# CORRECTOR BASED DETERMINATION OF QUADRUPOLE CENTRES 

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#### Abstract

A corrector magnet based method to determine the quadrupole magnet centres for storage rings has been tested on the MAX III synchrotron light source. The required corrector magnet strengths for the corrected beam orbit are used to determine the quadrupole magnet centre positions.


## INTRODUCTION

The 700 MeV MAX III electron storage ring [1][2] finished commissioning during 2007. The MAX III magnet cells were initially aligned using invar wires to an estimated accuracy of $100 \mu \mathrm{~m}$. After a stable beam was stored in the ring the offsets between the Beam Position Monitors (BPMs) and the centre of the neighbouring quadrupole magnets were determined to an accuracy of $50-100 \mu \mathrm{~m}$ using the quadrupole shunt method [3]. The corrector magnet strengths needed to adjust the orbit to the measured offsets were considerable and in some cases corrector saturation at 0.5 mrad was observed. Realigning MAX III to minimize the corrector magnet strength was therefore of interest.
In order to do so, the location of the magnetic quadrupole centres had to be determined as dislocated quadrupoles are a source of dipole field errors in the MAX III. This was done using a method that relies on the corrector strengths required to correct the MAX III orbit to the centre of the quadrupoles, to compensate for dipole errors present in the ring.

## METHOD DESCRIPTION

The closed orbit in a storage ring at a number of different locations $i$, given by the placement of the BPMs, is in the linear optics approximation given by

$$
\begin{align*}
& {[\bar{u}-\bar{q}]+\bar{\delta}=\mathbf{M} \bar{\theta}+\mathbf{N}(k L \bar{\delta})+\bar{R}+\bar{\eta}\left(\frac{\Delta p}{p}\right)} \\
& R_{i}=\int_{s_{i}}^{s_{i}+c}\left(\frac{f(\sigma) \sqrt{\beta_{i} \beta(\sigma)} \cos \left(v\left[\Delta \varphi_{i}(\sigma)+\pi\right]\right)}{2 \sin \pi v}-\frac{\eta(\sigma) \eta_{i}}{\alpha C}\right) d \sigma \tag{1}
\end{align*}
$$

where the integration is done for the entire ring except for quadrupoles and corrector magnets, and the following definitions are used

[^0]$\bar{u} \equiv$ Beam position vector as reported by the BPM system
$\bar{q} \equiv$ BPM offset error
$\bar{\theta} \equiv$ Corrector magnet kicks
$\bar{\delta} \equiv$ Quadrupole magnet field position error
$k \equiv$ Normalized quadrupole field strength
$L \equiv$ Quadrupole magnetic length
$v \equiv$ Betatron tune
$f(s) \equiv$ Dipole field perturbation at position $s$
$\bar{\eta} \equiv$ Dispersion vector
$\mathbf{M} \equiv$ Storage ring corrector kick response matrix
$\mathrm{N} \equiv$ Storage ring quadrupole kick response matrix
It should be noted that the above expression assumes that there is one BPM next to each quadrupole for which the magnet field position error is to be determined.
Energy shifts due to corrector magnet kicks or misalignments are included in the $\mathbf{M}$ and $\mathbf{N}$ matrices.
The $\bar{R}$ vector contains the orbit distortion at all BPMs due to unknown dipole errors distributed around the ring, except for those due to dislocated quadrupoles.
Correcting the closed orbit to the measured BPMquadrupole offsets, which sets $\bar{u}=0$, the corrector kicks and the beam energy deviation due to cavity radio frequency (RF) changes will be adjusted to states $\bar{\theta}^{\prime}$ and $\Delta p^{\prime} / p$ respectively. Thus
\[

$$
\begin{equation*}
-\bar{q}+\bar{\delta}=\mathbf{M} \bar{\theta}^{\prime}+\mathbf{N}(k L \bar{\delta})+\bar{R}^{\prime}+\bar{\eta}\left(\frac{\Delta p^{\prime}}{p}\right) \tag{3}
\end{equation*}
$$

