Two-Dimensional Studies of the Laser Wakefield Accelerator

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Abstract

Two-dimensional effects are investigated for the Laser Wakefield Accelerator (LWA) Concept[1,2,3,5]. The nonlinear regime is emphasized[7,8,9,10,11]. A fully three-dimensional and nonlinear fluid equation is derived. Using computer simulations, we find the nonlinear, 1-D theory to be a good guide for calculating the acceleration field even for narrow driving pulses. The possibility for relativistic optical guiding is also considered in light of the recent work by Sprangle *et al*[11].

Introduction

In the laser wakefield accelerator concept a short intense laser pulse of length $\sim \pi c/\omega_p$ is sent through a plasma to excite a plasma wave wake. A trailing bunch of electrons is accelerated by "surfing" on the wake. This concept was first proposed by Tajima and Dawson[1] and was subsequently studied using computer simulations by Sullivan and Godfrey[2] and Mori[3]. The necessary laser technology was not available at that time so an alternative concept called the plasma beat wave accelerator was proposed[4]. However, beginning with the linear analysis of Sprangle *et al.*,[5] there has been a renewed interest in the LWA owing to recent advances in laser technology[6].

Most of the recent research on the LWA has been concerned with 1-D nonlinear effects[7,8,9,10,11]. There are three reasons for considering the nonlinear regime. First, nonlinear drivers lead to an increase in the wake's phase velocity, thereby increasing the dephasing length for the accelerated particles. Second, nonlinear plasma waves lead to an increase in the wake's wavelength, enabling the use of longer laser pulses for a given plasma density, or the use of higher density plasmas (hence higher accelerating gradients) for a given pulse length[7]. Third, it is necessary for the drivers to be relativistically self-focused[12,13,14] (optically guided) so that wakes can be excited over many Rayleigh lengths. When a light pulse self-focuses in plasma, its radius reduces to a size on the order of a collisionless skin depth c/ω_p. For this spot size the value of $\frac{eE_o}{mc\omega}$ is greater than unity when the self-focusing power threshold ($P > 20\,\frac{\omega^2}{\omega_p^2}$ GW) is exceeded. Therefore the laser amplitude is typically nonlinear.

These nonlinear analyses have been limited to one dimension. However, since the laser spot size is in general on the order of a c/ω_p, then transverse derivatives can no longer be neglected. A two-dimensional analysis is therefore required. In this paper we will use particle-in-cell computer simulations in order to examine the LWA in the nonlinear, two-dimensional regime. We first derive a fully nonlinear three-dimensional fluid equation[8,11,15]. We reduce this to the one-dimensional limit and discuss some important consequences for self-focusing deduced from the 1-D equations by Sprangle *et al.*[11] We then present two-dimensional simulations which show qualitative agreement with these conclusions for spot sizes as narrow as $2\,c/\omega_p$.

Nonlinear Fluid Equations

In this section we outline the derivation of a single equation for the fluid momentum $\vec{p}$. We start from Maxwell's equations, the continuity equation and the relativistic Euler's equations for a cold plasma. By substituting Faraday's law into Euler's equation we obtain

$$\frac{\partial}{\partial t}\vec{V} + \nabla\times\vec{V}\times\vec{v} \qquad (1)$$

where $\vec{V} \equiv \nabla\times(\vec{p} - \frac{e\vec{A}}{c})$ is defined as the vorticity and $\vec{A}$ is the vector potential. The importance of eq. (5) is that it implies $\vec{V} = 0$ forever if it vanishes at $t = 0$ over all space.

To derive the nonlinear equation for $\vec{p}$ we substitute Ampere's law into the curl of $\vec{V}$:

$$\nabla\times\nabla\times\vec{p} = \frac{e}{c}\left[\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}\right] \qquad (2)$$

An expression for $\frac{\partial\vec{E}}{\partial t}$ can be obtained by differentiating Euler's equation:

$$\frac{\partial}{\partial t}e\vec{E} = -\frac{\partial^2}{\partial t^2}\vec{p} - m\frac{\partial}{\partial t}\nabla\gamma \qquad (3)$$

Finally, replacing $\vec{j}$ by $-eN\vec{p}\gamma$ and using Gauss' law in eq. (2) gives

$$\left[\nabla\nabla\cdot - \nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{p} + \frac{\omega_p^2}{c^2}\frac{\vec{p}}{\gamma}\left[1 + \frac{\partial}{\partial t}\nabla\ \ \vec{p} + \nabla^2\gamma\right] + m\frac{\partial}{\partial t}\nabla\gamma = 0 \qquad (4)$$

where $\gamma = (1 + \frac{p^2}{m^2c^2})^{1/2}$ and $\omega_p^2 = \frac{4\pi e^2 N}{m}$.

