STUDY OF EDDY ELECTROMAGNETIC FIELDS GENERATION PROCESSES IN LINEAR INDUCTION ACCELERATORS (LIA)

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Abstract. Methods for analysis and synthesis of circuits for accelerating voltage formation in linear induction accelerators (LIA) are given. The results of computations, physical simulation and experiment are presented.

Introduction

High cost and labour consumption of full-scale tests and many other factors make necessary to perform thorough computer analysis and synthesis /1/, physical simulation /2/ and optimization in the region of eddy currents fields generation in LIA. In this report the computational and experimental results on electron source accelerating voltage formation in applied linear induction accelerators are given. The computational procedure of the current threshold value for the beam transverse instability evolution is also described.

1. Solution for a synthesis problem

Let's suppose that the voltage pulse $U(t)=I(t)\cdot R$ should be formed at load R.

Let's represent the generator equivalent scheme as the seriesly connected forming circuits Z(p), voltage source $\frac{E}{p}$, commutator inductance pL, load R.

Then the forming circuits parameters (Fig.1) will be computated by the following formulae:

1) for 2-cell circuits

$$C_0=\frac{0{,}493}{R\,\omega_1};\quad L_1=\frac{R}{1{,}905\cdot\omega_1};\quad C_1=\frac{1}{L_1\,2{,}42\,\omega_1^2}\qquad(1)$$

where W_1 - is the main harmonics for expansion into a series;

2) for 3-cell circuits:

$$C_0=\frac{0{,}34}{\omega_1 R};\quad L_1=\frac{R}{1{,}636\,\omega_1},\quad C_1=\frac{1}{L_1\cdot 1{,}73\,\omega_1^2};\qquad(2)$$

$$L_2=\frac{R}{1{,}376\,\omega_1},\qquad C_2=\frac{1}{L_2\cdot 1{,}64\,\omega_1^2}$$

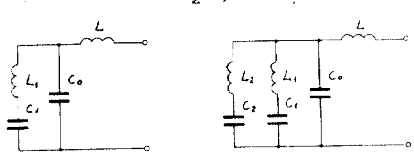

Fig.1. Pulse forming circuits

The synthesized circuit parameters depend upon load-R resistance, including nonlinear inductor parameters and beam current. R-magnitude, in its turn, depends upon forming circuit configuration and parameters. Therefore, forming circuit parameters are defined more exactly by iterations, including aboveconsidered circuit synthesis and analysis by obtained parameters.

2. Analysis of voltage pulse formation

Fig.2 shows the physical model principal scheme. Inductance L displays total thyratron and storage capacitance spurious inductance. Further two or three cells forming line model was used instead of L_n.

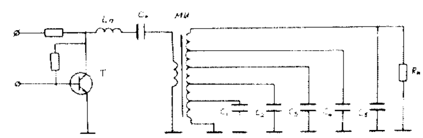

Fig.2. Principal scheme of model of pulse generator for accelerator LIA-I, 25-200 injector transformer module

Inductor secondary winding is divided into five sections connected with the case by means of C_1-C_5 - capacitances presenting the distributed capacity of inductor secondary turn, connected with the electron source and electron source capacitance. Investigation carried out on the physical model permitted to define exactly necessary elements of accelerating module equivalent scheme and their parameters. Then the accelerating module equivalent scheme for computer analysis was constructed by the results of physical simulation. Further each synthesis iteration included analytical computation by the formulae (1) or (2) and numerical simulation by the equivalent scheme shown on Fig.3.

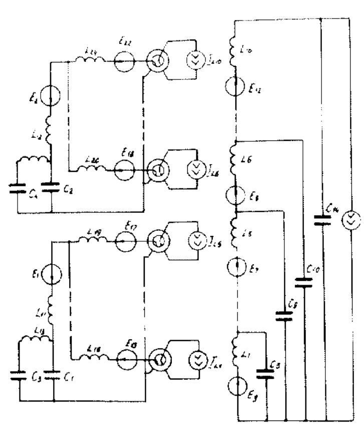

Fig.3. Equivalent scheme of accelerator LIA-I,25-200 injector transformer module

3. Algorithm for circuit analysis

As there are a large number of elements in equivalent scheme of accelerating voltage formation system (up to 48 in studied scheme), the computer was used to form and integrate the state equations.

