# EXPERIMENTAL EVIDENCE OF BEAM TRAPPING WITH ONE-THIRD AND ONE-FIFTH RESONANCE CROSSING 

S. Gilardoni, A. Franchi, M. Giovannozzi, CERN, Geneva, Switzerland


#### Abstract

Beam trapping in stable islands of the horizontal phase space generated by non-linear magnetic fields is realized by means of a given tune variation so to cross a resonance of order $n$. Whenever the resonance is stable, $n+1$ beamlets are created whereas if the resonance is unstable, the beam is split in $n$ parts. Experiments at the CERN Proton Synchrotron showed protons trapped in stable islands while crossing the one-third and one-fifth resonance with the creation of 3 and 6 stable beamlets, respectively. The results are presented and discussed.


## INTRODUCTION

In 2002 a new extraction scheme was proposed to eject the beam over a few turns by means of non-linear magnets, such as sextupoles and octupoles, rather than slicing it onto an electrostatic septum [1]. During this new multi-turn extraction, the beam is split and trapped inside stable islands of the horizontal phase space, that are generated and separated by sweeping the horizontal tune through a non-linear resonance while sextupoles and octupoles are powered.
The number of generated beamlets depends on the order of the crossed resonance. In general, a resonance of order $n$ generates $n+1$ beamlets if stable, whereas if it is unstable, the beam is split in $n$ parts, the centre becoming an unstable fixed point in the latter case. In the case of the beam transfer between the CERN Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS) at $14 \mathrm{GeV} / c$, the most suitable resonance is the stable one-fourth, i.e. $Q_{x}=6.25$ A loss-free beam splitting in five beamlets (four islands plus the beam core) was already proved [2] and the new five-turn extraction is due to be commissioned this year [3]. The choice of crossing the stable one-forth resonance to transfer the beam from the CERN PS to the SPS is dictated by the fact that two PS cycles are used to fill the SPS ring, whose circumference is 11 times the PS one. At the end of each PS cycle the beam would be ejected in five turns. The SPS would be then filled in 10/11 of its length (one empty slot is left to avoid interference between the circulating beam and the transient times of the SPS kickers). The delicate point of such scheme concerns the equalization of the beam parameters between the four beamlets trapped in the islands and the one remaining in the central area. The latter is unavoidably ejected with root-mean-square (RMS) optical parameters slightly different from to ones of the islands. As far as the intensity is concerned, the best beam sharing so far achieved is of $\sim 18 \%$ in the four islands and $\sim 28 \%$ in the central core. To avoid unwanted transient effects in the SPS, a maximum difference of about 05 Beam Dynamics and Electromagnetic Fields
$5 \%$ in the sharing between islands and core shall be provided. Another delicate point is the need of an additional kick to extract the beam core that might induce a sligthly different trajectory in the trasfer line, compared to the ones of the islands, not to mention the impact on the hardware requirements.

Dedicated experiments at the CERN PS were carried out in 2007 at $14 \mathrm{GeV} / c$ to exploit the possibility of generating islands by crossing new resonances: the unstable onethird, generating three beamlets only, and the stable onefifth, thus creating six beamlets, five in the islands plus the beam core. In the former case, three PS cycles might be used to fill the SPS, leaving two gaps. In the case of the one-fifth resonance, $N$ repeated crossings during the flat top of the same cycle can be envisaged, in order to extract $N$ batches of five identical beamlets each, see Ref. [1]. In both cases, the beamlets injected in the SPS would have exactly the same intensities and RMS optical parameters. The beam remaining in the central area after $N$ crossing of the one-fifth resonance might be eventually ejected towards a beam dump.


Figure 1: Schematic layout of the PS ring with the elements used during the experiments.

The magnetic elements and the beam instrumentation used in the experimental campaign are shown in Fig. 1. The tune is changed to cross a resonance by means of two families of focusing and defocusing quadrupoles, normally used to tune the machine at low energy. Two sextupoles and one octupole are used to generate the stable islands. Wire scanners are then used to measure the horizontal beam distribution and to monitor its evolution. The determination of islands' position, beamlets' size, and fraction of trapped particles is inferred from the recorded beam profile. This is performed by fitting $n$ or $n+1$ Gaussian functions, according to the resonance order $n$ and its stability type. The functions used to fit the islands are constrained to have the same intensity. The reason for such a constrain is twofold: First, D02 Non-linear Dynamics - Resonances, Tracking, Higher Order
the resonance symmetry guarantees that all islands have the same probability of trapping particles; Second, each measured island profile is actually an average over all islands profiles, the wire time of flight being much longer than the revolution period $(2.1 \mu \mathrm{~s})$ and the islands swapping position turn by turn.

## CROSSING THE 1/3 RESONANCE

Three stable islands were created for the first time on August 10th 2007 using a single-bunch of $\sim 3 \times 10^{12}$ protons, whose horizontal RMS normalized emittance at $1 \sigma$ before the resonance crossing was of $5.7 \mu \mathrm{~m}$. The horizontal tune was initially set to 6.330 , then increased linearly until the value of 6.337 in about 100 ms ( $\sim 4.8 \times 10^{4}$ turns), and eventually brought back to the initial value, as depicted in the right plots of Fig. 2. The vertical tune was set to 6.205 throughout the double resonance crossing.

