HIGH ORDER SUPER-PERIODIC STRUCTURAL RESONANCES*

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Abstract

High order super-periodic structural resonances, which arise from the study of SSRF storage ring beam dynamics, are found to have large effects on beam dynamics. The mechanism and feature of this kind of resonances are described in this paper. The super-periodic structural resonance diagram instead of generic structure resonance diagram is thought to be essential for tune choosing.

INTRODUCTION

For the single particle beam dynamics of ring accelerators, perturbations can take effect through the mechanism of resonance. The rule of thumb is that the structural resonances are stronger than generic ones, and resonances with lower orders are always more dangerous than that with higher orders. Thus before lattice design, one usually plot structure resonance diagram and choose the nominal working point in the area clear of low order structure resonances so as to make the subsequent nonlinear optimization easier. However, a kind of structure resonances with specific harmonics, even with high orders, can have relatively large effects on the beam dynamics. We name these resonances as "high order super-periodic structural resonances (HOSSR)". The HOSSR rises from the study of the Shanghai Synchrotron Radiation Facility (SSRF) storage ring beam dynamics, and in Section 2, taking the test lattice based on SSRF, the mechanism and feature of the HOSSR are investigated and described detailedly. The super-periodic structural resonance diagram instead of generic structure resonance diagram is proposed for tune choosing and applied to analyze other third generation light sources in Section 3. Some concluding remarks are given at last.

HOSSR STUDY BASED ON SSRF

Origin of HOSSR

SSRF is a third generation light source and consists of three major parts, a full energy injector, a storage ring and synchrotron radiation experimental facilities. The storage ring is 4-folded, with circumference 432m. The nominal working point is (22.22, 11.32). We apply the frequency map analysis (FMA) [1] to analyze the transverse beam dynamics, it shows that globally the beam dynamics are largely affected by a 5th-order structure resonance $3Q_x - 2Q_y = 44$, as shown in Figure 1. Particles encounter the resonance at horizontal amplitude $x \approx 10$ mm; the tunes do not cross directly but move along the resonance, leading to obvious orbit diffusion. Moreover, the particles at the edge of the horizontal dynamics aperture (DA) behave chaotic motion, with tunes dramatically changing around the resonance.

In common sense, resonance of 5th and higher order is harmless to beam dynamics. However, in this case the resonance does affect the particle motion obviously. We think it is not occidental, but due to that the perturbation component of 44th harmonics is relatively large.

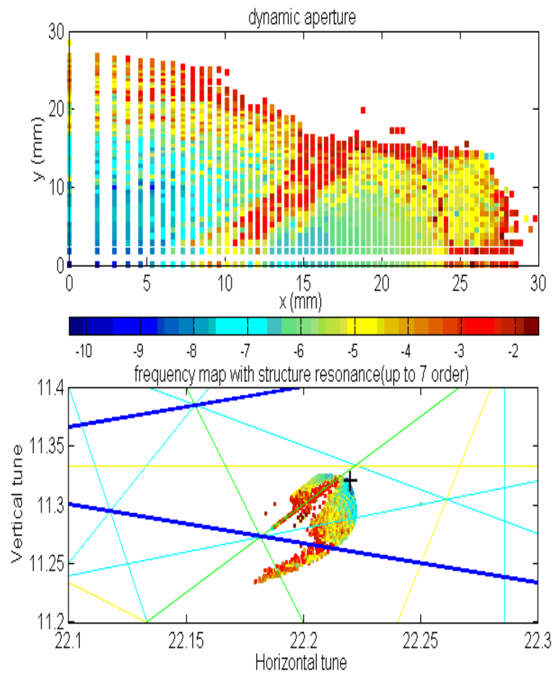

Figure 1: Dynamic aperture and frequency map for SSRF strong ring.

