A Strong Focussing Scheme for Hollow Beams in the RWT–Collider

N. Holtkamp, P. Schütt, R. Wanzenberg, T. Weiland

Technische Hochschule
Fachgebiet Theorie Elektromagnetischer Felder
Schloßgartenstr. 8, 6100 Darmstadt

W. Bialowons, M. Bieler, F.-J. Decker*, H.-C. Lewin

Deutsches Elektronen-Synchrotron DESY
Notkestr. 85, 2000 Hamburg 52, Germany

Abstract

For the proposed Resonant Wake Field Transformer (RWT)-Collider the hollow beam driver linac is an essential part. Applying the principle of *Strong Focussing* to the hollow beam has the great advantage of having field free driftspaces and small transverse beam dimensions. The presented hollow beam lens consists of special arrangement of permanent magnets, which nowadays can have remanent fields of more than 1.2 Tesla. The following transverse hollow beam dynamics can be analytically described by the well known formalism of amplitude- and phasefunctions in a comoving frame on a reference hollow beam ring. Calculations concerning a preliminary design of the lens are presented together with the results from hollow beam tracking through this lens.

Introduction

Wake Field accelerators need highly charged hollow beam electron rings to excite electromagnetic waves, which are spatially focussed in resonant *Wake Field Transformer* sections providing high gradients for test beams [1]. In a first experimental set up at DESY hollow electron beams with a diameter of $\simeq$10 cm are accelerated to an energy of 7 MeV in a linac with four 3-cell 500 MHz normalconducting driver cavities [2]. The whole linac is mounted within solenoidal coils of 0.7–0.9 m diameter running with currents of 500–1500 A. The beam is generated in a special gun and guided by an axial magnetic field of $\simeq$0.2 T. Due to the variation of the longitudinal field component, single electrons execute a helical motion and therefore the radial thickness of the hollow beam is increasing along the linac until the particles hit the aperture. But in the final stage the RWT–Collider has to have superconducting cavities and long driver sections which makes this kind of unflexible beam transport impossible to use.

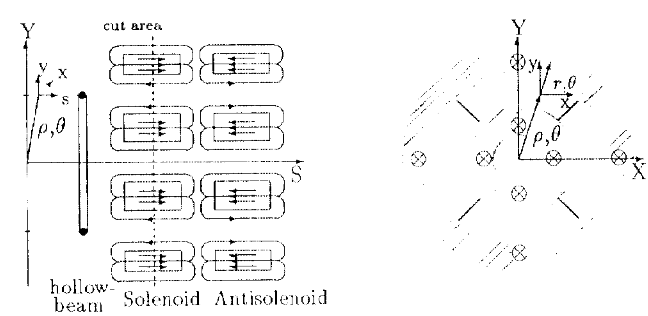


Figure 1 : Principle view of an azimuthal symmetric hollow beam lens.
Permanent magnets with remanent fields of up to 1.2 T can be used to focus hollow beams. The axially symmetric arrangement provides a special soleniod antisolenoid field, which acts like a lens with fokal length $\leq$ 10 m at hollow beam energies $\geq$200 MeV. The equation of motion is described in the comoving frame on the radius ρ. The right picture shows the local coordinates (r,θ) in this frame. Four tags, fixing the inner magnets, can be seen. In both pictures the direction of polarization of the permanent magnets is given by the arrows.

*Now Stanford Linear Accelerator Center, SLAC, California, USA

The alternative is, to have *Strong Focussing* for hollow beams as described by Courant and Snyder [3] for solid beams in storage rings and transport lines. Therefore it is necessary to construct a lens with a field gradient proportional to the distance of the particles from a reference ring as shown in figure 1. This kind of focussing can be provided by permanent magnets which nowadays have remanent fields of more than 1.2 T. The proposed new type of hollow beam lenses allows a focal length of less than 10 m with hollow beam energies of more than 200 MeV.