This nonlinear equation for only the fluid momentum completely describes the evolution of the plasma since all the other fields can be derived from $\vec{p}$.

1-D Limit

We assume that $\vec{p}$ varies in only the $\hat{x}$ direction and that it is a function of the single variable $\zeta = x - ct$. We assume $v_o = c$ which is equivalent to assuming $\frac{\omega^2}{\omega_p^2} \gg 1$. The x component of eq. (4) reduces to

$$\frac{d^2}{d\zeta^2}(\gamma - p_x) = \frac{1}{2}\left[\frac{\gamma_\perp^2}{(\gamma - p_x)^2} - 1\right] \qquad (5)$$

where $\gamma_\perp^2 = 1 + \frac{e^2E_o^2}{m^2c^2\omega^2} = 1 - \frac{v_o^2}{c^2}$ and p_x is normalized to mc. From eq. (3) we find $\gamma - p_x = 1 + \phi = \frac{\gamma}{N}$ where ϕ is normalized to $\frac{mc^2}{e}$ and N to the ion density $\bar{n}_o$. Eq.(5) has been solved analytically and numerically by various authors. Berezhiani and Murusidze[8] have shown analytically that for square shaped driving pulses the maximum value of $1+\phi$ is $\sim\gamma_\perp^2$ and the maximum vlaue of E_x is $\sim\gamma_\perp$. For gaussian shaped pulses, we find $E_x \sim \frac{\gamma_\perp}{2}$. These results can be found in ref. 7. Results from 1-D computer simulations also agree with these scaling laws.

In the 1-D limit the transverse component of $\vec{p}$ is due solely to the driving pulse and it is described by the transverse part of eq. (4).

$$\left[\frac{-\partial^2}{\partial x^2} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{p}_\perp = \frac{-\omega_p^2}{c^2}\frac{N}{\bar{n}_o}\frac{\vec{p}_\perp}{\gamma}$$

If $\frac{N}{\bar{n}_o}\frac{1}{\gamma} = 1$ (linear theory) the phase velocity of light in plasma is $v_\phi = c/(1-\frac{\omega_p^2}{\omega^2})^{1/2}$. The phase velocity is reduced when $\frac{N}{\bar{n}_o}\frac{1}{\gamma}$ becomes less than unity. Wavefronts self-focus in plasma because $\frac{N}{\bar{n}_o}\frac{1}{\gamma}$ is typically smaller at those positions where $P_\perp$ is largest.

It was thought that the entire portion of a light pulse which exceeded the self-focusing power threshold would be optically guided, since γ and N individually respond on ω^{-1} (laser) time scales. However, Sprangle et al. pointed out that the ratio $\frac{N}{\bar{n}_o\gamma}$ responds on ω_p^{-1} time scales. This is seen by noting that $\frac{N}{\bar{n}_o}\frac{1}{\gamma} = \frac{1}{1+\phi}$ and $1+\phi$ is described by eq. (5). Consequently, they concluded that it is not possible to optically guide (self-guide) laser pulses in the LWA because the pulses are only $\pi c/\omega_p$ in length. Their analysis was essentially one-dimensional. For narrow laser pulses the quantity $\frac{N}{\bar{n}_o\gamma}$ will no longer be equal to $\frac{1}{1+\phi}$. The importance of the difference is discussed in the next section.

Computer Simulations

In this section we present 2-D PIC simulation results. The simulation code WAVE was used. The laser pulses are injected from the left-hand boundary with a frequency $\frac{\omega}{\omega_p} = 10$. The light is polarized in the z-direction (out of the simulation plane) with a transverse field profile $\cos^2\frac{\pi}{2}\frac{y}{y_o}$ and a longitudinal pulse width $\pi c/\omega_p$. All lengths are in units of c/ω_p and fields are normalized to $\frac{mc\omega_p}{e}$.

We begin by examining 2-D effects on the plasma wake. In figs. 1a, b, and c, we plot the longitudinal electric field vs. position for $y_o = \infty$, 10 and 4 respectively. The amplitude of the driving pulse was $\frac{v_{osc}}{c} = 4$ and its field is plotted in fig. 1d. The numerical results given in ref. 7 predict a maximum value of 1.8 for E_x. We find reasonable agreement in fig. 1a where E_x is nearly 1.6. It should be noted that better agreement is obtained when longer system sizes are used. In these simulations the system was only 15 c/ω_p long. As the driver's width is reduced, it is seen in fig. 1 that E_x is only slightly reduced. Therefore, even when the driver's width approaches c/ω_p the 1-D predictions are still a good guide for determining the accelerating field strength. This is a significant result because the transverse derivatives in eq. (4) can no longer be neglected.

