LOW EMITTANCE PHOTON BEAM FOR A SYNCHROTRON RADIATION SOURCE*

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Abstract

The opening angle of the light cone emitted by a particle stored in a synchrotron radiation source is usually given as

$$\Psi \approx \frac{1}{\gamma}\sqrt{\frac{\lambda}{\lambda_c}}$$

Other expressions are proposed, which take into account the machine parameters, and the actual distribution of the photons.

The distribution of the radiation emitted by a circulating beam is the convolution of the distribution of the particles in the beam and of the abovementioned distribution. An emittance can then be defined for the emitted photon beam, and a limit set to the attainable brightness.

The spectral brightness of a light source measures the density of the energy emitted in the phase space of the photon beam. This brightness is obviously related to the emittance of the storage ring, but one has then to take into account the opening angle of the emitted photons, especially in the case of the so called low emittance rings.

Opening of the Light Cone

The well known formula :

$$\psi \approx \gamma^{-1}\,(\lambda/\lambda_c)^{1/2} \qquad (1)$$

describes the angular width of the radiation emitted by a storage ring. Fig. 1 to 3 show the actual dependance of the r.m.s. values of $\gamma\psi$ vs λ_c/λ for the total flux, and for its polarized components, perpendicular and parallel to the medium plane.

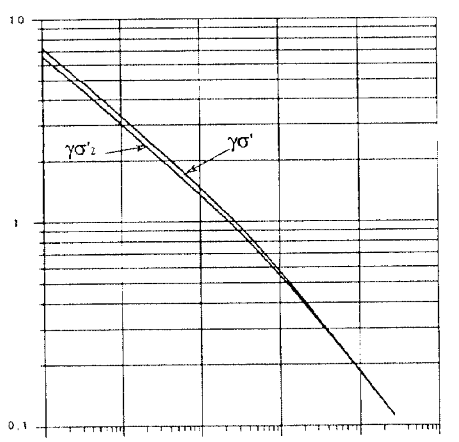

Fig. 1 : Total opening angle
σ' is the r.m.s. value
$\sigma'_2 = \sigma'$ for a gaussian distribution.

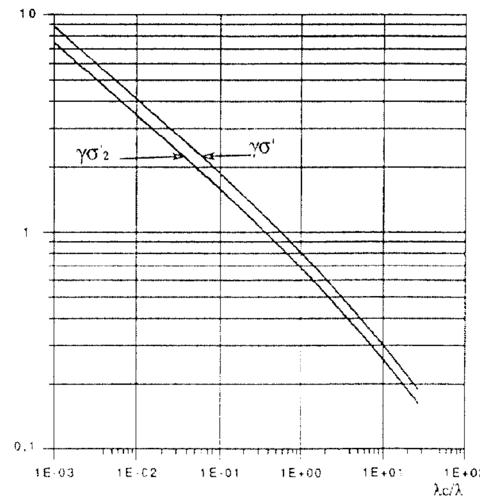

Fig. 2 : Opening angle of the normal component.

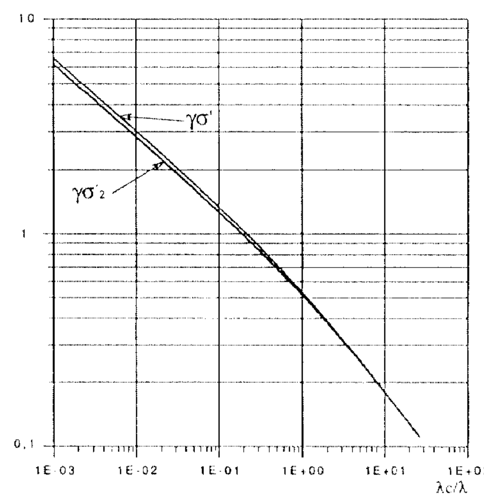

Fig. 3 : Opening angle of the parallel component.

At long wavelengths, the total r.m.s. width is :

$$\sigma' \cong 0.7235 \text{ rd} \times \gamma^{-1}(\lambda/\lambda_c)^{1/3} \qquad (2)$$

and for short wavelengths,

$$\sigma' \cong \gamma^{-1}(\lambda/3\lambda c)^{1/2} \qquad (3)$$

the graphs of $\sigma'_2 = (3.\langle\psi^4\rangle)^{1/4}$ show that the distribution of the radiation is nearly gaussian for the parallel component, especially towards the short wavelengths.

