SHORT RANGE WAKEPOTENTIALS COMPUTED IN A MOVING FRAME

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Abstract

It is proposed to compute short range wakepotentials in a moving frame. The frame shall move with velocity $v = \beta c$ in positive z-direction, following the exciting charge. In that frame, the relativistic charge still moves with the velocity of light, but its length is expanded by the γ- factor of the moving frame. Because of the longer charge, one can use a larger gridspacing. This allows a saving in CPU-time by a factor of γ^3.

In the moving frame, the device is shortened by a factor of γ and it moves opposite to the exciting charge. The time to traverse the structure therefore is decreased by a factor of $\gamma(1+\beta)$.

The larger possible gridspacing together with the decrease in time to traverse the structure allows a saving in computation time by a factor of $\gamma^4(1+\beta)$.

THE PROBLEM

Today the machine designers ask for wakepotentials of bunches with length of less than $\sigma = 1$ mm in devices which are longer than L=1 metre. They want to know the wakepotentials in s-ranges up to $s = 20\sigma$. For computing such wakepotentials, one needs a grid spacing h which is much less than the bunchlength, eg. $h = \sigma/10$. Even when computing with a moving mesh, the number of needed gridcells is proportional to $1/h^3$, as the number of z-planes which need to be used in a moving mesh is $n_z = s/h$. $N_{cells} \propto 1/h^3$. Because the maximum stable timestep is proportional to h, the number of timesteps needed is proportional to the length of the device divided by the gridspacing, $N_{time} \propto L/h$. In total, the computational load scales as

$$\begin{aligned} N_{time}N_{cells} &\propto \frac{L}{h}\frac{s}{h^3} \\ &\propto \frac{L}{\sigma^4} \end{aligned}$$

THE WAY OUT

Compute in a moving frame which travels with $\beta < 1$ in the same direction as the exciting charge.

When one computes the wakefield in a frame which moves in the same direction as the exciting charge, but with a velocity less than c, the lengths are changed. The device becomes shorter by a factor of γ. The exciting charge becomes longer by a factor of γ. Because of the length transformation of σ, one may compute with a larger gridspacing, now proportional to the transformed σ. Eg. $h' = \gamma\sigma/10$. Because the needed z-extension of the moving mesh also grows by γ, the number of gridcells in a moving mesh computing in a moving frame scales like $N_{cells} \propto \gamma s/(\gamma\sigma)^3$.

The larger gridspacing also allows a timestep.

Because of the length transformation of the device, the needed simulation time is reduced by a factor of γ. Together with the larger timestep, the number of timesteps it takes to simulate a relativistic charge traveling over the length of the device now is $N_{time} \propto L/\gamma/h' = L/\gamma/(h\gamma)$.

But the device is not at rest in the moving frame. It travels with velocity βc in negative z-direction. Because the end of the device moves toward the exciting charge, the number of timesteps needed until the relativistic charge has traveled from the beginning of the device to its end is:

$$N_{time} = \frac{1}{1+\beta}\frac{L/\gamma}{\gamma h} \tag{1}$$

When computing in a moving frame, the computational load scales like

$$\begin{aligned} N_{time}N_{cells} &\propto \frac{1}{1+\beta}\frac{L/\gamma}{\gamma h}\frac{\gamma s}{(h\gamma)^3} \\ &\propto \frac{1}{\gamma^4(1+\beta)}\frac{L}{\sigma^4} \end{aligned}$$

In order to have a substantial saving in computational load, γ should be 2 or larger.

THERE IS NO FREE LUNCH?

The argument of the previous section why computing in a moving frame is cheaper is that one may use a larger gridspacing because of the longer wavelengths excited by the longer charge. But that is not absolutely true. The wakefields are the primary fields of the exciting charge which are scattered by the device. When the device moves in negative z-direction, the scattered fields traveling in positive direction have wavelengths which are expanded by γ, but the scattered fields traveling in negative z-direction will have wavelengths which are shortened by γ. When one computes with a larger gridspacing, the waves traveling in negative z-direction will suffer severe dispersion errors. Fortunately, these waves do not contribute much to the wakepotentials because the witness charges see frequent sign changes of these fields.

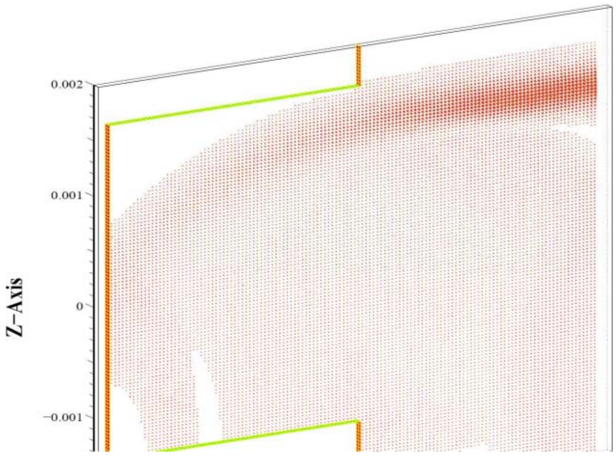

Figure 1: Primary field and scattered field when a beam exits a cavity. This is computed with the standard frame, ie the material boundaries are at rest, $\gamma = 1$.

