

## Beam Position Measurement in the CEBAF Recirculating Linacs by use of Pseudorandom Pulse Sequences\*

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### Abstract

The recirculating linear accelerator at CEBAF presents unique problems in beam position measurement. As many as five beams with different energies may be simultaneously in the linac. Modulation of the beam intensity by pseudorandom pulse sequences offers a simple, effective method for distinguishing between the individual beamlets.

### Introduction

In CEBAF the beam will pass through the linac five times before the maximum energy of 4 GeV is reached. In order to correct the trajectory of each beam individually, its position must be measured with a precision of 100  $\mu\text{m}$ . The measurement itself must not disturb the emittance, energy, or bunch length.

As described in previous papers,<sup>1,2</sup> one way to measure each beam individually is to modulate the beam for a period of less than the revolution time, 4.2  $\mu\text{sec}$ . If the monitor electronics are tuned to be sensitive only to the modulation frequency, then the signal will appear sequentially as the modulated pulse traverses the monitor on each of the passes through the linac. The first signal corresponds to the lowest-energy pass, the second to the next higher energy pass, and so on. The separation of the signals is equal to the time required for the beam to go completely around the accelerator. Although this method provides a straightforward measurement of the position of the beam in each pass, it has the disadvantage that each measurement has to be completed within 4.2  $\mu\text{sec}$ .

Recently another solution to the problem of individual beam position measurement has been devised by the authors. We use the fact that the modulation signal is sent both to the gun, where the beam is modulated, and to the monitors. In the monitor electronics, the signal picked up by the monitor is mixed with the modulation signal. The mixer output is a dc level that depends upon the strength of the input and its phase relative to the modulation. It has its highest absolute value when the signals are in phase, and is lowest when they are out of phase by 90°. Such a detection system is called "coherent." We propose to modulate the beam sinusoidally as before, but also to alter the phase of the modulation signal by 180° at intervals in a manner that allows detection of the relative phases. The sequence of shifts is designed so that the accelerator operator can identify the position of each beam independently of the others.

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The separate identification of each beam can be illustrated for the simple case of two passes. Let us assume a modulation amplitude

$$f(t) = A \sin(\omega t) x(t/\tau), \quad (1)$$

where  $\omega$  is the modulation frequency,  $\tau$  is the beam circulation period in the machine (4.2  $\mu\text{s}$  for CEBAF), and  $x(t/\tau)$  is a function with values  $\pm 1$ , changing at integer values of its argument. Let us consider the sequence

$$x = 1, 1, -1, -1, 1, 1, -1, -1, \dots \quad (2)$$

If the signal is mixed with itself, the resulting autocorrelation function is

$$f(t)f(t) = A^2 \sin^2(\omega t), \quad (3)$$

representing the detected signal for the first pass. The time average is nonvanishing and is equal to  $A^2/2$ . If  $f(t)$  is delayed by  $\tau$  and then mixed with the detected signal, however, the correlation is

$$f(t)f(t - \tau) = A^2 \sin^2(\omega t) x(t/\tau) x(t/\tau - 1). \quad (4)$$

The time average is then

$$\begin{aligned} \frac{1}{T} \int_0^T f(t)f(t - \tau) dt &= \frac{A^2}{T} \int_0^T \sin^2(\omega t) dt (1 - 1 + 1 - 1 \dots) \\ &\approx 0. \end{aligned} \quad (5)$$

Thus, the correlation function in this case picks out the signal from the first pass and suppresses that from the second. To suppress the first and observe the second, the signal is mixed with the delayed sequence  $f(t - \tau)$ .

The sequence  $x(t/\tau)$  that has been given for illustrative purposes is not sufficient for three or more passes, but it is easy to extend the argument. It is necessary only to generate a sequence with values  $\pm 1$  such that the autocorrelation function with delay will vanish. Such sequences are already well known in coding theories for communications systems, where they are referred to as pseudorandom sequences.

### Pseudorandom Sequences (PRS)

We define the correlation function  $R(x, y; n)$  by

$$R(x, y; n) = \frac{1}{T} \int_0^T x(t/\tau) y(t/\tau + n) dt, \quad (6)$$

where  $x(t/\tau)$ ,  $y(t/\tau)$  are pulse sequences with values  $\pm 1$ ,  $T$  is the total time of the sequence, and  $n\tau$  is the time delay. We are particularly interested in the autocorrelation,  $x(t/\tau) = y(t/\tau)$ , which should satisfy the orthogonality condition in order for

the sequence to act as a filter:

$$R(x, x; n) = \frac{1}{T} \int_0^T x(t/\tau) x(t/\tau + n) dt = 1 \text{ if } n = 0, \quad (7)$$

$$= 0 \text{ if } n \neq 0.$$

Perfect sequences,<sup>3</sup> which satisfy Eq. 7 exactly, are hard to find and implement. Shift register sequences, however, which will satisfy Eq. 7 to within an arbitrarily small residual error determined by the length of the sequence, can be easily implemented by shift registers with appropriate feedback connections. Programmable array logic (PAL) devices can produce shift register sequences in a single device that needs only a clock pulse. A length-63 PRS generator is indicated in Figure 1; we have used a similar register in our evaluations.

