# COMPACT DESIGN OF RACE-TRACK MICROTRON MAGNETS* 

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## Abstract

A novel design of race-track microtron (RTM) end magnet with four poles and an REPM material as a source of the magnetic field is proposed. For a proper choice of parameters such magnetic system can provide both the first orbit beam reflection to the linac axis and required focusing properties. It is shown that such end magnet can be made quite compact thus allowing to build miniature RTMs. The procedure of design of the fourpole magnetic system and its optimization using the ANSYS code is described. We also analyze focusing properties of end magnets with two poles and show their limitations.

## INTRODUCTION

For applications of low energy electron beams (from $\sim 10$ to $\sim 100 \mathrm{MeV}$ ) which require compact, low weight and low power consumption machines the race-track microtron (RTM) with the end magnets made of rare earth permanent magnet (REPM) material is a promising choice. Examples are 35 MeV and 70 MeV RTMs described in $[1,2]$ and a miniature 12 MeV RTM which is under construction at the UPC (Barcelona) in collaboration with several Spanish institutions and SINP (Russia) [3].

The main function of the $180^{\circ}$ RTM end magnets is to provide the electron beam recirculation through the accelerating structure (linac), however their design influences strongly the vertical beam focusing and method of linac bypass (first orbit problem). A reverse field pole introduced at the magnet entrance [4] solves the beam focusing problem. The end magnet focusing power can be decreased sufficiently by adjusting the reverse field amplitude and pole position, however at the expense of decrease of the 1st orbit distance from the linac axis thus leaving no space for linac bypass. A standard procedure to solve this problem [5] is to install a pair of dipoles at both linac ends. One of these pairs closes the 1st orbit loop and reflects the beam back to the linac axis and the other one compensates the beam displacement at higher orbits. However these dipole pairs increase about two times the distance between the end magnets, thus increasing the RTM dimensions.

REPM materials as a field source permit to build magnets with a strong field variation within a short length interval and therefore get field properties unattainable within designs with electromagnets. An example is the 70 MeV RTM end magnet [2] with the reverse field pole paled at a very short distance from the main pole. In

[^0]combination with narrow rectangular accelerating structure this permitted to solve the 1st orbit problem in a simple way.

## FOCUSING PROPERTIES OF TWO-POLE MAGNETIC SYSTEMS

First we will study a two-pole ( 2 P ) end magnet. It is known that its fringe field can be optimized to provide 1st orbit beam reflection to the linac axis [6], however then its focusing power can be varied only in a very restricted way. To analyze these features we introduce a coordinate system with the origin placed at the position of the main pole face, xz-plane coinciding with the magnet medium plane, and z-axis directed outside the main pole. Suppose that the magnetic field of the reverse pole is localized in the interval $0 \leq z \leq z_{0}$ and that the field of the main pole has a fringe field region $l-d \leq z \leq l$ and a uniform region for $z \geq l$ with $B_{y}=B_{0}$. The focusing power of a localized magnetic field extending from $z_{i}$ to $z_{f}$ is given by [7]:

$$
\begin{equation*}
\frac{1}{f}=-\left(\frac{e}{p}\right)^{2} \int_{z_{i}}^{z_{f}} B^{\prime}(z) d z \int_{z_{i}}^{z} B(t) d t=\left(\frac{e}{p}\right)^{2}\left(\int_{z_{i}}^{z_{f}} B^{2}(z) d z-B\left(z_{f}\right) \int_{z_{i}}^{z_{f}} B(z) d z\right) \tag{1}
\end{equation*}
$$

where $B_{y}=B(z)$ is the magnetic field profile and $p$ is the particle momentum. Using these formulas we calculate the end magnet focusing power $1 / F$. The complete expression is rather cumbersome and will be published elsewhere, here we present the two first leading terms only which will be sufficient for our analysis:

$$
\begin{equation*}
\frac{1}{F}=2\left(\frac{1}{f_{1}}+\frac{1}{\widetilde{f}_{2}}\right)-\pi R\left(\frac{1}{f_{1}}+\frac{1}{\widetilde{f}_{2}}\right)^{2}+\ldots \tag{2}
\end{equation*}
$$

where $R$ is the radius of the particle orbit in the uniform field, $f_{1}$ is the focal length of the reverse pole and $\widetilde{f}_{2}$ is that of the main pole fringe field,

$$
\begin{equation*}
\frac{1}{\widetilde{f}_{2}}=\frac{1}{f_{2}}-\left(\frac{e}{p}\right)^{2} B_{0} \int_{0}^{l-d} B(z) d z \tag{3}
\end{equation*}
$$

The second term is a correction due to the non-zero angle of the particle trajectory at the entrance to the fringe field region. The focusing powers $1 / f_{1}$ and $1 / f_{2}$ are calculated from Eq. (1).

