DIAGNOSTICS AND INSTABILITY STUDIES OF COOLED ION BEAMS

I. Hofmann, K. Beckert, S. Cocher and U. Schaaf
GSI Darmstadt
P.O.B. 110552
6100 Darmstadt, Germany

Electron cooling in storage rings leads to phase space densities, at which beam transfer function and Schottky noise signals can have unexpected features. They come mainly from the fact that for nonrelativistic energies the impedance is dominated by the imaginary space charge impedance and that the threshold of instabilities is approached during cooling. Theoretical results for Schottky spectra and BTF measurements are used to discuss the interpretation of presently available experimental data for beams cooled by electrons. These data indicate that the conventional Keil-Schnell-threshold has already been exceeded by a factor larger than five. The relevance of these methods to the diagnosis of ultracold beams, where the friction force plays a role, is discussed.

Introduction

The analysis of the noise from the random distribution of particles in circular accelerators has become a standard diagnostics technique since the first observations at the CERN ISR [?]. In low-intensity (more precisely, low phase space density) beams the phases of particles in a coasting beam are uncorrelated. Since each particle excites an electromagnetic signal at the harmonics of its revolution frequency, the longitudinal noise spectrum of a beam at low phase space density directly yields the distribution of momenta. This was the case in the early ISR measurements, where the microwave instability of the injected beam was believed to lead to phase space dilution and thus to an uncorrelated noise spectrum. The thus obtained momentum distribution was used to determine the transverse coupling impedance from the measurement of the transverse beam transfer function (BTF). Due to rf stacking the transverse phase space density was high enough and lead to a considerable "collective" shift of the stability diagram proportional to the coupling impedance and the current. For some assumed value of the impedance a momentum distribution could be calculated. If it agreed with the Schottky measurement, the correct impedance was found.

The development of electron cooling and, most recently, of laser cooling has lead to high phase space densities in the longitudinal as well as the transverse direction. The Schottky spectra are considerably distorted by collective effects (double peaks or "ears"), as has been shown by measurements of several groups [1,2,3]. Effective cooling leads to the boundaries of longitudinal and transverse stability, in which case the consistent interpretation of the data of Schottky noise and beam transfer function measurements is not straightforward, and a comparative support from theoretical calculations becomes necessary.

It is interesting to note here that much earlier a behaviour analogous to the double peaks has been observed in scattering of electromagnetic waves from plasmas. Calculations pertaining to the backscattering of radar from the ionosphere have shown that the scattered spectrum for a "hot" electron plasma is Gaussian ("incoherent scattering"), whereas for "low" temperature there is a shifted peak at positive, and another one at negative frequencies [4]. This peak has been related to "coherent scattering" at the frequency of plasma oscillations. The oscillations cause spatial correlations over distances of the wavelength of the oscillation, which then lead to enhanced scattering at these particular frequencies.

Analytical Treatment

In the following we present some basic relationships for Schottky noise spectra and BTF of cooled beams (see also Ref.[5,6,7]) as well as numerical evaluations. The model used here is that of a beam described as a collisionless ensemble of particles, with correlations among particles only due to the (macroscopic) electric field of coherent oscillations. We deviate from this "collisionfree plasma" approach only in the last section, where the strong friction force from the cooling electron beam is discussed.

Longitudinal Schottky Spectrum

The origin of the electromagnetic fluctuations in a circulating beam is the statistical noise with no correlations in the case of low phase space density. On a longitudinal pick-up one observes at each harmonic p of the revolution frequency a power spectrum, which is given by the familiar expression [8]

$$P_{\|}(\Omega, p) = \frac{q^2 e^2}{\pi} \frac{N}{p} \Psi_0(\Omega/p) \tag{1}$$

with Ω the observed frequency, q the charge state, N the total number of ions and $\Psi_0(\omega)$ the equilibrium distribution of revolution frequencies.