Mechanism of HOSSR

First it should be mentioned that the HOSSR is related closely with the first and second order super-periodic structural resonance (SSR). Fang and Qin apply the conventional stability diagram (necktie diagram) to study the modern accelerators which usually adopt complex lattice structure, such as the Chasman-Green structure and the quasi-FODO cell, instead of simple FODO cell. Some integer and half integer resonances exhibit as "stopbands" in the $K_f - K_d$ space, implying stronger effect than others. The harmonic number of the super-periodic resonance in the lattice configuration contributes to the formation of the structural resonance stopband [2].

Take the simplest case, i.e. horizontal resonance, to describe the mechanism of the HOSSR. For $mQ_x = N$, the equation of particle motion can be written as

$$\ddot{w} + v_0^2 w = \overline{p}_n(\varphi) w^{n-1} \qquad (1)$$

Generally one separate the perturbation term $\overline{p}_n(\varphi)$ with betatron oscillation term w. In fact, w can be decomposed to be an average term and a fluctuant term. Rewrite right hand of the equation

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$$\begin{aligned}\bar{p}_n(\varphi)w^{n-1} &= \bar{p}_n(\varphi)(<w> + \tilde{w})w^{n-2} \\ &= U_{n-1}(\varphi)w^{n-2} + \bar{p}_n(\varphi)\tilde{w}w^{n-2}\end{aligned} \quad (2)$$

where $U_{n-1}(\varphi) = \bar{p}_n(\varphi) < w >$.

Note that the new perturbation term $U_{n-1}(\varphi)$ contains the betatron oscillation term <*w*>. If the working point is close to a super-periodic structural stopband, in our case, Q_x close to $2Q_x = 44$, the particle will experience about 22 time beta oscillations in one revolution period (just 22 semi-periods, but already enough), thus the perturbation with harmonics number of 22 will be much superimposed, and thus the corresponding resonance with the same or multiple of the harmonics generally will be excited easily and impose strong perturbation force on the regular particle motion.

Feature of the HOSSR

It has been mentioned above that the HOSSR is related to the harmonic number of the super-periodic resonance in the lattice configuration, thus HOSSR may change with linear optic parameters. Lattices with five different working points along the 5th-order SSR $3Q_x - 2Q_y = 44$ are chosen (see Figure 2) and put into subsequent analysis. Two cases for working point (22.16, 11.23) are presented, with different sextupole arrangements.

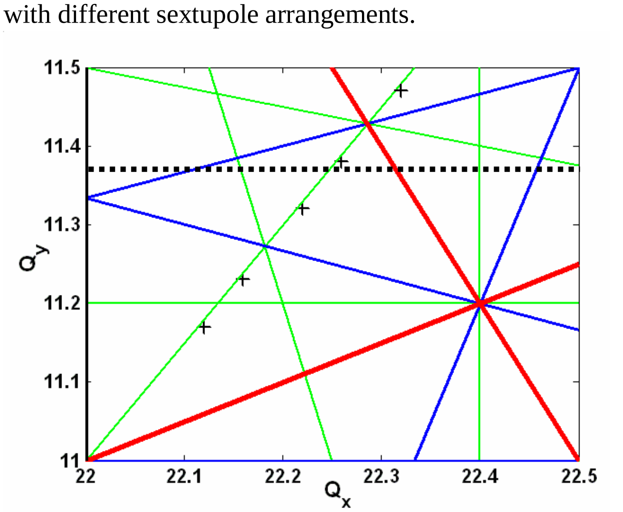

Figure 2: Tune scanning path in resonance diagram, marked with black dashed line and cross.

The typical DAs of the on-momentum particles for five working points are obtained using FMA, as shown in Figure 3. The resonance $3Q_x - 2Q_y = 44$ affects the DA mainly in horizontal plane, corresponding to the bars with high diffusion rates marked by circles. We can foresee that the motion of trapped particles will be unstable, even leading to beam loss and DA shrink, if this resonance is further excited, for instance, by magnetic imperfections. We compare the resonance position, the number and the average diffusion rate of the trapped particles for different working points. We also calculate the resonance strength and position using Lie Algebra method for cross-check. The results obtained with the two approaches accord with each other very well [3].

