotropic dielectric or ferrite-vacuum condition to make tangential components of fields continuous must be satisfied. Problem formulation (3) with permeability tensor (2) describes so called 'perpendicularly biased' state of the ferrite. External constant magnetic field is along z axis and RF magnetic field is in azimuth direction.

MONOPOLE MODES IN ISOTROPIC LOSSY MEDIUM

If external magnetic field and RF one are parallel or external field is switched off, the permeability tensor becomes diagonal

$$\hat{\mu} = \begin{vmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{vmatrix} \qquad (4)$$

In this case (it may be considered as partial case of (3) with $\mu_\alpha = 0$) usual equation for H_φ (or E_φ for TE modes) is valid:

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r H_\varphi}{\partial r} + \frac{\partial}{\partial z}\frac{1}{r}\frac{\partial r H_\varphi}{\partial z} - \omega^2 \varepsilon \mu H_\varphi = \frac{\partial j}{\partial r} \qquad (5)$$

METHOD OF SOLUTION

We assume each component of the permeability tensor to be complex value and solve problem relative to complex amplitudes of H_φ and E_φ. Dividing real and imaginary parts in equations and boundary conditions, we have numerical problem for four independent variables in (3) and for two variables in (5).

Problems, described above, are solved with using finite elements method in Galerkin formulation. In this formulation the continuity of tangential electric field at the boundaries between materials is natural. To satisfy tangential magnetic field continuity special treatment of the boundaries is done. Eight-node quadrilateral isoparametric elements are used. To solve large algebraic system, which is the result of FEM discretization, effective methods are used. Input of the ferrite properties may be done with using either external functions or table data.

IMPEDANCE CALCULATION

Impedance of the cavity is defined as

$$Z = \frac{\int E_z j_z^* dV}{I_0^2} \qquad (6)$$

where I_0 is total current.

To avoid the losses in precision with numerical differentiation, let us transform (6) to:

$$Z = -\frac{4\pi^2 i}{\sigma^2 \varepsilon\omega}\int r H_\varphi \frac{\partial j^*}{\partial r} dr dz - \frac{L\pi i}{\sigma^2 \varepsilon\omega} \qquad (7)$$

where L is the cavity length.

APPLICATION

As the example for calculation, consider possible design of the ferrite- tunable cavity (Fig. 1) for kaon factory synchrotron. There is LANL proposal [3] to use for this purpose perpendicular bias field for changing effective ferrite permeability. It is known, that if perpendicularly biased ferrite is near saturation ($Re\mu_z = 1$ in (2)), RF losses in the ferrite drops drastically [3]. It allows to construct RF cavities with high accelerating voltage at the gap. Consider the cavity without ferrite we shall obtain modes spectrum and impedance values as it shown in Fig. 2a. Suppose ferrite to have properties of the isotropic dielectric with constant permeability and permittivity ($Re\ \mu = 2.6$, $Im\ \mu = 2.6*10^{-3}$, $Re\ \varepsilon = 14$, $Im\ \varepsilon = 1.4*10^{-3}$) we find new spectrum (Fig.2b). Calculations for these examples were made with using MULTIMODE [4] code (it is 'usual' code for RF cavity calculation and losses in the dielectric are calculated using usual perturbation theory).

If we try to approximate real properties of the ferrite in the case of external magnetic field in z direction, components of the tensor in (2) are [2]:

$$\begin{aligned} Re\ \mu &= 1 + 4\pi D^{-1}\gamma M_0 \omega_H (\omega_H^2 - \omega^2) \\ Im\ \mu &= D^{-1}\alpha\omega\gamma M_0 \frac{\omega}{\omega_H}(\omega_H^2 + \omega^2) \\ Re\ \mu_\alpha &= D^{-1}\omega\gamma M_0 (\omega_H^2 - \omega^2) \\ Im\ \mu_\alpha &= D^{-1}\alpha\omega\gamma M_0 2\omega^2 \\ D &= (\omega_H^2 - \omega^2)^2 + 4\alpha^2\omega^4 \end{aligned} \qquad (8)$$

where M_0 is inductance of saturation of the ferrite, α - loss parameter, γ - gyromagnetic constant. Modes spectrum and impedance values for this case are shown at Fig. 2c for $M_0 = 210$ G, $\omega_H = 420$ MHz, $\alpha = 0.1$.

If the external field is switched off and for the dependence of the permeability versus frequency shown in Fig. 3 modes spectrum and impedance values are shown at Fig. 2d.

Comparison shows that results are essential different in frequency regions where ferrites properties strongly depend on frequency.

If losses in the ferrite are relatively small, we can neglect with imaginary parts of μ, μ_α and ε in (3) and solve problem for two variables. From the theory of cavity excitation it is known, that maximum of the field amplitude corresponding to cavity eigenmode with frequency ω_k reaches at:

$$\omega = \omega_k \sqrt{1 - \frac{1}{4Q^2}} \qquad (9)$$

where Q is quality factor. If Q is large enough (losses are small) we can calculate resonance position in lossles approximation. Because for this approximation only two variable are needed in general case, it results in CPU time reduction. Special version of the code for solving (3) in lossles approximation was developed.

SUMMARY

Special code was developed to calculate in 2D approximation electromagnetic field distribution exited with the beam given in the cavities filled with gyrotropic lossy material. Taking into account the dependence of material properties versus frequency allows to estimate more reliable cavity characteristics.