For a cooled beam this expression has to be modified in the following sense: the electromagnetic fluctuations due to the random noise act as a source term to which we must add the collective response of the beam.

$$E_{total} = E_{source} + E_{collective} \tag{2}$$

As usual in electromagnetic theory this can be done by introducing a dielectric function connecting the total electric field with the source term according to

$$E_{source} = \epsilon E_{total} \tag{3}$$

The dielectric function is calculated from the collisionless Vlasov equation by first order perturbation theory. For longitudinal perturbations one obtains the familiar expression for the one-dimensional plasma dielectric function

$$\epsilon_{\|}(\Omega, p) = 1 - iZ^* D(\Omega, p) \tag{4}$$

where D is the dispersion integral given by

$$D = \int \frac{\partial \Psi_0/\partial \omega}{\Omega - p\omega} d\omega = PV \int \frac{\partial \Psi_0/\partial \omega}{\Omega - p\omega} d\omega + i\frac{\pi}{p} \partial \Psi_0/\partial \omega \tag{5}$$

and Z^* is related to the coupling impedance and the (electrical) current I according to

$$Z^* = Z_{\|} \frac{\eta q I}{\beta^2 \gamma A m c^2/e} \tag{6}$$

with A the ion mass, and η introduced as $\delta\omega/\omega = \eta \delta p/p$. It is important to note here that for nonrelativistic energies the space charge impedance

$$\frac{Z_{\|}}{p} = -i\frac{1 + 2ln(R_p/R_b)}{2\beta\gamma^2} 377 \quad (Ohm) \tag{7}$$

is in general much larger than the pure machine impedance, in particular the resistive (real) part of the impedance.

With the dielectric function and using Eq.3 we thus obtain the general expression for the Schottky power spectrum

$$P_{\|}(\Omega, p) = \frac{q^2 e^2}{\pi} \frac{N}{p} \frac{\Psi_0(\Omega/p)}{|\epsilon_{\|}|^2} \tag{8}$$

It is noted that for small impedance or current we have $\epsilon \approx 1$, and thus Eq.1. On the other hand, if we decrease the momentum width of a beam with given current, ϵ can become very different from unity indicating a large collective response as will be calculated in the next section. Here we only note that for small momentum spread the spectrum develops two sharp peaks. They become (formally) infinite, if for some real Ω

$$\epsilon_{\|}(\Omega, p) = 0 \tag{9}$$

which is the condition for the existence of real eigenfrequencies (i.e. the dispersion relation). Eq.9 is at the same time the boundary to instability to be discussed next.

BTF, Stability Curve and Keil-Schnell-threshold

The beam transfer function $r_{\|}$ is defined as ratio of beam response to the excitation voltage on a kicker, where we have to add the additional collective response as above. It is usual to compare the collective response with a feedback loop given by an impedance. The relationship can be written in the same fashion as Eq.8, where the source term is the response in the absence of the impedance, rsp. intensity. Hence we have

$$r_{\|} = \frac{r_{\|,0}}{\epsilon_{\|}} \tag{10}$$

where as usual

$$r_{\|,0} = -i\frac{\eta q I}{\beta^2\gamma A m c^2/e}\int\frac{\partial\Psi_0/\partial\omega}{\Omega - p\omega}d\omega \tag{11}$$

The inverse response is the stability diagram, which is written as

$$\frac{1}{r_{\|}} = \frac{1}{r_{\|,0}} + Z_{\|} \tag{12}$$

hence the shift of the stability diagram gives directly the impedance The boundary of stability is reached, if the inverse response vanishes, in accordance with Eq.9. In Fig.1 we plot in the complex $Z_{\|}$ plane the curve $1/r_{\|,0}$ for three distinct momentum distributions, but normalized to the same $(\Delta p/p)_{fwhm}$: a quadratic $(1 - x^2)^2$; a Gaussian e^{-x^2}; and a bi-Gaussian $e^{-x^2} + \frac{\alpha}{4}e^{-(x/2)^2}$, where a fraction α (here $\alpha = 0.4$) of the main distribution is contained in the broader Gaussian. For stability the point $-Z_{\|}$ must be inside the respective curve. Due to its sharp edge the quadratic distribution has the smallest stability region, whereas the extended tails of the bi-Gaussian lead to an enlarged stable area near the positive imaginary axis [9]. This is the direction, which is relevant for the space charge impedance below transition energy, whereas above transition the stability margin extends only to the lower boundary.