1 Design of a Hollow Beam Lens

The advantages of using permanent magnetic materials are well known in accelerator physics, especially for wigglers and undulators which become more and more important for the next generation of synchrotron light sources. A Strong magnetic field produced by small pieces of rare earth cobalt (REC) is the essential feature for the construction of the proposed lens, where some part of the magnetic material has to be fixed within the the hollow beam. The goal to have the same focussing characteristics for particles independent if they are above or below the reference trajectory, which is given by a cylinder with radius ρ, claims for this geometry (compare figure 1).

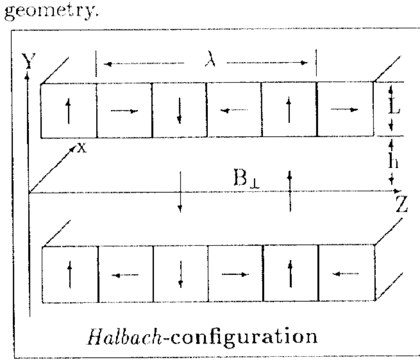
Analytical treatment

The so-called *Halbach*-configuration [4] is a twodimensional model of a REC device normally used to calculate the magnetic field components of an undulator. As shown in figure 2 the direction of polarisation is changed by ninety degree from block to block which gives a strong transverse magnetic field in the middle of the gap of the undulator. The complex analytical formular to express the field is given by:

$$B^*(\xi) = i \cdot B_{rem} \sum_{\nu=0} c_n \cdot \cos(n\,k\,\xi) \tag{1}$$
$$c_n = 2 \cdot \exp^{-n\,k\,h}(1 - \exp^{-n\,k\,L}) \sin(n\,\pi/m)/(n\,\pi/m).$$

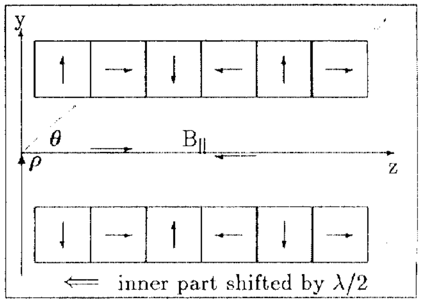
With $B^* = B_z - i\,B_y$, $\xi = z + i\,y$, m = number of blocks per period, ν = index, n = ν·m and c_n = the coefficients of the series that are constant for a chosen geometry.

Figure 2: *Halbach*-configuration with strong $B_\perp$ on the beam axis.
For analytical calculations the configuration is thought to be infinitely long in x-direction. One period is devided into four blocks with a shift in the direction of polarisation by 90°.



Shifting the lower part of the geometry by half a period and estimating the radius ρ to be much greater than two times h in order to have only a small local deviation from the cartesian frame in x direction, the same calculation gives a strong

Figure 3: Hollow Beam lens with longitudinal magnetic field on axis. *The whole structure is azimuthal symmetric with radius ρ in the middle of the gap. The inner part is shifted by half a period compared to Halbachs configuration.*



longitudinal magnetic field in the gap of the hollow beam lens at radius ρ.

$$B^*(\xi) = B_{rem} \sum_{\nu=0} c_n \cdot \sin(n\,k\,\xi), \tag{2}$$

with the same coefficients c_n. The corresponding geometry is presented in figure 3. Splitting the function into the real and the imaginary part leads to the components of magnetic field B_z and B_y ($= B_r$ in this case):

$$\begin{aligned} B_z &= +B_{rem} \sum_{\nu=0} c_n \cdot \sin(n\,k\,z) \cdot \cosh(n\,k\,y) \\ B_y &= -B_{rem} \sum_{\nu=0} c_n \cdot \cos(n\,k\,z) \cdot \sinh(n\,k\,y) \end{aligned} \tag{3}$$

A taylor expansion of the two series displays the required field characteristics for small distances from the middle of the aperture ($|y| \ll h$):

$$\begin{aligned} B_z &\approx +B_{rem} \sin(n\,k\,z) \\ B_y &\approx -B_{rem} \cos(n\,k\,z) \cdot k\,\underline{y} \end{aligned} \tag{4}$$

B_z is independent of y and B_y is proportional to the distance from ρ.