The state equations are set up defining the circuit directed graph, described by complete incidence matrix /Aa/. Matrix dimensional representation is n x b, where n - is the number of units, b - number of elements. /Aa/ matrix columns are filled with the following priority: E,C,R,L,I. Writing the Kirckhoff's equations in the matrix form: $|B_u||\vec{u}| = C$, $|D||\vec{i}| = C$, where $\vec{u}$ is the voltage drop vector, i - currents vector, the following matrices were used: $|B_u| = |B_T 1_\mu|$, $|D| = |1_\rho D_L|$, where T and L mean belonging to the tree and communications ρ=n-1, μ=b-n+1, b - number of graph branches. To obtain the system of differential equations let's introduce such matrix /A_1/ that:

$$|A_1| = |\vec{u}_L \vec{i}_C \vec{i}_R \vec{i}_E \vec{u}_I \vec{u}_R \vec{i}_L \vec{u}_C|^T = |B_1| \left|\begin{matrix}\vec{E}\\ \vec{I}\end{matrix}\right| \qquad (3)$$

Matrices /A_1/ and /B_1/ are obtained by block filling from matrices /Ba/ and /D/. Having solved the system of algebraic equations (3) relatively to $\vec{u}_L, \vec{i}_C, \vec{i}_R, \vec{i}_E, \vec{u}_I, \vec{u}_R$, we find:

$$|\vec{u}_L \vec{i}_C \vec{i}_R \vec{i}_E \vec{u}_I \vec{u}_R|^T = |B_2||\vec{E}\vec{I}|^T - |A_2||\vec{i}_L \vec{u}_C|^T \qquad (4)$$

As $u_L = L\frac{di_L}{dt}$ and $i_C = \frac{du_C}{dt}$, then /4/ may be rewritten as follows:

$$\left|L\frac{d\vec{i}_L}{dt}\, C\frac{d\vec{u}_C}{dt}\right|^T = |B_{21}||\vec{E}\vec{I}|^T - |A_{21}||\vec{i}_L \vec{u}_C|^T \qquad (5)$$

On the base of this equality the system of differential equations in normal form, integrated by the 4-th order RUNGE-KUTTA method has been found.

4. Computational and experimental results

As computations have shown the synthesis process converges after two or three iterations. The forming circuit obtained from these computations was tested on the accelerating module physical model. Two-cell circuit version for voltage pulse formation at the electrons source cathode with 300 kV amplitude at 250 A - loading was realized in accelerator injector. For obtained pulses the top fall in the middle part is typical.

For pulse better shape series of computations has been performed and 3-cell forming line scale-model has been developed and operated. The equivalent scheme for computation differs from that on Fig.3, third cell is added. Several iterations aimed at definition of forming line parameters have been made. The computational result of the transient process with synthesized forming line is shown on Fig.4,a; pulse shapes, obtained on the physical model - on Fig.4,b and experimental results obtained on real accelerating module - on Fig.4,c. These results testify to good correlation of computational and experimental data.

5. Gradient-ravine optimization

For various applications different time dependencies of accelerated particles energy are needed. To realize the preset pulse shape gradient-ravine optimization method has been developed.

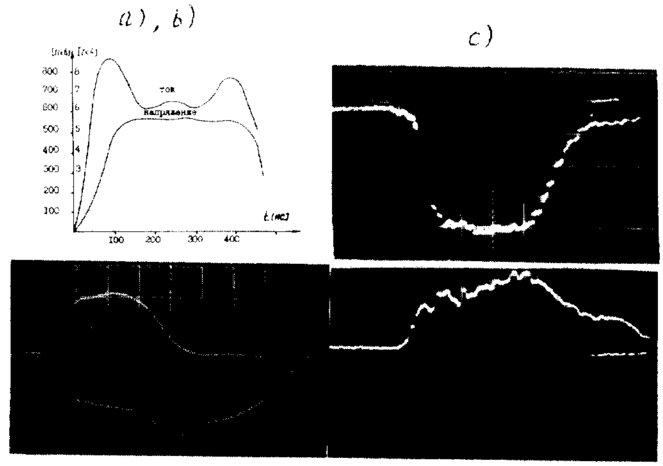

Fig.4. Pulse shapes (3-cell line): a - graphs of computated load voltage and thyratron current pulses; b - load voltage and inductor current pulse waveforms (physical model); c - pulse waveforms of inductor primary winding voltage and thyratron current.
Scale: horizontal - 100 ns/division (b,c);
vertical - 5 V/division (b);
5 kV/division (c)

Let's consider the electric circuit with the preset configuration and parameters. Introduce the following vectors:

$$\vec{L} = (L_1, L_2, \ldots L_{NL})^T,\ \vec{C} = (C_1, C_2, \ldots C_{NC})^T,\ \vec{R} = (R_1, R_2, \ldots R_{NR})^T,$$
$$\vec{E} = (E_1, E_2, \ldots E_{NE})^T,\ \vec{I} = (I_1, I_2, \ldots I_{NI})^T,\ \vec{B} = (B_1, B_2, \ldots B_{NB})^T$$

where L,C,R.E,I,B - vectors components represent the inductances, capacitances, resistances, voltage and current sources, core inductions, respectively; NL,NC,NR,NE,NI,NB - their number.

The following dependence:

$$P = \int_{t_1}^{t_2} |X_3(t) - X_K(t)|\,dt \qquad (6)$$

will be considered as a functional where $X_3(t)$ - is a preset dependency and $X_k(t)$ - value of "K"-th vector of the component.