\]

which is rewritten as

$$
\begin{align*}
& \bar{\delta}=(\mathbf{I}-k L \mathbf{N})^{-1} \mathbf{M} \bar{\theta}^{\prime}+\bar{\varepsilon} \\
& \bar{\varepsilon} \equiv(\mathbf{I}-k L \mathbf{N})^{-1}\left(\bar{q}+\bar{R}^{\prime}+\bar{\eta}\left(\frac{\Delta p^{\prime}}{p}\right)\right) \tag{4}
\end{align*}
$$

where $\mathbf{I}$ is the unity matrix. When initially adjusting the cavity RF, it can be chosen to either minimize the corrector magnet sum or to match the theoretical orbit circumference.
The quadrupoles are then adjusted using the misalignment estimate $\bar{\delta} \approx(\mathbf{I}-k L \mathbf{N})^{-1} \mathbf{M} \bar{\theta}^{\prime}$. Equation (1) then becomes

$$
\begin{equation*}
[\bar{u}-\bar{q}]+\bar{\varepsilon}=\mathbf{M} \bar{\theta}+\mathbf{N}(k L \bar{\varepsilon})+\bar{R}+\bar{\eta}\left(\frac{\Delta p}{p}\right) \tag{5}
\end{equation*}
$$

At the new magnet positions, beam is stored and the orbit corrected to the BPM-quadrupole offsets. This changes the corrector magnet kicks and possibly the beam energy deviation to new values, given by $\bar{\theta}^{\prime \prime}$ and $\Delta p^{\prime \prime} / p$. Thus

$$
\begin{equation*}
-\bar{q}=\mathbf{M} \bar{\theta}^{\prime \prime}-(\mathbf{I}-k L \mathbf{N}) \bar{\varepsilon}+\bar{R}^{\prime \prime}+\bar{\eta}\left(\frac{\Delta p^{\prime \prime}}{p}\right) \tag{6}
\end{equation*}
$$

Using the definition of $\bar{\varepsilon}$ in (4) yields

$$
\begin{align*}
& -\bar{q}=\mathbf{M} \bar{\theta}^{\prime \prime}-\left(\bar{q}+\bar{R}^{\prime}+\bar{\eta}\left(\frac{\Delta p^{\prime}}{p}\right)\right)+\bar{R}^{\prime \prime}+\bar{\eta}\left(\frac{\Delta p^{\prime \prime}}{p}\right) \\
& \mathbf{M} \bar{\theta}^{\prime \prime}=-\left(\bar{R}^{\prime \prime}-\bar{R}^{\prime}\right)-\bar{\eta}\left(\frac{\Delta p^{\prime \prime}-\Delta p^{\prime}}{p}\right) \tag{7}
\end{align*}
$$

It should be noted that the elimination of $\bar{q}$ assumes that the BPM heads are fixed relative to the quadrupoles, which will prevent the quadrupole offsets changing during the magnet adjustment. This would require a new quadrupole-BPM offset measurement, resulting in a new value of $\bar{q}$.
After the realignment the corrector strengths for a corrected orbit will correspond to changes in $\bar{R}$, i.e. any changes in the accumulated orbit distortion from dipole errors outside of the quadrupoles, and any change in the RF relative to the value before the realignment. Keeping the RF constant is thus advised unless compensation for temperature changes or other effects is necessary.
It should be noted that when using the approximation $\bar{\delta} \approx(\mathbf{I}-k L \mathbf{N})^{-1} \mathbf{M} \bar{\theta}^{\prime}$, the approximation error consists of two parts; corrector strength noise, and the $\bar{\varepsilon}$. While the corrector noise contribution can be minimized by measuring over a sufficiently long time period, the contribution from $\bar{\varepsilon}$ cannot. It can thus be considered the accuracy limit to which the quadrupole misalignments can be determined.
Further, the dimensions of the $\mathbf{N}$ and $\mathbf{M}$ matrices determine the nature of the solution for $\bar{\delta}$. In order not to have an underdetermined equation system, the number of BPMs should equal or exceed the number of quadrupoles for which the misalignments are being calculated. This limits the usefulness of the method; while it will still work to minimize corrector strengths for any ring by adjusting the quadrupoles neighbouring the BPMs, it will need to be generalized in order to be provide misalignment estimates for the more common case where there are more magnets than BPMs. The method does however rely on the ability to correct the closed orbit to $\bar{u}=0$, which requires the number of corrector magnets to be equal to or larger than the number of BPMs.

## MAX III REALIGNMENT

The method described above was applied to the MAX III ring. In the case of MAX III, certain design decisions had an impact on the realignment accuracy as well as how the method was applied.