Numeric Field Calculation

A very similar geometry with the radius $\rho = 5\,\text{cm}$ was calculated with PROFI [5] to optimize the shape of the different field components beside the reference trajectory with radius ρ and especially the field around the entrance and exit of the magnet. A compromise is made, choosing a gap of $\rho \pm 1\,\text{cm}$, between a large aperture for the hollow beam within the lens and the corresponding decreased field strength in the gap. Iron is mounted above and below the configuration, but is mainly used to return the flux. Only at the entrance of the aperture iron blocks of different size are inserted to form a radial symmetric magnetic field with respect to ρ. The result can be seen in a contour plot from the magnetic vector potential in figure 4. The polarisation is indicated by arrows and shifted by 90° from block to block as in the two-dimensional model. The length of the magnet is $\simeq$40 cm and the maximum longitudinal field on axis, which will give us the focussing strength, is: $B_z \simeq 0.85\,\text{T}$.

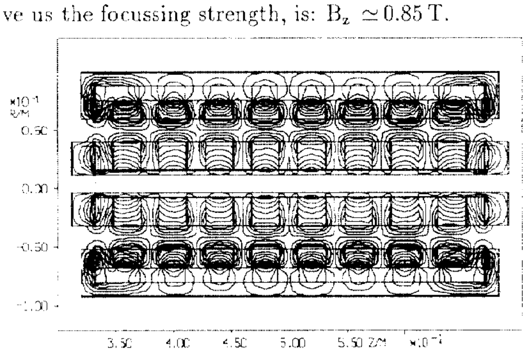


Figure 4: Contour lines of the magnetic vector potential.

In figure 5 the field components B_z and B_r on the reference trajectory and with different displacements are plotted. To reach the symmetric shape for the radial magnetic field different block sizes and remanent field strength' must be chosen in the upper and inner part of the lens. Therefore the geometry has to be

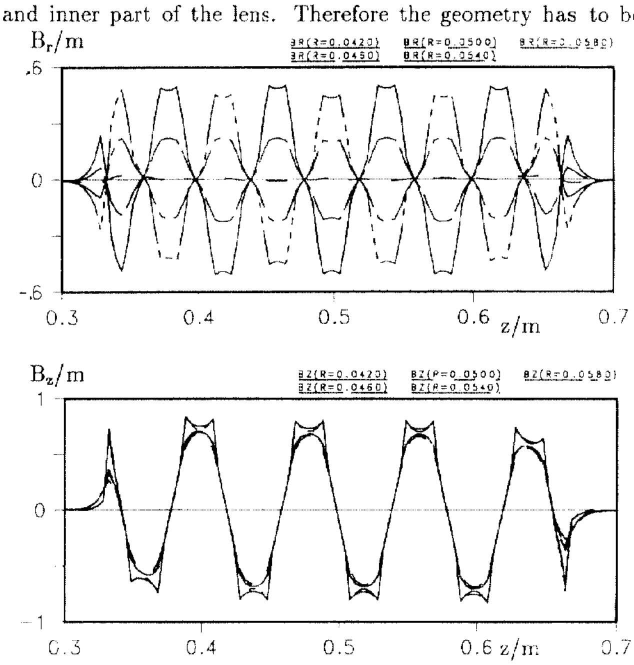


Figure 5: Magnetic field components B_z and B_r along z.

varied until, with $R=\rho$, the radial magnetic field is zero along the entire path in the lens and the focussing force vanishes on the reference trajectory as it is expected. The longitudinal magnetic field increases with the distance from ρ and leads to a symmetric focussing error which is 10% over an aperture of $\pm 5\,\text{mm}$.