To define the antigradient direction the following procedure was used:

1. Transient process in a circuit was computated with initially preset controls.
2. Components of gradient vector was defined by the difference formula:

$$\partial P/\partial Y_i \approx \Delta P/\Delta Y_i = (P_{NEW} - P_{old})/\Delta Y_i \qquad (7)$$

All the having been found, the gradient vector was normalized per unit:

$$\partial P/\partial Y_i|_n \approx \partial P/\partial Y_i \Big/ \sqrt{\sum_n (\partial P/\partial Y_n)^2} \qquad (8)$$

and then new controls were found:

$Y_{iNEW} = Y_{iold} - h\cdot\partial P/\partial Y_i$, where h > C and "h" numerical value defines the step length in the antigradient direction.

The initial computations have shown that P-function spatial relief in a control space is of ravine character. Therefore gradient method appeared to be low efficient. So, the minimum search strategy was changed. Supposing approximate symmetry of ravine slopes in proximity to its bottom, the vector of ravine gradient will be found as the sum of normalized gradients at ravine opposite slopes.

Computational results are illustrated on Fig.5. To rise the functional sensitivity to controls change, weight coefficients, depending upon time, were used. Obtained results show that such method is effective for electrophysical apparatus forming circuits synthesis.

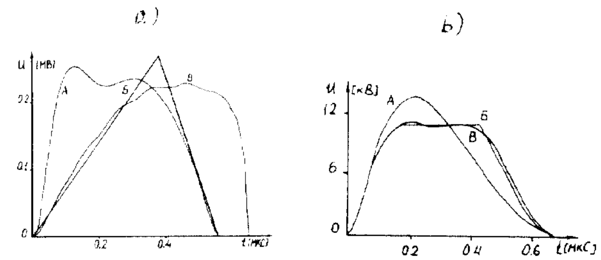

Fig.5. Pulse shapes: a - accelerating voltage at the rod; b - at the core; A - initial, Б - preset, B - resulting

6. Analysis of the beam interaction with asymmetric modes of an accelerating tract

Particles in linear induction machines are accelerated with azimuthally symmetric electric field E_0, excited by pulsed generator, taking into account the beam loading effect. Besides, RF-oscillations of dipole type (m=1) are possible, what may result in transverse beam instability /4/.

For pulse shortening phenomenon further study applied to LIA design, developed in Efremov Institute, cavity model was used /5/.

The equation for field amplitude is:

$$\dot{\tilde{P}} + \alpha \tilde{P} = \frac{i}{2\omega} \underset{t}{M} \left(e^{i\omega t} \int_V \vec{j} \vec{L}\, dv \right) \qquad (9)$$

where $\alpha = \omega/2Q$; w - frequency; Q - quality factor; $\vec{L}$ - vector eigenfunction.

Particles dynamics was defined from equation:

$$\ddot{\tilde{X}} = \ell_1 e^{-i\varphi} - \Omega \dot{\tilde{X}} \qquad (10)$$

where $X = x + jy;\quad \ell_1 = \frac{a_{ic}}{2i\beta_0 u m} \frac{e}{} \tilde{P},$

$$\Omega = j\frac{e}{m} B_z,\quad \varphi = \omega t - \beta_0 z;\quad a_{ic}^2 = -\frac{\omega^2 a}{2\pi c^3} \frac{\lambda}{h} \frac{R_{\omega\perp}}{\varphi}$$

h - cavity length; a - channel radius; B_z - focusing field.

From (9),(10) follows that instability evolution depends upon current constant component with the threshold value for a single section:

$$I_{thr} = \frac{2c}{\pi R_{\omega\perp}} \frac{|B_z|}{Re\, \tilde{z}} \frac{\lambda}{h} \qquad (11)$$

where $\tilde{z} = \frac{1}{\theta}\left[\frac{1-\cos\theta}{\theta} + i\left(\frac{\sin\theta}{\theta} - 1\right)\right];\ \theta = h\left(\beta_0 - \frac{\omega}{v}\right)$

The field amplitude for n-th section is varied according to ratio /5/:

$$\dot{\tilde{P}}_n + \alpha_1 P_n = \varkappa_1 \sum_{K=1}^{n-1} \tilde{P}_K(\tau) \qquad (12)$$

where $\tau = t - (n-1)\frac{h}{v}$

Values of ω, $R_{\omega\perp}$ and Q were defined experimentally, measurements were performed at low-power level. For 13.0 cm multisectional vacuum tube made of X-22C ceramics with covar cones, applied in LIA-5/5000, dipole-type resonance was revealed at 3.555 GHz - frequency with $Q \approx 10$ and $R_{\omega\perp}/Q$ = 1 k /m. These values at h=1 m; γ_0 =2 and B_z=0.05 T correspond to threshold current I_{thr}=500 A.

Conclusion

As the experimental investigations show, described methods for accelerating voltage pulse formation permit to reduce significantly the time for tuning of LIA pulsed generators forming circuits. Developed method of gradient-ravine optimization makes possible to define parameters of generators forming voltage pulse of arbitrary shape. Relationship obtained analyzing the beam interaction with RF-modes of an accelerating tract permit to evaluate threshold current for the beam transverse instability evolution.

References

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