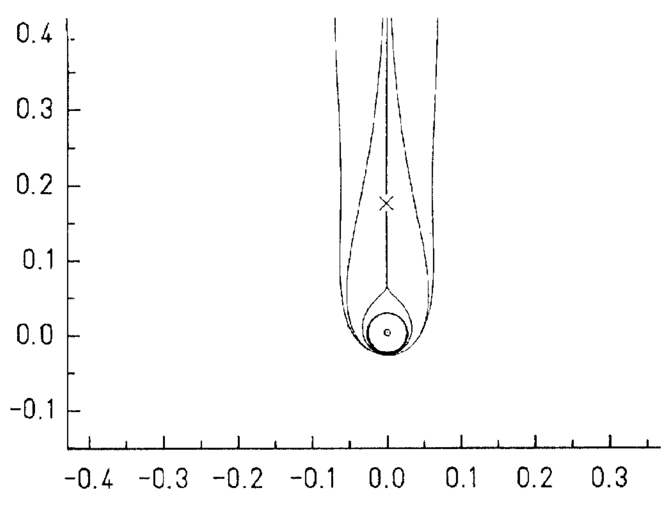

Fig.1 Stability Curves and Keil-Schnell-Circle

It is noted that the distance to the lower boundary is practically the same for all three distributions, hence it is a measure for $(\Delta p/p)_{fwhm}$. In order to relate this to real numbers it is convenient to define a circle in the complex $Z_{\|}$ plane with a radius extending to the lower boundary as indicated in Fig.1, which is usually understood as "Keil-Schnell-threshold". This we can readily calculate by inserting into Eq.11 the frequency at the center of the distribution, $\Omega = p\omega_0$, and using the Gaussian. Hence we find:

$$|Z_{\|}| \leq |1/r_{\|,0}|_{p\omega_0} = \frac{|\eta|\beta^2\gamma A m c^2/e}{qI}\frac{\pi}{ln2}\left(\frac{\Delta p}{p}\right)_{hfwhm} \tag{13}$$

where the momentum spread is the half fwhm value.

How to Interpret Measured Schottky Spectra?

Longitudinal Schottky spectra reflecting the characteristic double peaked features have been obtained by all groups working with electron cooling after successful control of electron and ion beam parameters. Fig.2 shows the measured current (i.e. the square root of the power) for a 2.5 emA beam of $^{12}C^{6+}$ at 11.7 MeV/u in the Heidelberg TSR before and after cooling [10]

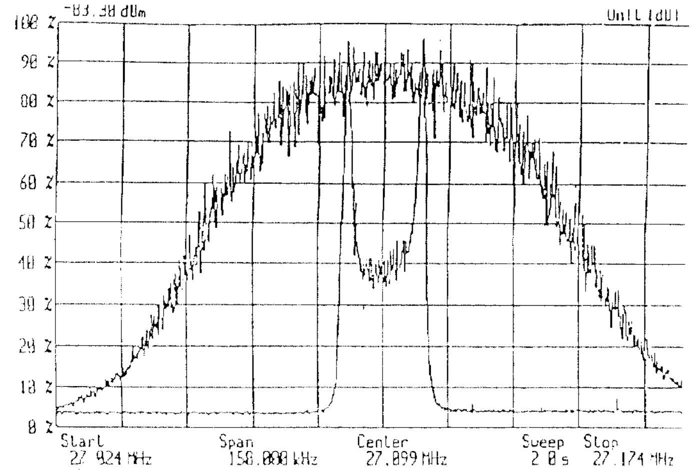

Fig.2 Measured Schottky current (TSR Heidelberg)

It is noted that the cold beam spectrum has developed two peaks, which are related to the slow and fast coherent waves. A straightforward evaluation would be to calculate spectra for different momentum spreads and an assumed distribution function and compare them with the measured spectra. The almost equal heighth of the slow and fast wave peaks of Fig.2 indicate that the resistive impedance is quite small. We have thus arbitrarily assumed a small resistive impedance of 1 % of the space charge impedance, the precise value of which is of no importance for the rest of the discussion. The space charge impedance we have taken according to Eq.7 approximating the logarithmic term by 2 (for a more consistent determination see the later section on peak separation). For a Gaussian momentum distribution the Schottky current (i.e. the square root of the expression of Eq.8) is shown in Fig.3 for the beam of Fig.2 and using different momentum spreads (half fwhm).

A comparison between the measured and calculated spectra suggests that the case with $\Delta p/p = \pm 0.00015$ agrees best with the measured spectrum of the cold beam.