We next consider the tendency of the driving beam to self-focus when $\vec{E}$ is polarized in the translationally invariant direction. For this polarization nonlinear term in eq. (4) is simply $\frac{-\omega_p^2}{c^2}\frac{N}{\bar{n}_o\gamma}\vec{P}_\perp$. Sprangle *et al.* argued that for $y_o \gg c/\omega_p$, $\frac{N}{\bar{n}_o\gamma} = \frac{1}{1+\phi}$ and that the quantity $\frac{N}{\bar{n}_o\gamma}$ therefore responds on ω_p^{-1} time scale. We performed 1-D simulations for which the incident laser had a rise time of $.5\,\omega^{-1}$. The laser then maintained its peak amplitude for the duration of the simulation. The quantity $\frac{N}{\bar{n}_o}\frac{1}{\gamma}$ was carefully monitored. The results are summarized in fig. 2. The ratio $\frac{\omega}{\omega_p}$ was 5, 10 and 20 for the simulations shown in figs. 2a, b and c respectively. The x axis in fig. 2c is normalized to $\frac{c}{2\omega_p}$ rather than c/ω_p. The results show that, although $\frac{N}{\bar{n}_o\gamma}$ begins to respond after a single laser cycle, it takes $\sim \omega_p t \cong \frac{\sqrt{8}}{\gamma_\perp^2 - 1}$ to reach its asymptotic value. We note that even when the plasma wave gets large enough for a significant nonlinear frequency shift, $\frac{N}{\bar{n}_o}\frac{1}{\gamma}$ still responds

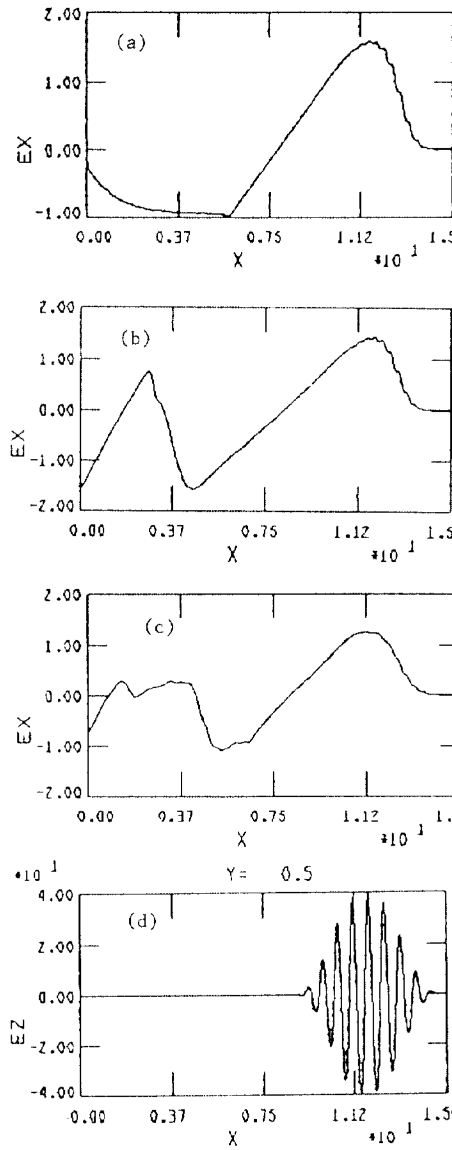

Figure 1

on a time scale of the linear plasma frequency. This is important because in the nonlinear LWA concept the driver pulse is $\frac{\pi}{\omega_{pNL}} = \frac{\gamma_\perp}{\omega}$ long. Therefore, if $\gamma_\perp$ is large a substantial fraction $(1-\frac{\sqrt{8}}{\gamma_\perp^2})$ of the driving pulse is initially optically guided.

In fig. 3 we plot $\frac{N}{\bar{n}_o\gamma}$ from 2-D simulations for which the driver's pulse length was $\pi c/\omega_p$. The simulation had $y_o = 10$ and 4 and $\frac{v_{os}}{c} = 4$. We find that initially along the axis of the laser $\frac{N}{\bar{n}_o\gamma}$ behaves as it did in the 1-D simulation. However, after $\omega_p t \cong 1$ it becomes considerably smaller because of the transverse blow out of the plasma electrons. This would seem to indicate that narrow nonlinear LWA pulses may be more easily guided.

