# FLEXIBLE MOMENTUM COMPACTION RETURN ARCS FOR RLAS\*

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#### Abstract

Neutrino Factories and Muon Colliders require rapid acceleration of short-lived muons to multi-GeV and TeV energies. A Re-circulating Linear Accelerator (RLA) that uses a single Linac and teardrop return arcs can provide exceptionally fast and economical acceleration to the extent that the focusing range of the RLA quadrupoles allows each muon to pass several times through each high-gradient cavity and the cost of the return arcs is appropriate. Flexible Momentum Compaction (FMC) lattice designs for the teardrop return arcs provide sufficient momentum acceptance to allow multiple passes of each sign of muon in one string of magnets to improve cost-effectiveness.

### **INTRODUCTION**

Future neutrino factories or muon colliders require a large number of muons to be accelerated in a very short time due to the muon's short lifetime. Recirculating linear accelerators (RLA) have become the most likely solution for muon acceleration, where two options now under consideration are the racetrack and "dog bone" [1] configurations. Both the low (2.5-32.5 GeV) and the high energy RLAs (32.5-753.5 GeV) assume about seven passes through superconducting RLAs. Each configuration requires multiple magnetic arcs to return the beam back to the linac as it is done for the existing Jefferson Lab CEBAF racetrack. In this report, nonscaling Fixed Field Alternating Gradient (NS-FFAG) arcs are proposed to simplify the racetrack by reducing the number of separate magnetic channels by using arcs with wide momentum acceptance, each capable of multiple passes. This approach will also work for a single linac with teardrop-shaped return arcs in the presently preferred dog bone solution. It might even allow easier matching.

While a NS-FFAG [2] uses superconducting RF cavities, the arcs for an RLA have the additional constraint that the beam phase must be accurately controlled due to the fixed RF frequency. The arc design is described in the next section, followed by a description of the matching module to the RLA and the complete racetrack lattice.

## **NON-SCALING FFAG ARCS**

The major advantage of a NS-FFAG with respect to any other fixed field accelerator is its small transverse aperture. A scaling FFAG would require almost two orders of magnitude larger aperture. The aperture for low muon momentum is determined by the muon emittance, which depends on six-dimensional muon beam cooling. The NS-FFAG provides extreme focusing, resulting in a

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very small beam amplitude functions and very small dispersion.

In the discussion below, we have designed NS-FFAG lattices with the idea that half of the ring could provide multiple return arcs for the racetrack RLA. This report is focused on the energy range between 2.5 and 10 GeV, with an attempt to replace the multiple arcs with a single arc. The NS-FFAG used requires an aperture larger than 80 mm as the orbit offsets during acceleration oscillate between  $-11 \text{ mm} < x_{\text{off}} < 67 \text{ mm}$ . The momentum acceptance of  $-60\% < \delta p/p < 60\%$  is obtained by raising the number of cells to N=142. The NS-FFAG ring used for arcs is presented in Fig.1.



Figure 1: The NS-FFAG ring used for racetrack RLA return arcs, using 142 densely populated FODO cells of combined function magnets. The magnet lengths are  $L_D=0.9$  m and  $L_D=0.3$ m, with a drift of  $l_3=0.2$  m.

#### Arc Cell and Magnet Properties

Magnet properties are described in Table 1. The FODO cell amplitude functions are relatively small due to very small magnet lengths and the strong focusing.

Table 1: Combined function magnet properties

Mag.	L(m)	B(T)	$B_{max}(T)$	G(T/m)	$\theta(rad)$	X <sub>off</sub> (mm)
B <sub>D</sub>	0.9	3.2	3.2	-23.0	0.038	-10 <x<41< td=""></x<41<>
$\mathbf{B}_{\mathrm{F}}$	0.3	-3.2	-3.6	38.6	-0.047	-0.5 <x<67< td=""></x<67<>

The major advantages of the NS-FFAG with respect to the scaling one are the small orbit offsets as shown in Fig. 2. The betatron functions are shown in Fig. 3 and the dispersion function during acceleration is shown in Fig.4.

The defocusing combined function magnet has a positive bending angle of  $\theta$ =0.1382 rad, while the focusing combined function magnets has opposite bending angle of  $\theta$ =-0.047 rad. The magnetic field of each magnet type varies linearly with displacement on the horizontal axis.



Figure 2: Orbit offsets during acceleration. The cell length is  $L_0=1.9$  m.