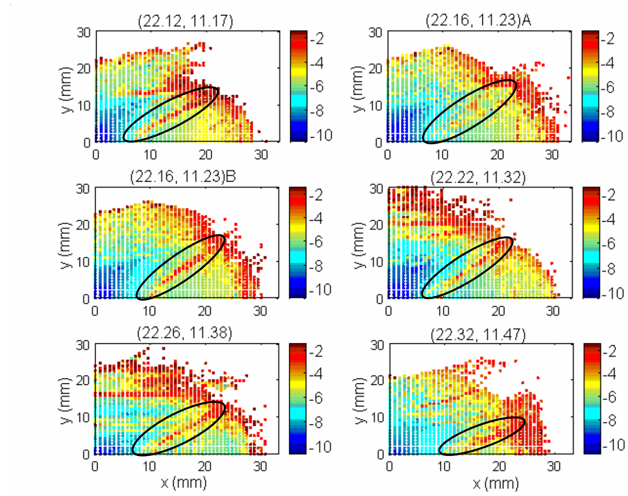

Figure 3: DAs for different working points (22.12, 11.17), (22.16, 11.23), (22.22, 11.32), (22.26, 11.38) and (22.32, 11.47) obtained using FMA.

From the analysis, the conclusion can be made that the resonance can affect the beam dynamics significantly and generally speaking, the effect will be weaken along with the working point moving away from the super-periodic structural resonance stopband.

Scanning in Tune Space

Before the final perorating, one should compare SSRs with non-SSRs. For DA is a good token for the beam dynamics, we investigate the effects of the resonances on beam dynamics by tracking the DA while scanning the horizontal working points, from 22.02 to 22.48, as shown in Figure 2. The vertical working point is chosen to be 11.36, so as to make the scanning path away from the resonances-crossing area and easily identify and compare the effects of each resonance. We use MAD and OPA for linear matching, chromaticity correction (slightly larger than 0) and nonlinear optimization, and try to find a good result of the DA for each tune.

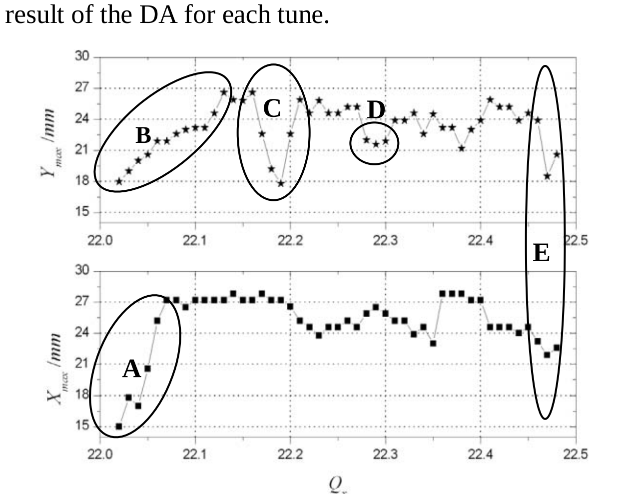

Figure 4: On- and off-momentum dynamic aperture for different horizontal working points, while keeping the vertical working point as 11.36.

The on-momentum DA variations with the horizontal working point are plotted in Figure 4. The relationship between the DA shrinks and the resonances is listed in Table 1.

It shows that the super-periodic structural resonances, including the second order SSR stopband $2Q_x = 44$ and the 5th-order SSR $3Q_x - 2Q_y = 44$, do have large effect on the beam dynamics compared with other resonances.