Finally, we compared $\frac{N}{\bar{n}_o\gamma}$ to $\frac{1}{1+\phi}$ for the narrow beam, $y_o = 4$, simulation. In fig. 3c and d we plot $\frac{N}{\bar{n}_o\gamma}$ and ϕ vs. y at an x position within the driving pulse. We find that the relative phases of $\frac{N}{\bar{n}_o\gamma}$ and $\frac{1}{1+\phi}$ are in agreement, but their amplitudes are not. This

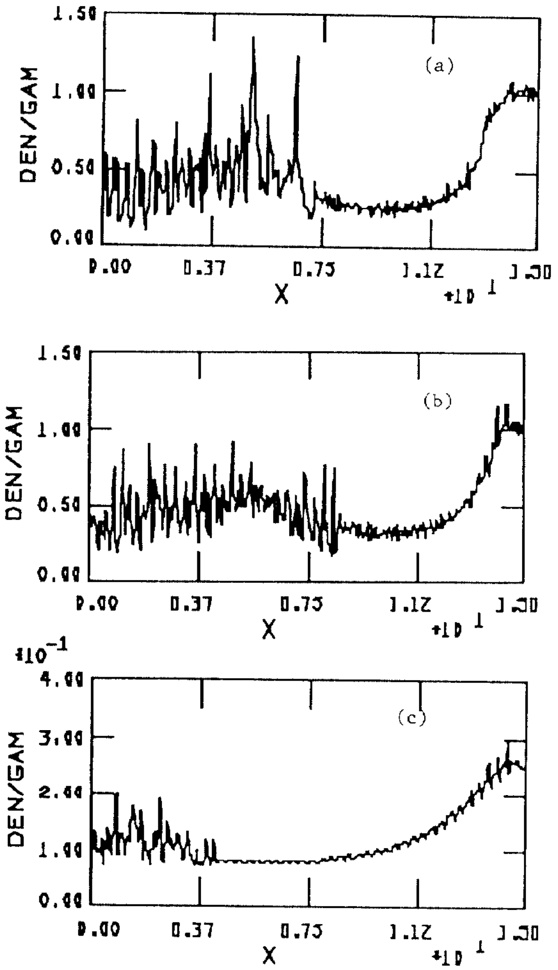

Figure 2

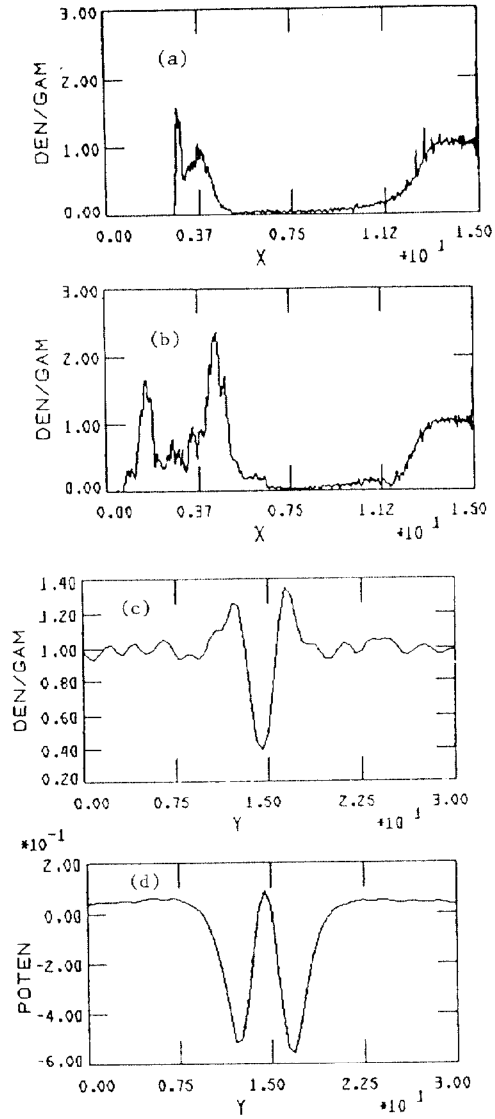

Figure 3

indicates that the 1-D arguments which equated $\frac{N}{\bar{n}_0\gamma}$ to $\frac{1}{1+\phi}$ give the correct qualitative behavior, but a more rigorous 2-D analysis is necessary for narrow $(y_o \sim 1)$ driving pulses.

Summary

In this paper we have presented preliminary results from 2-D simulations. These simulations were done to study the nonlinear, LWA concept for laser pulses with c/ω_p spot sizes. We found that the 1-D nonlinear theory gives reasonably accurate estimates of the accelerating field. We also found qualitative agreement between the conclusions of Sprangle et al.[11] and the 2-D simulation results. We have not performed short pulse simulations over many Rayleigh lengths for large values of ω/ω_p. This is an area for future work. Lastly, we note that experiments are being planned jointly between UCLA and LLNL in the USA to test both wakefield generation and relativistic optical guiding. This laser is a 10TW 1 μm in laser with a pulse width of 1 ps.

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