"Figure of Merit" of Longitudinal Cooling

In the stability diagram of Fig.1 the shift corresponding to this cold beam result is represented by a cross. Hence this case is well within the stability boundary of the Gaussian, but a factor of 5.7 above the Keil-Schnell-threshold. This is an important result with respect to the design of heavy ion fusion storage rings [9,11]. It is interesting to note that the excess factor over the Keil-Schnell-threshold is also related to the suppression of the Schottky current at the band center (where $\Omega = p\omega_0$), which is given by the dielectric function ϵ. This suggests to use this factor as the "figure of merit" of longitudinal cooling, by using Eqs.10,12, with all functions evaluated at the band center:

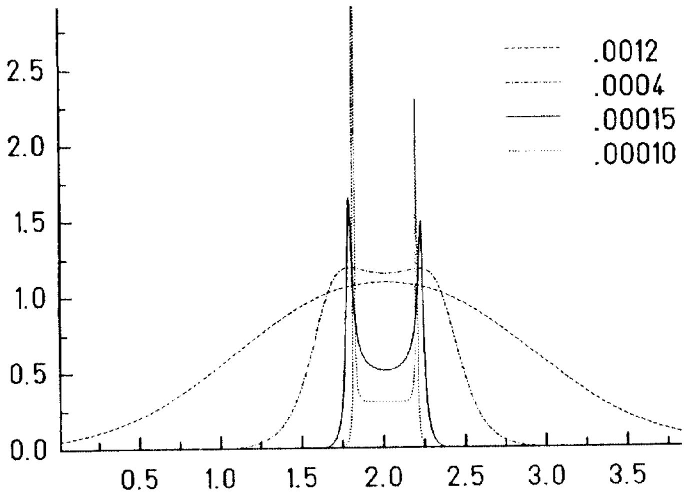

Fig.3 Calculated longitudinal Schottky current with decreasing momentum spread for Gaussian momentum distribution and beam of Fig.2

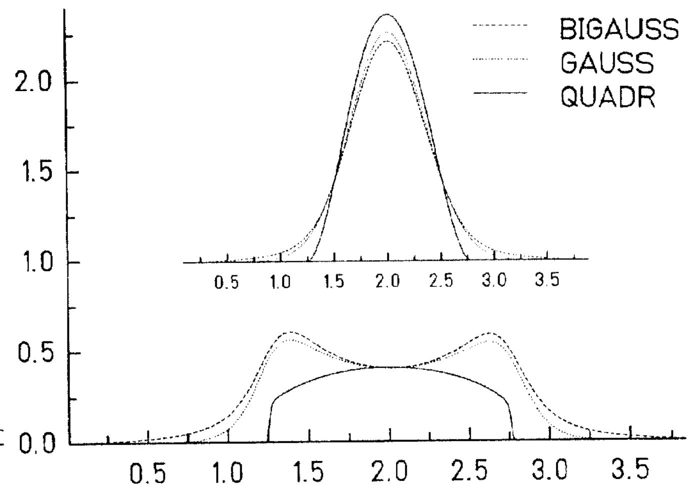

Fig.4 Schottky power for different forms of distributions and $\Delta p/p = \pm 0.0003$

$$"FM" \equiv \epsilon_{\|b.c.} - 1 = \frac{|ImZ_{\|}|}{|1/r_{\|,0}|_{b.c.}} \tag{14}$$

Hence the Keil-Schnell-threshold corresponds to "FM"=1 and a Schottky current suppression at the band center by 2. This is also in agreement with Fig.3, where the different curves corresponds to "FM"= 0.09, 0.8, 5.7 and 12.8.

For completeness we note that above transition energy the r.h.s. of Eq.14 acquires a negative sign. Approaching the threshold of the "negative mass instability" ($\epsilon_{\|b.c.} = 0$) thus leads to an infinitely high Schottky power at the band center.

Information on the Distribution Function

In the preceding section we have shown that within sufficient accuracy the suppression of Schottky current at the band center of the cooled beam is a reliable measure for $(\Delta p/p)_{fwhm}$. Using Eqs.13,14 and comparing the central suppression in the spectra of the uncooled and cooled beam one can thus directly determine $(\Delta p/p)_{fwhm}$.

In Fig.4 we compare calculated power spectra (i.e. squared Schottky currents) for the three momentum distributions introduced in the preceding section, again normalized to the same $(\Delta p/p)_{fwhm}$, and all other parameters as in Fig.3.