The so-called maximal-length shift register sequence has nearly perfect autocorrelation characteristics. It is referred to as pseudorandom because it has several randomness characteristics<sup>4</sup>:

1. the numbers of +1 and -1 pulses are approximately equal, with a maximum difference of 1,
2. the number of runs of length  $l$  is proportional to  $2^{-l}$ ,
3. the autocorrelation function has only 2 values.

The first of these characteristics is convenient because it means that an electronic implementation will not have to compensate for large shifts in the voltage or current levels introduced by the PRS modulation. The second implies that the levels will not shift appreciably within the time span of the sequence; for example, the longest possible run for a PRS of length 1023 is 10 consecutive +1s or -1s, and this longest run will occur only once. The third is the required autocorrelation condition. The autocorrelation values for maximal-length sequences are

$$R = \frac{1}{N} \sum_{r=0}^N x(r) x(r+n) = 1, \quad n = 0, \quad (8)$$

$$= -\frac{1}{N}, \quad n \neq 0.$$

For sequences of reasonable length, the deviation from perfect autocorrelation is insignificant.

For a PRS that is generated by a shift register of  $n$  stages, the maximum length is  $2^n - 1$ . Note that the orthogonality condition is satisfied only for a complete sequence; a sum of less than a complete sequence will have spurious peaks. It is usually not difficult to use a complete sequence. The characteristics of a three-stage shift register that generates a length-7 pseudorandom sequence are illustrated in Figure 2.

Shift registers also provide another benefit in that they are easily expanded. Addition of another register stage will double the length of the maximal sequence, so with small expenditure one can select a sequence length that will give the processing time required to process the signal and average out the interference.

### Signal Processing with Pseudorandom Pulses

The block diagram for the modulation is shown in Figure 3. The beam is modulated with the frequency  $\omega$  and in addition by a pseudorandom sequence  $Ax(t)$ ,

$$I = I_0 + Ax(t/\tau) \sin(\omega t), \quad (9)$$

where  $x(t/\tau)$  is either +1 or -1. For five beam passes through

the detector, the output signal at the probe is

$$S_d = \sum_{r=0}^4 (k_r Ax(t/\tau - r) \sin(\omega t - r\tau)) + \text{noise}, \quad (10)$$

where  $k_r$  is a constant for each beamlet which depends upon its position. Because noise is generally quite system-specific and cannot be easily handled analytically, we will henceforth neglect its effect; as usual, however, noise will determine the processing time needed for a given signal/noise ratio. When the detector signal is mixed with the input delayed by  $j$  circulation periods, the output is

$$S = \sum_{r=0}^4 k_r A^2 x(t/\tau - r) x(t/\tau - j) \sin^2(\omega t). \quad (11)$$

Integration over  $m$  complete sequences will remove the high-frequency time-varying terms, so the signal average will be

$$\bar{S} = \left[ \frac{1}{2} k_j A^2 - \frac{1}{2N} \sum_{r \neq j} k_r A^2 \right] \left[ 1 - \frac{\sin 2\omega T}{2\omega T} \right], \quad (12)$$

where  $\omega T$  is equal to  $mN\tau$ ; it is seen that the last term in the second brackets is  $O(1/T)$ .

### Development

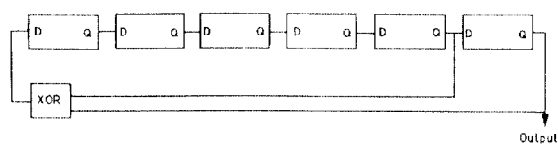
We have done some preliminary evaluation of a PRS generator developed at CEBAF (Figure 4). It is a good simulation of the interaction between a beam position monitor and two beamlets, one delayed relative to the other. Our data show the orthogonality ("delta") characteristic of the PRS autocorrelation function. The results (Figures 5-7) show great promise for arriving at a firm design that will be able to distinguish individual beams in the accelerator. In particular, Figure 5 shows that two uncorrelated signals have little influence upon one another's signals; this corresponds to two beamlets' simultaneous signals on one wire of one beam position monitor. Figure 6 shows the orthogonality of an uncorrelated beam to a particular probe delay; all uncorrelated probe delay values show this same resistance to interference. Finally, Figure 7 shows the output of the integrator as the probe sequence is stepped through the entire sequence of delays with both channels at full amplitude. The two correlation peaks are located at the delays at which the probe sequence is exactly correlated with the two channel sequences.

### Summary

Pseudorandom pulse modulation of the beam current allows measurement of the position of each beam in the presence of other beams. This technique is useful not only for recirculating linacs but for example can also be used in storage rings when the beam position of the injected beam has to be detected in the presence of the stored beam.