- The MAX III focusing quadrupoles and main dipoles containing a defocusing gradient are contained within a single solid iron block, as shown in Figure 1. Hence there was no option to adjust the quadrupoles separately.
- BPM heads are fastened to the cell blocks, rather than the supporting girders. As a result, the BPM offsets relative the quadrupoles stayed constant while adjusting the magnet cells.
- The MAX III corrector magnets are integrated into the focusing quadrupoles. While the method was tested on MAX III, the $\mathbf{M}$ and $\mathbf{N}$ matrices were considered identical as an approximation.


Figure 1: Overview of a MAX III magnet cell.
The offsets between BPMs and corresponding quadrupole centres were measured through quadrupole shunting. The offset accuracy was empirically determined to $50 \mu \mathrm{~m}$ and $100 \mu \mathrm{~m}$ for the horizontal and vertical plane, respectively. BPM signal noise was the largest influence on how accurately the offsets could be determined. The signal noise was estimated by the $1 \sigma$ standard error for 1000 orbit measurements, and was determined $4.8 \mu \mathrm{~m}$ and 1.3 $\mu \mathrm{m}$ for the horizontal and vertical plane respectively. Most of the signal noise was shown to be due to beam motion. In order to avoid any possible systematic error due to current dependency of the BPM signal all measurements during the realignment procedure utilising the BPMs were performed in the $30-35 \mathrm{~mA}$ current range. BPMs and correctors were also calibrated by response matrix analysis using MATLAB LOCO [4]. This provided BPM gain factors, corrector kick strengths as well as a measured response matrix $\mathbf{M}$ for use in the calculation of $\bar{\delta}$.
Orbit correction to minimise $\bar{u}$ was done using the MAX III Singular Value Decomposition (SVD) based orbit feedback system. The correction system used all 16 correctors and 16 BPMs to adjust the closed orbit to the measured quadrupole magnet centres, with a correction frequency of 1 Hz . The feedback system did however not adjust the RF. The RF was set to 99.926 MHz , which for the 700 MeV MAX III ring corresponds to an orbit circumference of 36.001717 m . This frequency corresponds to the usual operating point of the ring.
The procedure described in the preceding section was then applied twice in the horizontal plane. The RF was kept constant throughout the entire procedure.

## MAX III RESULTS AND DISCUSSION

The required root mean square (RMS) corrector strength for a fully corrected orbit roughly halved after each iteration; from the starting value of 0.234 mrad the RMS strength decreased to 0.120 mrad and 0.067 mrad after the first and second iteration, respectively.
This indicates that there were substantial changes to the $\bar{R}$ vector, as the RMS corrector strength did not immediately go to zero. The likely explanation lies in the
approximation $\mathbf{M}=\mathbf{N}$; the change in dislocation kick strength due to the unavoidable shift of the dipole gradient was not taken into account in the $\mathbf{N}$ matrix. As convergence towards zero RMS corrector strength was still observed, ignoring this shift in the misalignment calculations was a valid approximation though it came at the cost of a reduced rate of convergence.
The individual correction kicks required to bring the closed orbit to the measured BPM quadrupole offsets after each iteration are shown in Fig. 2. The misalignment estimates for each quadrupole after each iteration are shown in Fig. 3.


Figure 2: Change of required horizontal corrector magnet kicks for correcting the MAX III closed orbit.


Figure 3: Change of estimated horizontal misalignments of the MAX III quadrupoles.

Some attempts were made to apply the method in the vertical plane. However, due to the construction of the MAX III magnet cell supports this had to be temporarily
abandoned; a vertical position adjustment caused horizontal shifts to appear. While it was possible to compensate for this effect, time constraints meant the MAX III method test had to be restricted to the horizontal plane.

## CONCLUSIONS

The required MAX III corrector magnet strengths to correct the closed orbit to the quadrupole centres as determined by standard beam-based alignment (BBA) have been considerably relaxed in the horizontal plane. The method used here relies on corrector magnet strengths as input. When applying the method iteratively on the MAX III storage ring it decreased the RMS horizontal corrector strength by roughly $50 \%$ per iteration. The prospects for increasing this rate for the MAX III ring are good, as the effects of shifting the MAX III combined function dipole were initially ignored.
Estimates of magnet misalignments could be obtained, although the error margin is not quantifiable without other measurements.
The method requires a BPM located close to the quadrupoles that will be realigned. Generalization of the method to include quadrupoles without neighbouring BPMs is currently being investigated.

## REFERENCES

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