2 Hollow Beam Dynamics

The axially symmetric arrangement of permanent magnets shown in fig. 1 leads to a special *solenoid–antisolenoid* combination. Solving the equations of motion in the frame (r, θ, z) under the assumption of a thin lens:

$$\begin{aligned} \gamma\, m_0\, \dot{v}_z &= 0 \\ \gamma\, m_0\, \dot{v}_r &= -q\, v_\theta\, B_z + \gamma\, m_0\, v_\theta^2 \,/\, r \\ \gamma\, m_0\, \dot{v}_\theta &= -q\, v_z\, B_r - \gamma\, m_0\, v_r\, v_\theta \,/\, r \end{aligned} \tag{5}$$

the net azimuthal rotation vanishes if going through the whole magnet [6].

$$\Delta\theta \simeq \frac{q\,B_z}{2\,\gamma\, m_0\, v_z} \cdot \Delta z \tag{6}$$

This is very important, because the inner REC-pieces could be fixed with tags at the entrance and the exit without loosing a significant portion of electrons. Particles with r not equal zero are always focussed towards the axis because the focussing strength is proportional to $(\gamma / B_z)^2$ and therefore independent of the sign of B_z.

$$f = -\frac{r}{r'} \simeq 2 \left(\int dz \left[\frac{q\,B_z}{\gamma\, m_0\, v_z} \right]^2 \right)^{-1} \tag{7}$$

A tracking calculation through the magnetic field of the lens as calculated with PROFI is presented in figure 6. 200 particles with an energy of 200 MeV and a charge of 1 μC are used. It follows from the calculation, that the kinetic energy is high enough to overcome space charge force. The blow up in the focal point is affected by the field errors in the lens.

Amplitude Functions for Hollow Beams

The azimuthal motion of particles in the hollow beam ring can be neglected, because the locations of particles in this direction

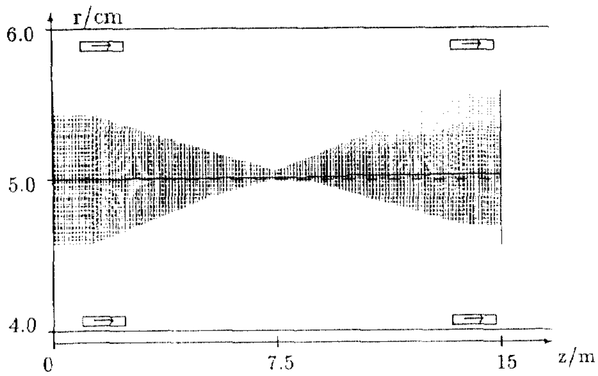


Figure 6: Tracking through the lens with WAKTRAK.
From left to right 200 particles are tracked through the magnetic field of the lens (see fig. 4). The momentum is 200 MeV/c *and the radial thickness of the hollow beam is* 1 cm *In the picture the aperture of* $\rho \pm 1$ cm *is shown along a straight section of* 15 m. *The lenses are marked by* ▭.

are not changed to first order by the lens. Therefore only the radial coordinate has to be regarded.

A hollow beam linac with the proposed magnets consists of driftspaces and focussing lenses. To determine the trajectories in r, a matrix representation of this component in the frame $r, \theta, z = s$ (see fig. 1) can be calculated and is completely analogue to the one known from solid beams in cartesian coordinates.

$$\begin{pmatrix} r_f \\ r'_f \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} r_i \\ r'_i \end{pmatrix} \quad \text{drifspace} \tag{8}$$

$$\begin{pmatrix} r_f \\ r'_f \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\mathrm{K}^2 l & 1 \end{pmatrix} \cdot \begin{pmatrix} r_i \\ r'_i \end{pmatrix} \quad \text{lens}$$

The meaning of the symbols is:

$$\begin{array}{rcl} l & = & \text{length of the } \textit{solenoid antisolenoid} \\ \mathrm{K} & = & q\,c/(2\,p\,c) \cdot \mathrm{B}_z \\ \mathrm{K}^2 \cdot l & = & 1/f \text{ , f = focal length} \end{array}$$