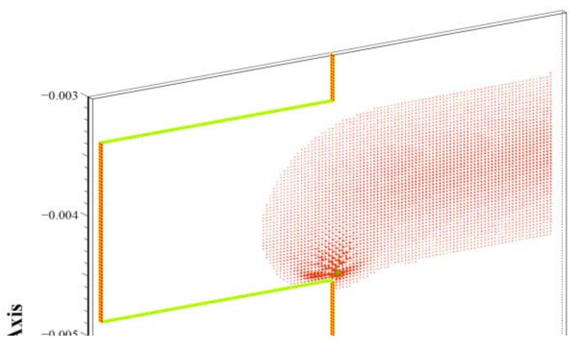

Figure 2: This is computed in a moving frame. The material boundaries move. The beam length is expanded by $\gamma = 2$. The length of the cavity is shrunk by γ.

RELATIVISTIC BOUNDARY CONDITIONS

The field computation within the volume does not change. Only cells where material enters or exits need a modification in their fields update procedure.

The boundary condition at perfect conducting planes moving with $\vec{v}$: $\vec{n} \times \vec{E} = (\vec{n} \cdot \vec{v})\vec{B}$.

The boundary condition at straight waveguide sections where the plane normal is perpendicular to the velocity of the device do not change. Tangential electric fields there continue to be zero.

Finite Difference Implementation

As an example how the relativistic boundary condition can be implemented in a FDTD-code, we show the equation for computing the next H_x component from the old one and the surrounding E-components.

At metallic planes with plane normal in $-z$-direction, the boundary condition for E_y is $E_y = \beta c B_x$. In Finite Difference terms where h_x, h_y, u_x, u_y are integrated magnetic and electric voltages between cellcenters and gridpoints, the update for h_x reads

$$\begin{aligned} h_x^{n+1/2}(i,j,k) &= h_x^{n-1/2}(i,j,k) \\ &\quad -F_x\left(u_z^n(i,j+1,k) - u_z^n(i,j,k)\right. \\ &\quad \left. -u_y^n(i,j,k+1) + u_y^n(i,j,k)\right) \end{aligned} \quad (2)$$

where

$$F_x = \Delta t \frac{\Delta x}{\mu \Delta y \Delta z} \quad (3)$$

Below a metallic plane, the u_y-voltage above the gridcell must be computed from the h_x-voltage within the gridcell. In a equally spaced grid, $\Delta x = \Delta y = \Delta z$, and also assuming $\varepsilon = \mu = c = 1$:

$$u_y^n(i,j,k+1) = \frac{\beta}{2}\left(h_x^{n-1/2}(i,j,k) + h_x^{n+1/2}(i,j,k)\right) \quad (4)$$

This gives an update for h_x:

$$\begin{aligned} h_x^{n+1/2}(i,j,k) &= h_x^{n-1/2}(i,j,k)\frac{1+\beta F_x}{1-\beta F_x} \\ &\quad -\frac{F_x}{1-\beta F_x}\left(u_z^n(i,j+1,k) - u_z^n(i,j,k)\right. \\ &\quad \left. +u_y^n(i,j,k)\right) \end{aligned} \quad (5)$$

The factor F_x, which depends on Δz, must also be modified. Instead of using the normal grid-spacing, one must use the z-length of that part of the gridcell which is not yet overrun be the moving material boundary.

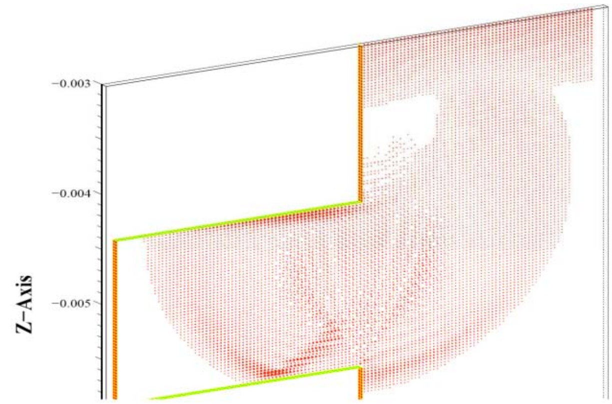

Figure 3: The beam is well above the moving cavity.

EXAMPLE

Figure 1 shows the known field pattern of a wakefield computed without moving boundaries. Figures 2 and 3 show the aequivalent field of a 2 times longer beam in a two times shorter geometry moving with $\beta = 0.866$ in -z-direction.

ACKNOWLEDGEMENT

The idea to compute in a moving frame is from Jean-Luc Vay.