Both the octupole and the sextupoles were powered with a constant current of $-421 \mathrm{~A}\left(K_{3}=-155 \mathrm{~m}^{-3}\right)$ and 330 A ( $K_{2}=0.9 \mathrm{~m}^{-2}$ ), respectively. The main results are plotted


Figure 2: Evolution of the horizontal beam profile during the crossing of the one-third resonance. The profile is shown before the resonance is crossed (upper), right after the crossing (centre), and at the end of the crossing (lower). The corresponding phase space portrait obtained from the MADX PS model is superimposed to the measured profile. 05 Beam Dynamics and Electromagnetic Fields
in Fig. 2, where the horizontal beam profile is shown before the resonance is crossed (upper), right after the crossing (centre), at the end of the crossing (lower). Next to each profile the corresponding position on the tune ramp is indicated. In the background of the measured profile, in gray, the phase space portrait computed by the MADX program [4] for the corresponding working point is shown. For completeness, below the window with the curve of the horizontal tune the temporal evolution of the beam intensity is reported. While the resonance is crossed without measurable losses, at the end of the capture a loss of about $10 \%$ occurs, probabily due to a too fast tune variation around 1020 ms . The steep descent of the tune after 1050 ms might be the reason of the further beam loss, that at the end of the process is of about $20 \%$. From the measured profiles it is not possible to conclude whether losses occur because particles trapped in the islands drift away as in the normal slow extraction, or whether they are due to non-trapped (or de-trapped) particles in the chaotic region between the core and the islands.

The beam profiles measured during the capture were fitted with a superposition of four Gaussians. Results are shown in Fig. 3. The beam profile measured at the end of the capture ( 1070 ms ) is well fitted by three main Gaussian curves (in green in the picture), each one containing a fraction of particle of about $31.8 \%$, see left plot of Fig. 3. The remaining $4.6 \%$ of the beam remains in the central area (in red in the picture). Both the 2D Hénon model [1] and pure 4D multi-particle simulations predict a complete depletion of the beam core and well separated islands. This is not the case when chromatic effects $\left(Q_{x}^{\prime}=1.2\right)$ and tune modulation induced by the synchrotron motion (synchrotron tune $Q_{s}=1 \times 10^{-2}$ and momentum spread $\left.\Delta p / p=1 \times 10^{-3}\right)$ are included: Simulations in this case show that about $6 \%$ of the beam remains in the core, while about another $6 \%$ is confined in the region between islands and core. The comparison between the measured and simulated profiles is shown in the right plot of Fig. 3. Even though the threepeak structure is well reproduced, the beamlets' positions slightly differ: the matching is improved is the profile is computed 1.4 m upstream the actual position of the wire scanner.


Figure 3: Measured horizontal beam profile at the end of the capture with superimposed multi-Gaussian fit (left), and comparison with multi-particle MADX simulations including chromaticity and tune modulation (right). D02 Non-linear Dynamics - Resonances, Tracking, Higher Order

## CROSSING THE 1/5 RESONANCE

A similar manipulation was performed on July 27th, when five stable islands were created for the first time. The beam parameters were the same of the one-third resonance crossing. The horizontal tune was initially set to 6.190 , then increased linearly until the value of 6.213 in about 300 ms ( $\sim 1.3 \times 10^{5}$ turns), as depicted in the left plot of Fig. 4. The vertical tune was kept constant to about 6.3 throughout the the splitting process. Conversely to the one-third resonance crossing experiment, the octupole was programmed to impart a large gradient $K_{3}=99 \mathrm{~m}^{-3}(-280 \mathrm{~A})$ during a fraction of the crossing ( 85 ms ), to be then reduced at the end of process to $K_{3}=15 \mathrm{~m}^{-3}(-40 \mathrm{~A})$. The sextupole current was fixed to $350 \mathrm{~A}\left(K_{2}=1.0 \mathrm{~m}^{-2}\right)$ in order not to perturb the machine chromaticity. Data from the current transformer clearly show that no losses occur throughout the resonance crossing, the beam intensity remaining constant at about $\sim 2.95 \times 10^{12}$ (see right plot of Fig. 4).


Figure 4: Left: measured horizontal fractional tune and time evolution of the octupole current in the interval where the one-fifth resonance is crossed. Right: number of protons per pulse measured by the current transformer.

A simple way to scan over the octupole strength without a repeated and time-consuming programming of the octupole current curve was found by delaying the octupole turn-on. Several measurements of the horizontal beam profile were taken at a fixed time ( 1160 ms ) and tune ( $Q_{x}=6.212$ ), after retarding the octupole pulsing each time of 10 ms . Three cases are shown in Fig. 5. In the first case (top row) the resonance is crossed (at 1050 ms ) with the octupole weakly powered, while the measurement is taken when its strength is maximum: five islands close to the beam centre are then expected. In the second case (centre row) the octupole is set to impart the maximum strength ( $I=-280 \mathrm{~A}$ ) when the resonance is crossed. The profile in this case is measured when the octupole current is halved ( $I=-140 \mathrm{~A}$ ) yielding more separated islands. In the third and last case (bottom row) the octupole curve is further shifted towards left and the resonance is crossed when the octupole current is of about -50 A. The lower current generates less populated, but more separated, islands.

The above considerations are confirmed by displaying the phase space portraits computed by the MADX program 05 Beam Dynamics and Electromagnetic Fields


Figure 5: Horizontal beam profiles (left) measured at the end of the one-fifth resonance crossing ( 1160 ms ) for three different settings of the octupole currents (right).
on top of the the measured profiles, and by performing the multi-Gaussian fit, as shown in the left pictures of Fig. 5. The agreement between the fitted Gaussians and the islands' position is excellent and unambiguous for the third case, when the islands are well separated. When instead the distance is reduced the fit may not be unique (first case), and the phase space portrait can be used to set some constraints, such as the islands centroids.

## REFERENCES

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