Figure 3: Betatron functions during acceleration.



Figure 4: Dispersion function variation with energy.

#### General Layout

A major obstacle of the NS-FFAG is the variation of tunes with energy during acceleration, as shown in Fig. 5. In the case of many turn acceleration, crossing resonances could be a problem, but is not expected to be a problem for 7 passes. In addition to the tune variations with energy there is an unavoidable problem of a path length dependence on energy. This relationship is a parabolic function, which is shown in Fig. 6.



Figure 5: Variation of the tunes with energy in the arc cell.



Figure 6: Path length variation with energy in the arc cell.

The path length variation with energy is a major challenge of this study. Muon arrival at the beginning of the linac after the first pass through the arc is delayed by 0.149 m or  $\sim$ 0.5 ns. A length of one RF cycle is 5 ns for 200 MHz RF frequencies. The path length variation needs to be adjusted by chicanes.

#### Matching to the Linac

The arc cell is only 1.9 meters long and very densely packed. The superconducting 200 MHz cavity requires more than a meter. In addition the longer cell will have larger amplitude functions  $\beta_x$  and  $\beta_x$ . One cell at the end of the half of the NS-FFAG ring is replaced with a 5.7 m cell with the same bending angle as the arc cell. The bending angles of the combined function magnets in the matching cell are  $\theta_{DL}$ =0.102 rad and  $\theta_{FL}$ =-0.0435 rad. Properties of the magnets are shown in Table 2. The gradients and bending angles are adjusted to reproduce the orbit offsets at the middle of the major bend of the arc cells at each momentum. The available drift space in the "large" cell is 4.04 m long. Details of the cell with the beam offsets are shown in Fig. 7.

Table 2: Magnet properties of the large matching cell

Mag.	L(m)	B(T)	$B_{max}(T)$	G(T/m)	$\theta(rad)$	X <sub>off</sub> (mm)
B <sub>D</sub>	0.66	2.94	2.94	-19.6	0.093	0.0 <x<41< td=""></x<41<>
$\mathbf{B}_{\mathrm{F}}$	0.3	-2.7	-2.7	22.9	-0.039	-0.5 <x<44< td=""></x<44<>



Figure 7: Large matching cell with orbits.

The large ring with matching cells reproduces the same orbit offsets at the center of the defocusing magnets as at the arc cell. Figure 8 shows the orbits for a ring with matching cells, where muons are tracked over a momentum range of  $\delta p/p=\pm 60\%$ . The transverse displacements are magnified 100 times.



Figure 8: Closed orbits for the momentum range of  $\delta p/p=\pm 60\%$  in the racetrack made of arcs and large cells.

The length of the temporary racetrack is 330.65 m. It is made of 2x6 large cells and two arcs.

## The Full Racetrack with Linac

The linac is made of the same triplets as the 'large cells' but the magnets are quadrupoles without any bending. The available space for cavities is the same  $l_{cav}=4.04$  m. Two 1.78 m long cavities are placed in each cell. Two linacs on each side of the racetrack are made of

10 cells each with a total of 40 cavities. The racetrack with total length of 456 m is presented in Fig. 9.



Figure 9: The racetrack muon accelerator with two linacs.

Muons have stable closed orbits in the momentum range of  $\delta p/p=\pm 60\%$ , as shown in Fig. 10.



Figure 10: Closed orbits (x50) in the racetrack muon accelerator for the energy range from 2.5-10 GeV.

## SUMMARY

The initial results for Non-Scaling FFAG arcs to connect racetrack RLA linacs are very encouraging. Multiple arcs can be successfully replaced with a single arc with an energy acceptance of  $\delta p/p=\pm 60$ % for a factor of four in energy. Stable orbits were found for the racetrack within the whole energy range. The time of flight in the arcs for each pass has a difference at the lowest energies corresponding to a maximum path length difference of 3.5 mm. An additional chicane can be added to allow perfect time of flight matching to the linac. Corrections to the path length dependence with better chicanes, as well as better matching to the linac, are in progress. The dispersion matching of the linac at each energy pass will also be improved.

## REFERENCES

- [1] S. A. Bogacz, "Low Energy Stages 'Dogbone' Muon RLA", Nuclear Physics B, 149 (2005) pp. 309–312.
- [2] D. Trbojevic, E. Courant, and M. Blaskiewicz, Phys. Rev. ST Accelerator Beams 8, 050101 (2005).