REFERENCES

[1] K. Halbach, R.F. Holsinger, SUPERFISH- a computer program for evaluation of RF cavities with cylindrical symmetry, Particle Accelerators, v. 7, p. 213, 1976

[2] A.G.Gurevitch. Magnetic resonance in ferrites. Nauka, Moscow, 1973 (in Russian)

[3] W.R. Smythe et al. RF cavities with transversely biased ferrite tuning. IEEE Trans. on Nucl. Sci., v. NS-32, n. 5, 1985

[4] A.I.Fedoseev et al., MULTIMODE - a powerfull code for frequency spectrum computation of elrctromagnetic fields in axially symmetric cavities and longitudinally homogeneous waveguides of arbitrary shape. Nuclear Instruments and Nethods, v.227, n. 3, p. 411, 1984

[5] V.V.Nicolsky. Electrodynamics and propagation of radiowaves. Nauka, Moscow, 1973 (in Russian)

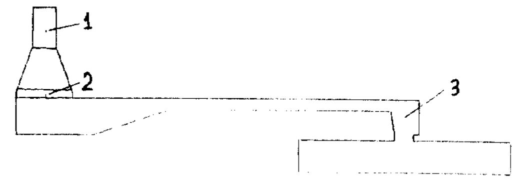

Fig. 1 Ferrite-tunable cavity, 1- ferrite ring, 2- RF ceramic window, 3- accelerating gap.

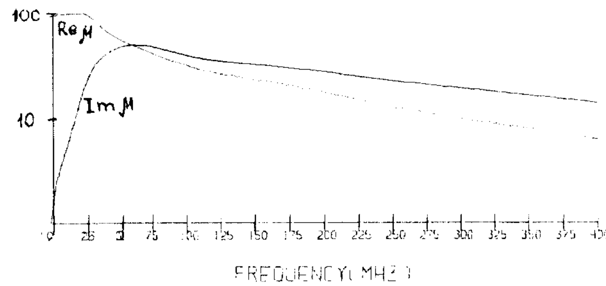

Fig. 3 Example of permeability vs frequency for dispersive ferrite.

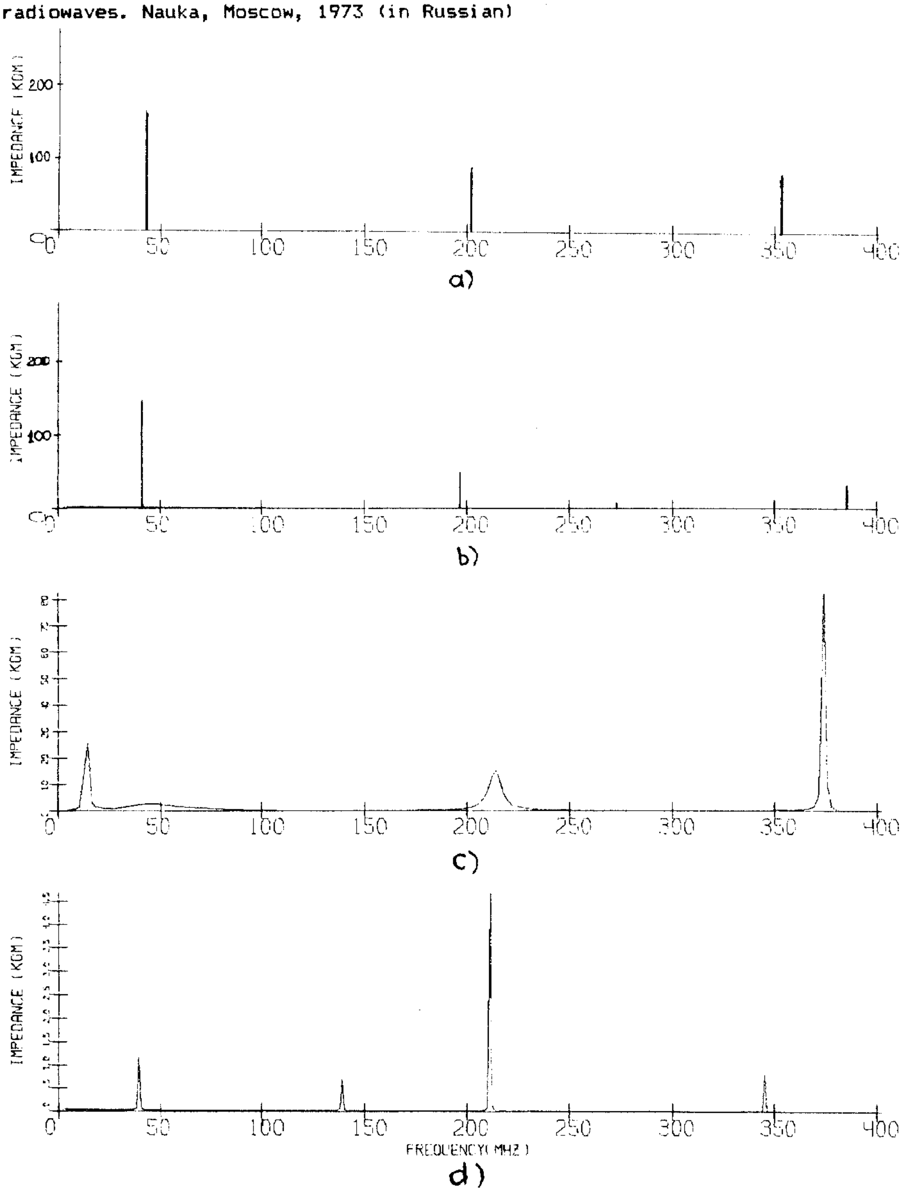

Fig. 2 a) Modes spectra in the cavity without ferrite, b) one in approximation of isotropic dielectric, c) in approximation of gyrotropic material, d) for dispersive lossy ferrite