References

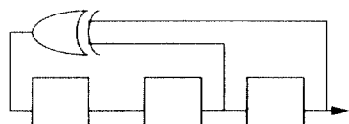
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Shift Register Pseudorandom Sequence Generator

(length 63)

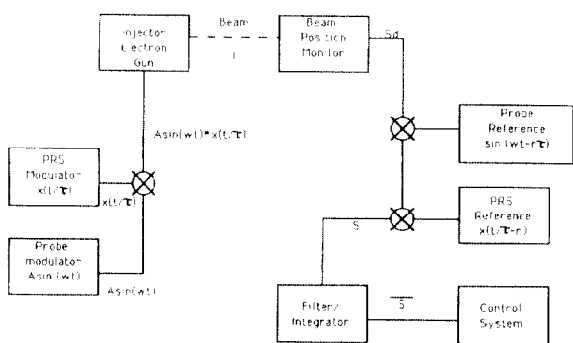
Figure 1



Initial condition	delay	sequence from t=0 to t=6	sequence of $x(t) \cdot x(t-\tau)$	$\sum_{t=0}^6 x(t) \cdot x(t-\tau)$
-1 -1 -1	t=0	-1 -1 -1 -1 -1 -1 -1	+1 +1 +1 -1 -1 -1 -1	2
-1 -1 -1	t=1	-1 -1 -1 -1 -1 -1 -1	+1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=2	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=3	-1 -1 -1 -1 -1 -1 -1	+1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=4	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=5	-1 -1 -1 -1 -1 -1 -1	+1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=6	-1 -1 -1 -1 -1 -1 -1	-1 -1 -1 -1 -1 -1 -1	-3
-1 -1 -1	t=0	-1 -1 -1 -1 -1 -1 -1	+1 +1 +1 -1 -1 -1 -1	2

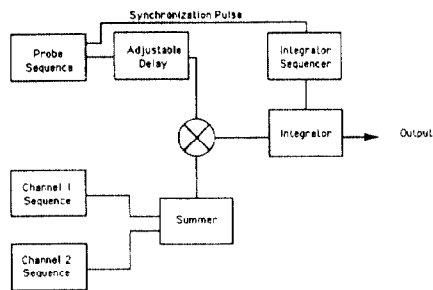
Orthogonality of shifted length-7 Pseudorandom Sequence

Fig 2



Beam Position Monitor Processing

Fig. 3



Pseudorandom Sequence Evaluator

Fig. 4

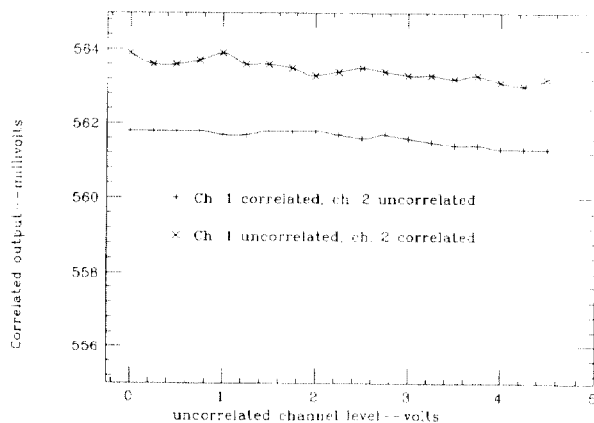


Figure 5

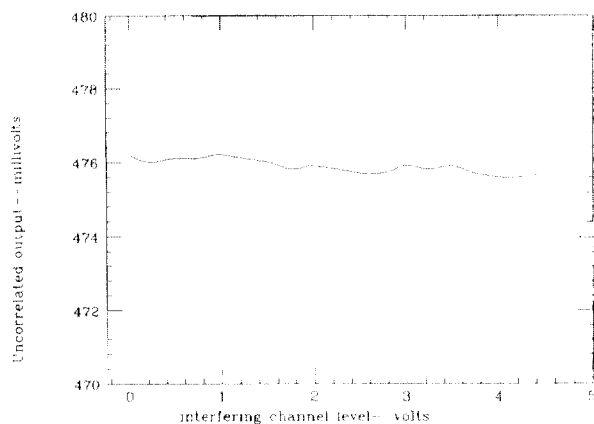


Figure 6

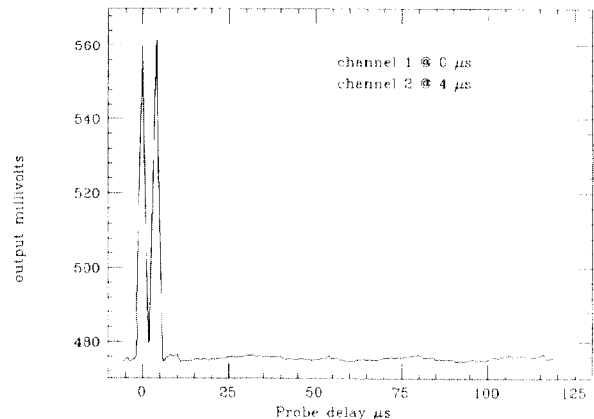


Figure 7