For practical uses, these formulae need to be worked out. Indeed, the energy dependance suggested by the explicit factor γ must be corrected by taking into account λ_c. Beeing interested in small emittances, I shall concentrate first on the high energy part of the spectrum, where the divergence is the smallest.

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The critical wavelength is :

$$\lambda_c = \frac{4\pi}{3} \times \frac{\rho}{\gamma^3} \tag{4}$$

or

$$\lambda_c = 5.59 \times \frac{\rho}{E^3} \tag{5}$$

or else

$$\lambda_c = 18.6 \times \frac{1}{B\,E^2} \tag{6}$$

where the units are Å for λ, m for ρ, T for B, and GeV for E. Using these formulae, equality (3) becomes :

$$\sigma' \cong \sqrt{\frac{\lambda}{4\pi} \times \frac{\gamma}{\rho}} \tag{7}$$

or

$$\sigma' \cong 68.3\sqrt{B\lambda} \tag{8}$$

in µrd. And if B ≈ 1.6 T,

$$\sigma' \approx 85\sqrt{\lambda} \tag{9}$$

Note that the energy does not enter any more into these formulae, unless the radius of curvature is fixed.

Photon Beam Emitttance

The phase space distribution of the stored particles is bigaussian, with r.m.s. values $\sqrt{\varepsilon\beta}$ in position, and $\sqrt{\varepsilon/\beta}$ in angle. At the emission point, the spatial distribution of the photons is the same as that of the emitting particles, but their divergence is the convolution of the angular distribution of the particles with the opening of the light cone. For the sake of simplicity, I shall assume this cone to have a gaussian profile. The photon flux will then also be bigaussian, and one can define an emittance ε_1 and a β_1 corresponding to the new r.m.s. values

$$\sigma_1 = \sqrt{\varepsilon_1\beta_1} = \sqrt{\varepsilon\beta}$$

$$\sigma'_1 = \sqrt{\varepsilon_1/\beta_1} = \sqrt{\varepsilon/\beta + \sigma'^2}$$

whence

$$\varepsilon_1^2 = \varepsilon^2 + \varepsilon\beta\sigma'^2 \tag{10}$$

The photon flux emittance appears to be the quadratic sum of the particle beam emittance and of the product $\sigma.\sigma'$. Since σ is the source size, and σ' the minimum divergence, $\sigma\sigma'$ can be described as the intrinsic emittance of the photons.

A Proposed Definition of "Low" Emittance

The formula (10) suggests the definition of two classes of storage rings according to their emittances, or more precisely according to the ratio of the ring emittance to the intrinsic emittance of the photons.

When this ratio is high, ε_1 is close to ε. The optics of the storage ring determines entirely the brightness of the source. One may then say that the ring has a high emittance.

When this ratio is low, ε_1 is close to $\sigma.\sigma'$. The optics of the storage ring determines only the size of the source. One may say that the ring has a low emittance. In this case, ε_1 varies like the square root of ε.

For typical conditions such as $\lambda \approx 1$ Å, B ≈ 1.6 T and $\beta \approx 5$ m, the low emittance condition reads $\varepsilon < 10$ nm.rd. This figure is easily achieved in the vertical plane. In the horizontal plane, for either a bend or an undulator, the horizontal width of the light beam is always greater than the divergence of the stored beam, and we still are in the low emittance situation defined above, although for different physical reasons.

Then, in both planes, the brightness is limited by the opening of the light cone and by the size of the source. The figure of merit of a storage ring is then the density of the beam, which varies like the square root of the emittances. According to the type of apparatus used, the relevant number is either the flux divided by the beam height, or the flux divided by the cross section. Unfortunately, the latter quantity is called "emittance" by the photometrists.

On the Longer Wavelength Side

The approximation (3) which is used above is valid for wavelengths smaller than about 3 λ_c. For longer wavelengths, (2) is better suited and equalities (7), (8) and (9) must be replaced by

$$\sigma' \cong 0.7235\left(\frac{3}{4\pi} \times \frac{\lambda}{\rho}\right)^{1/3} \tag{11}$$

$$\sigma' \cong 140\,(\lambda\ B/E)^{1/3} \tag{12}$$

$$\sigma' \cong 163\,(\lambda/E)^{1/3} \tag{13}$$

Comparing the two sets of equalities, one can tell their range of validity. The σ' given by (8) and (12) are equal for $E = 8.5\sqrt{\lambda\ B}$, that is, when B = 1.6 for $E = 6.7\sqrt{\lambda}$.