It will be noted that the double-peaked feature is practically absent for the quadratic distribution, which has a sharp edge. Although this distribution is not a very realistic one it may be of interest to observe the following: approaching the stability boundary is equivalent to moving (during cooling) the coherent frequency to the edge of the distribution function, where its derivative is large. This means large Landau damping, which suppresses the amplitude of the coherent oscillation. Due to the absence of tails, on the other hand, the stability region is much smaller than for the Gaussian distribution. Landau damping is abruptly lost if during cooling the edge of the distribution function moves across the coherent frequency.

Reducing the momentum spread further down to ± 0.0001 illustrates the importance of the tails: the Gaussian (same as coldest case in Fig.3) is almost at the stability boundary and therefore has higher, but narrower Schottky peaks (note that the quadratic is unstable at this momentum spread). We thus conclude that the heighth of the Schottky peaks is a quite sensitive function of the population of the tails of the distribution function, and of the vicinity to the stability boundary.

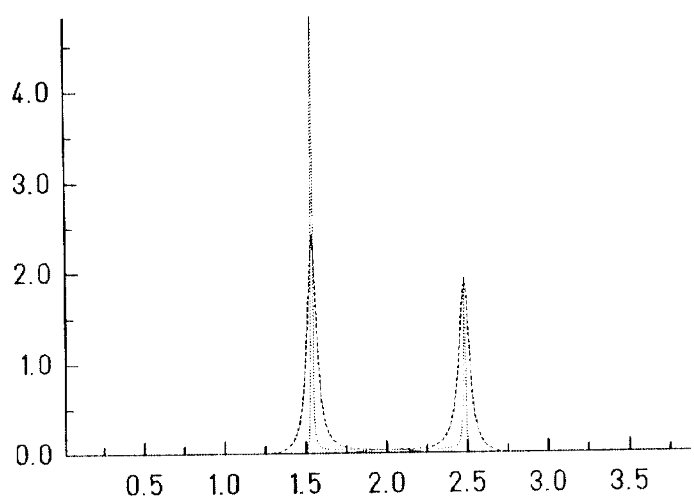

Fig.5 Same as Fig.4 but $\Delta p/p = \pm 0.0001$ *(note the reduction of vertical scale by 2)*

Figs.4,5 comfirm that all three distributions have in common an equally large suppressed Schottky power at the band center as described by Eq.14. The suppression of the integrated Schottky power (over one band) is also an indication of decreasing $(\Delta p/p)_{fwhm}$, yet this depends quantitatively more on the tail population as follows from inspecting the areas in Figs.4,5.

Peak Separation and Impedance

The Gaussian, and even more the bi-Gaussian, show the typical double peaks at practically the same separation. For sufficiently cold beams ("FM" comparable with or larger than unity) this separation can be thus used to measure the coherent frequency shift. The left peak is related to the "slow wave" and the right peak to the "fast wave". By inserting into Eq.4 a δ-function, $\Psi_0 = \omega_0^2/(2\pi)\delta(\omega - \omega_0)$, one obtains readily

$$\Delta\Omega = \pm p\omega_0 \left|\frac{\eta q I}{2\pi\beta^2\gamma A m c^2/e}\right|^{1/2} \left|Im\frac{Z_{\|}}{p}\right|^{1/2} \tag{15}$$

The peak separation can be used to determine the total intensity, if the impedance is known. Eq.15 can be used also to determine the impedance via the slope of the frequency separation vs. intensity. Measurements performed at the TSR have indicated with increasing intensities a decrease of the slope [10]. This is consistent with the asumption that the space charge impedance decreases, since with increasing intensity there is an increase of the emittance and thus of R_b.

BTF Measurements

If the impedance were known one could directly determine $r_{\|,0}$, the imaginary part of which gives the derivative of the distribution function. For unknown impedance one has to search for both, impedance and distribution function. We discuss two procedures depending on whether the Schottky spectrum is known or not.

Comparison with Schottky Spectrum

With an assumed value for the impedance one can determine a function $r_{\|,0}$, and from Eq.5 a corresponding distribution function Ψ_{btf}. With this impedance and the dispersion integral and by using Eqs.4,8 we obtain another distribution function $\Psi_{schottky}$. The true solution is found if both functions agree, i.e. if the functional

$$I = \int |\Psi_{schottky} - \Psi_{btf}|^2 d\omega \qquad (16)$$

vanishes or adopts a minimum near zero. This procedure has been implemented in the diagnostics concept of the ESR [12,13]. An appropriate computer program has been written herefore. With an analytical input for the BTF and Schottky power we have calculated in Fig.6 the variation of I by varying the real part, respectively the imaginary part of the impedance. We also show the modification assuming a 5% statistical error on the input Schottky signal, which indicates that the minimum search is still well-defined.