To gain an understanding of the properties of $2 P$ magnetic systems let us consider a simple analytic example with the reverse and main pole fields modeled by linear functions as it is shown in Fig. 1 so that $z_{0}=2 z_{1}$. Using Eqs. (1) and (3) one gets

$$
\begin{equation*}
\frac{l}{f_{1}}=\frac{2}{3 r^{2}} \kappa^{2} \mu, \quad \frac{l}{\widetilde{f}_{2}}=\frac{1}{r^{2}}\left(\kappa \mu-\frac{\delta}{6}\right) \tag{4}
\end{equation*}
$$

where we introduced dimensionless variables $\mu=z_{1} / l$, $\kappa=B_{1} / B_{0}, \delta=d / l$, and $r=R / l$.


Figure 1: Magnetic field profile for the analytic model.
Let the radius of the orbit with energy $E_{0}$ in the uniform field be $R_{0}$ and we denote $r_{0}=R_{0} / l$. The displacement of a trajectory exiting the end magnet with respect to the linac axis is given by

$$
\begin{equation*}
\Delta=-\frac{2}{B_{0} R} \int_{0}^{z_{\max }} \frac{A(z)}{\sqrt{1-\left(A(z) / B_{0} R\right)^{2}}} d z \tag{5}
\end{equation*}
$$

where $A(z)$ is the $x$-component of the vector potential, so that the vertical component of the magnetic field is equal to $B(z)=\partial A / \partial z$. Here $z_{\max }$ is the maximum depth of penetration of the particle inside the magnetic system, its value is determined by the condition $|e| A\left(z_{\text {max }}\right)=p$. The condition of the first orbit reflection to the linac axis, i.e. $\Delta=0$ for $R=R_{0}$, is fulfilled if the ratio of the magnetic fields of the reverse and main poles is equal, in the leading approximation, to
$\kappa_{0}(\mu)=\left(\sqrt{\chi^{2}+2 r_{0}^{2}+\delta^{2} / 12}-\chi\right) / \mu, \chi=1-\mu-\delta / 2$.
For $\kappa=\kappa_{0}$ the function $l / F$ characterizing the focusing power of the magnetic system as a function of $\mu$ has minimum (Fig. 2). It can be shown that the focusing power takes zero values if $r_{0}<r_{0 \max }=0.11$. For a fixed initial energy $E_{0}$ this means that the distance between the main and reverse poles must be larger than certain minimal value, namely $l>l_{\min }=0.11 R_{0}$. For example, for $E_{0}=2 \mathrm{MeV}$ and $B_{0}=0.8 \mathrm{~T}$ we get $l_{\text {min }}=8.67 \mathrm{~cm}$. We see that for a magnetic system with two poles to have long enough focal length and provide the 1 st orbit reflection the distance between the reverse pole and main pole must be larger than $l_{\text {min }}$, so that the system cannot be made compact enough.

The considered analytic model is too simple, real fringe field distributions are more complicated than that in Fig. 1. To gain a better understanding of 2 P end magnets we considered a system with a realistic fringe field and made numerical calculations of the beam trajectory using RTMTRACE code [8]. In Fig. 3 (a) a set of magnetic field profiles with the position of the reverse pole relative to main pole varied from 2 cm to 4.25 cm are shown (notice that here and in what follows the $z$-axis is directed outside the system). These curves were obtained in the electrostatic approximation using POISSON code [9] by calculating spatial potential distribution with fixed the main and reverse pole potentials.


Figure 2: Function $l / F(\mu)$ with for $\kappa=\kappa_{0}(\mu)$ and $l=6 \mathrm{~cm}, 8.67 \mathrm{~cm}$ and 10 cm (from top to bottom).