The anology is also valid for the well known amplitude- and phase functions ($\beta(s)$, $\phi(s)$), defined by Courant and Snyder in 1958. The two eigenvectors, calculated from the solution of Hill's differential equation are:

$$\begin{pmatrix} r_1 \\ r'_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\varepsilon\,\beta} \cdot \cos(\phi - \phi_0) \\ \sqrt{-\varepsilon/\beta} \cdot \sin(\Delta\phi) + \alpha \cos(\Delta\phi) \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} r_2 \\ r'_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\varepsilon\,\beta} \cdot \sin(\phi - \phi_0) \\ \sqrt{-\varepsilon/\beta} \cdot \cos(\Delta\phi) - \alpha \sin(\Delta\phi) \end{pmatrix}$$

With:

$$\begin{array}{rcl} \beta(s) & = & \text{amplitude function in s} \\ \varepsilon & = & \text{emittance of the holllow beam in } r \\ \alpha(s) & = & -\frac{1}{2} \cdot \beta(s) \\ \phi(s) & = & \phi_0 + \int_0^s 1/\beta \, dz \text{ phasefunction} \end{array}$$

In the case of hollow beams they are given on the beam radius ρ and they have azimuthal symmetry going at a given point s once around the reference ring with radius ρ.

With the concepts, described in the last section, it is possible to calculate the β-function along a normal cell of a resonant *Wake Field Transformer* linac including the lenses and the driftspaces for superconducting cavties and transformers. The linac can be mounted by a series of such normal cells. For the first calculations a hollow beam radius of 5 cm is given and the half-aperture in the lenses, where the β-function has its maximum, is taken to be 1 cm. Also in the cavities the hollow beam may have quite large dimensions, whereas in the transformer smaller radial diameters are required because of the thin slots in the disks. A possible layout for a normal cell is presented in figure 7.

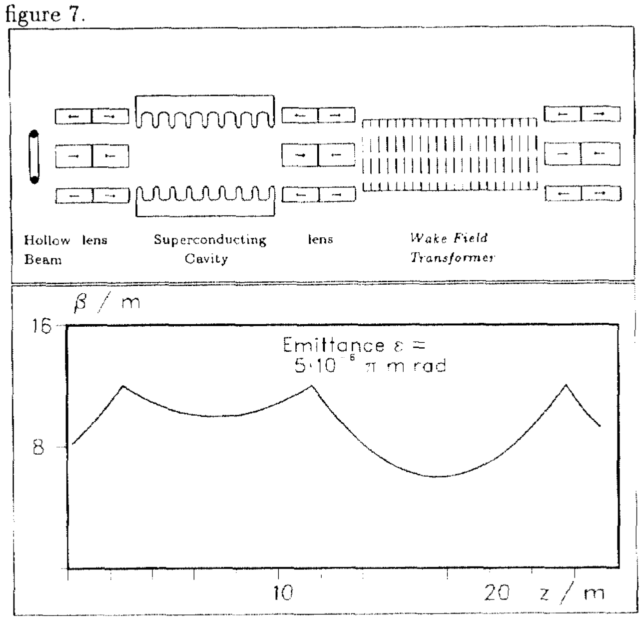


Figure 7: Section in the hollow beam driver linac
The collider consists of several normal cells like the one shematically given in the first picture. The corresponding radial β-function of the hollow beam on the cylinder with radius ρ is given below.

3 Conclusions

The presented hollow beam lens provides the possibility of *Strong Focussing* for hollow beam energies of a few hundred MeV. The focussing characteristics permit the beam transport over quite large distances and enables the design of long sections in the proposed RWT-collider. The lenses, which can be build with well known techniques used for example for REC-undulators, are cheap compared to the energy consuming solenoid–beam transport system in the DESY experiment. Control of the radial thickness of the hollow beam is a second advantage and necessary for the next stage of linear collider experiments.

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