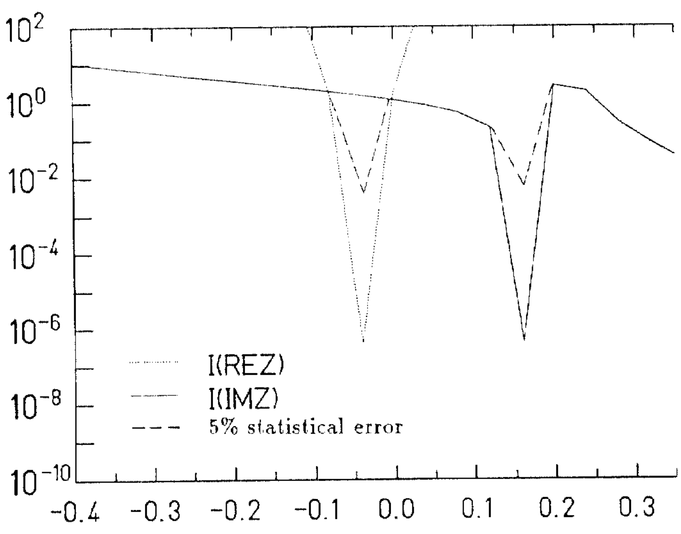

Fig.6 Variational functional to determine impedance

BTF evaluation alone

Due to signal suppression for cooled beams it may not always be possible to obtain sufficient resolution for the Schottky spectra. The longitudinal BTF measurement alone can be used to determine approximately impedance and distribution function, provided that the shift of the origin is large enough. A sufficiently large $ReZ_{\|}$ can be determined by checking the shift of the asymptotes of the stability diagram. This requires sufficient resolution for the frequencies outside of the beam distribution. The shift in direction $ImZ_{\|}$ can be approximated by noting from Fig.1 that the unshifted origin is approximately at the center of a circle fitted to the bottom of the stability curve (i.e. the "Keil-Schnell-circle").

Transverse BTF

In cooling experiments with O^{8+} the shift of the transverse stability diagram has been recently determined by the LEAR group [14] as the vector of displacement from the point of symmetry (Fig.7).

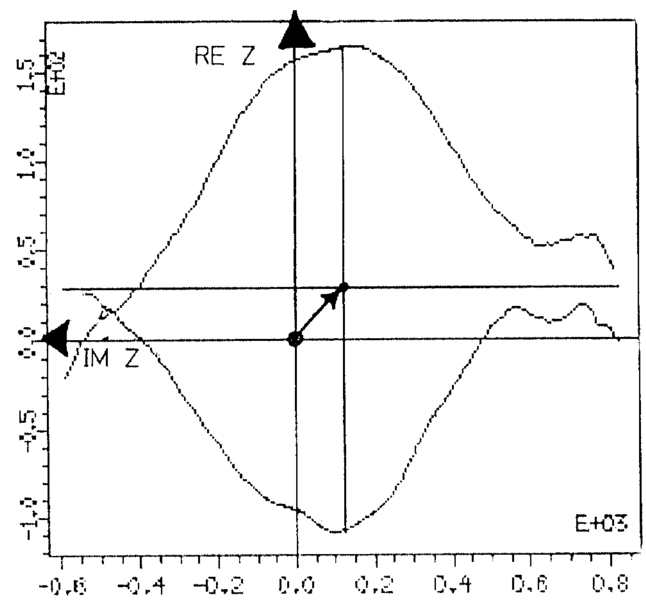

Fig.7 Transverse stability diagram of cooled O^{8+} (Ref.[14])

Having determined the impedance they have subtracted it from $1/r_{\perp}$ and plotted the thus obtained function $1/r_{\perp,0}$, which is the zero impedance response. As expected one finds that the curves corresponding to the slow and fast waves overlap (with zero chromaticity). This confirms that the correct impedance has been subtracted. Obviously this overlap method can be used directly for searching the impedance. Due to the separation of fast and slow wave stability curves in the transverse case this fitting method is more straightforward than the analogous procedure for the longitudinal case. The distribution function of momenta follows directly from the imaginary part of $1/r_{\perp,0}$. Here it must be assumed that there is no overlap with the transverse amplitude distribution, in which case the situation becomes considerably more complicated.