For each position the reverse field amplitude was adjusted to provide the orbit closure (see Fig. 3 (b)) for $E_{0}=2 \mathrm{MeV}$ and $B_{0}=0.8 \mathrm{~T}$ (UPC RTM parameters).

(a)

(b)

Figure 3: (a) Fringe fields for different positions and field amplitudes of the reverse pole. (b) Particle trajectory for variants 1 and 10 .

In Fig. 4 (a) the focal power dependence on the reverse pole position for $E_{0}=2 \mathrm{MeV}$ is shown (compare to Fig. 2). For a distance between end magnets of about 20-30 cm , as in UPC RTM, to avoid vertical over focusing the value of the focal power must be below $5 \mathrm{~m}^{-1}$, which is achieved for $z_{1} \approx 2.5 \mathrm{~cm}$ or for $z_{1}>10 \mathrm{~cm}$. As it can be seen from Fig. 4 (b) the choice $z_{1} \approx 2.5 \mathrm{~cm}$ (variant " 3 ") provides too strong focusing at the higher orbits (4, 6 and 8 MeV ) thus leading to instabilities of vertical beam oscillations. The choice $z_{1}>10 \mathrm{~cm}$ would increase the distance between the end magnets at least by the factor of two. In addition, because of this increase the focal power must be reduced appropriately, so it is not evident that stable vertical oscillations can be reached in this way.

## FOUR POLE DESIGN

To resolve the problem of overfocusing we separated the functions of beam reflection to linac axis and beam focusing by introducing in each end magnet an additional pair of poles with fields of equal amplitudes but of opposite signs. In this four-pole (4P) system the main pole and the pole with reverse field provide beam bending and focusing. The pair of additional poles in one end magnet provides the 1 st orbit beam reflection to the linac axis, whereas the pair of poles in the other magnet compensates the higher orbits beam displacement created by the first one.


Figure 4: (a) Focal power dependence on reverse pole position for $E_{0}=2 \mathrm{MeV}$. (b) Focal power dependence on energy for variants " 3 " and " 10 ".

Optimization of this four-pole (4P) system included the following steps. First, the geometry of the corresponding 2 P system (main and reverse poles) was chosen using POISSON and RTMTRACE to provide a required optical power. The distance between entering and exiting trajectories was defined and parameters of two additional pole pairs, namely the field amplitude and poles separation, were adjusted to get the beam reflection. Than the focusing properties of the initial 2 P system were adjusted to get the required optics for the whole magnet. After several iterations the optimal field shown in Fig. 5 (a) and pole potentials providing this field were found. Dependence of the 4P end magnet optical power on the beam energy is shown in Fig. 5 (b). Beam dynamics calculations show that in this case vertical oscillations are stable.

Strong field variation within a short interval, as shown in Fig. 5 (a), can be achieved using REPM material as a field source. In Fig. 6 (a) a $1 / 8$ of the end magnet is shown. The steel poles are marked as \#1-\#4, and M1-M10 are blocks of REPM material surrounded by a steel yoke. Following Ref. [10] from the pole potentials calculated within the electrostatic approximation the pole height and thickness and residual magnetization of the REPM blocks were estimated. These data was used as the initial approximation in the ANSYS [11] input file. By adjusting the magnet geometry and REPM magnetization the fringe field shown in Fig. 5 (a) was reproduced with sufficiently good accuracy.


Figure 5: Magnetic field profile of the 4P system (a) and focal power dependence on energy (b).

The yoke thickness was optimized to provide a minimal magnet weight without essential steel saturation. In Fig. 6 (b) field induction in the vertical symmetry plane is shown. As one can see the field induction in the steel parts of the magnet does not exceed 1.3 T .

(b)

Figure 6: (a) $1 / 8$ of the magnet geometry for ANSYS calculations, (b) field induction distribution in the symmetry plane.

## CONCLUSIONS

We have studied focusing properties of two-pole RTM end magnets and shown that for such magnetic system to have long enough focal length and provide the 1 st orbit reflection the distance between the reverse and main poles must be larger than some critical value $l_{\text {min }}$. This property does not allow to build compact end magnets. As a solution to this problem we have proposed a four-pole design with separated functions of beam focusing and reflection, which can be built using REPM material as a field source. An example of optimized 4P design of a compact RTM end magnet has been discussed.

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