Table 1: Relationship between the DA shrinks and resonances

Region	Tune range	Resonance
A	22.0~22.07	$2Q_x = 44$
B	22.0~22.13	$2Q_x = 44$
C	22.17~22.21	$3Q_x - 2Q_y = 44$
D	22.28~22.30	$Q_x + 3Q_y = 56$ & $Q_x - 2Q_y = 0$
E	22.46~22.50	$2Q_x = 45$

SUPER-PERIODIC STRUCTURAL RESONANCE DIAGRAM AND APPLICATION

Tune Diagram of SSR

According to the analysis above, give the complete description of the SSR as follows:

If the lattice is M periodic, i.e., composed of M identical sectors (this is the case for a perfect machine), the first and second order SSR is

$$R_Q = M \times l / 2 \text{ with } l \in \mathbf{N} \quad (3)$$

The resonance is first order when l is even and second order when l is odd. If the working point near the first or second order SSR stopband and the nearby structural resonance $mQ_x + nQ_y = k \times M$ satisfies

$$mQ_x + nQ_y = s \times R_Q \text{ with } s \in \mathbf{N} \quad (4)$$

Namely, the harmonic number of the resonance is the multiple of both M and R_Q, the resonance will be HOSSR.

In practical lattice design, one usually chooses the nominal working point in the area away from the integer and half integer resonances, which may be first or second order SSRs, but not always attentively avoid the HOSSRs. So it is necessary to plot the super-periodic structural resonance diagram near the working point, to examine possible perturbation induced by HOSSRs and guide the choice of working point and nonlinear optimization.

Application of the Tune Diagram of SSR to Other Light Sources

SOLEIL is a newly built light source in French, the analysis based on the Nadolski's FMA results [1]. The storage ring is 4 folded. The working point is (18.28, 8.38), near the first order SSR stopband $Q_y = 8$ and second order SSR stopband $2Q_x = 36$. The SSRs from first to seventh order are plotted, as shown in Figure 5. It shows that the nominal working point locates at the resonance crossing area.

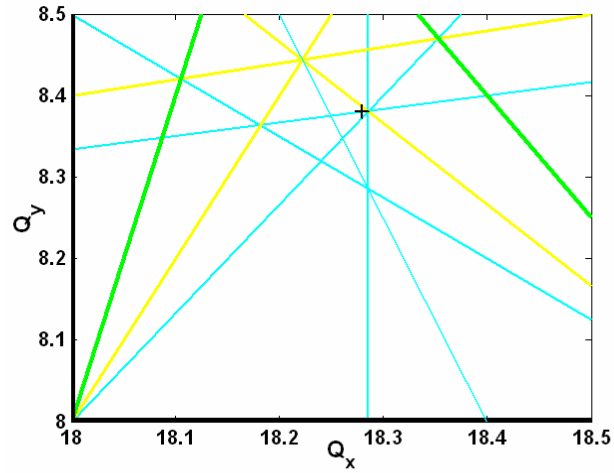

Figure 5: Super-periodic structural resonance diagram near the SOLEIL working point.

In reference [1], three lattices are presented in turn. For the first optics, it shows that the 7th-order SSRs $5Q_x + 2Q_y = 3 \times 36$ and $-Q_x + 6Q_y = 4 \times 8$ reach for the horizontal amplitude $x \approx 24$ mm. Keeping the same working point, modify the sextupolar strengths to make the frequency map folded, so as to keep away from the resonance $5Q_x + 2Q_y = 3 \times 36$, but the resonance $-Q_x + 6Q_y = 4 \times 8$ affects smaller horizontal amplitude $x \approx 10$ mm and another 7th-order SSR $7Q_x = 16 \times 8$ becomes the dominant resonance to the beam dynamics. For the third optics, the working point is moved to (18.30, 8.38) to avoid the resonance $7Q_x = 16 \times 8$, the frequency map twice folded on itself, thus particles encounter low number of resonances and allow of more regular particle motion. However, the beam dynamics is still limit by two SSRs $-Q_x + 6Q_y = 4 \times 8$ and $3Q_x + 3Q_y = 10 \times 8$ at the edge the DA.

Similar analysis to ESRF, SUPER-ACO and ALS are made as well, the performance of the beam dynamics can be successfully explained using tune diagram of SSR.

CONCLUSION

The existence and strong effect to beam dynamics of the super-periodic structural resonances are identified in this paper. The tune diagram of SSR is proposed for tune choosing and applied to explain the beam dynamics of light sources. However, the conclusions are obtained only basing on the computer simulation, and further checking beam experiments are needed.

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