Feedback Stabilization

At sufficiently high intensities several groups have reported unstable transverse oscillations of the beam [14,10]. This confirms theoretical estimates that transverse Landau damping is lost in the process of cooling [15].
At LEAR a transverse damper has been used successfully to stabilize the oscillations observed for the lowest harmonics [14]. The transverse position signal on an electrostatic pick-up is amplified and appropriately delayed, before it is applied on a kicker to correct the transverse displacement. The instability is usually observed for proton intensities exceeding 10^9. It manifests itself by rapidly growing coherent Schottky signals, which develop in the process of cooling and disappear with the damper turned on. Thus it has been possible to raise the intensity above 10^9. The equivalence of the feedback damper with an impedance is seen from the stability diagram (Fig.8). The damper nearly compensates the large shift from the impedance, which would have otherwise brought the origin into the unstable region.

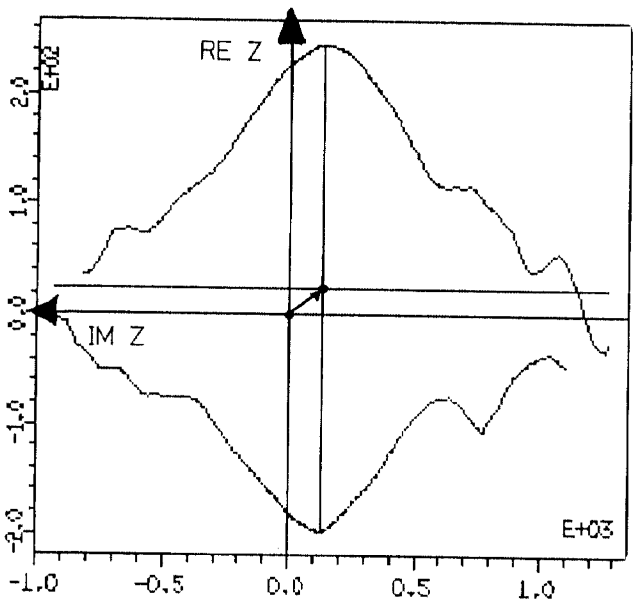

Fig.8 Compensation of impedance by a stabilizing feedback

Schottky Spectra for Very Cold Beams

The development of electron cooling and - for appropriate ions - of laser cooling towards achieving very low beam temperatures introduces some further complications for interpreting Schottky spectra. In the following we summarize several features that go beyond the model of collisionless particles with only collective response used so far:

1) The electron cooling acts as a friction force on coherent oscillations. This friction force can be included in the dynamical description of the response and widens the stability diagram, if the cooling rate is comparable or larger than the growth rate. In Ref.[7] it was found sufficient for stabilization to have a factor of two larger cooling rate than the growth rate. These calculations have shown that the Schottky double-peaks become lower due to this "collisional" damping of the coherent oscillations.

2) For very low intensities it is possible that the cooling rate becomes comparable with the frequency of coherent oscillation given by Eq.15 (the "beam plasma frequency" $\propto \sqrt{N}$) in which case the coherent response is entirely damped by friction. Calculations have shown that under such extreme conditions the Schottky power spectrum looses the double peaks and adopts again a single-peak shape [5,7] We note, however, that this requires cooling times, which are considerably below a millisecond, depending on the number of particles.

3) An entirely new phenomenon would arise, if the positions of the ions along the equilibrium orbit were ordered. The requirements on $\Delta p/p$ have been discussed first by the Novosibirsk group [1]. For intensities between 10^5 and 10^7 particles one would expect a "linear chain" ordering, which can probably be achieved more easily than three-dimensional "crystalline" ordering requiring much higher intensities [16]. A direct evidence of such a linear chain ordering could be given by exciting transverse or longitudinal oscillations and picking up the response signal. The coherent frequency shift would reflect whether the Coulomb force is due to ordered positions [17]. Obviously this requires frequencies far beyond standard techniques, i.e. as high as 100...1000 GHz.

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