# VERTICAL BEAM SIZE REDUCTION VIA COMPENSATION OF RESIDUAL TRANSVERSE COUPLING 

T. F. Roque, X. R. Resende, P. F. Tavares<br>LNLS, Campinas, SP 13085, Brazil

## Abstract

This report describes our recent efforts in understanding and controlling what residual effects there are in the Brazilian Synchrotron Light Source (LNLS) ring that dominate the its vertical beam size. In particular, we study the indirect effects of residual coupling perturbations on the beam size through the transfer matrix formalism and measured closed orbit distortions (CODs). A computer code (AIOIA) was developed to model Twiss parameters directly from measured data. With this tool we are able to propose new skew quadrupole (SQ) elements to the storage ring that should reduce local couplings and, as a consequence, the vertical beam size at the EPU.

## INTRODUCTION

The 1.37 GeV electron storage ring at LNLS is a DBAtype ring with a 93.2 m circumference and with a 100 nm.rad emittance. The ring had a six-fold symmetry which was lost after the installation of two insertion devices (IDs) in consecutive long straight sections: a 18 cm -period 2 T wiggler and a 5 cm -period EPU. In order to allow higher fields with reduced magnetic gap sizes and to reduce the impact of these fields on beam dynamics, a new low vertical beta $\left(\beta_{z}\right)$ mode at the two straight sections with IDs was calculated, implemented and is now the default mode for users shifts. The 2 T wiggler introduced noticeable effects: in particular, an additional coupling is easily observed experimentally.

Although there are a number of approaches in the literature deriving the relation between skew coupling fields and equilibrium vertical emittance[1, 2], none so far seems to have both conceptual simplicity and robustness. Instead of choosing one or another approach to study residual coupling in detail we decided to take a conceptually simpler path: we assume that overall reduction of the off-diagonal COD response functions implies reduction of global transverse coupling and consequent reduction of coupling contribution to the vertical beam size. In the limit when offdiagonal COD tend to zero everywhere in the ring, our assumption is certainly true since zero local coupling everywhere translates into no global coupling. Naturally, in real situation, we can only try to minimize the CODs on the BPMs positions, where the orbits can be measured, and with a small number of SQ correctors. The following paragraphs describe our recent preliminary study in this direction. Our basic assumption has to be justified experimentally a posteriori.
05 Beam Dynamics and Electromagnetic Fields

## COMPUTER CODE "AIOIA"

To better understand coupling fields introduced by the wiggler, as well as those which are due to spurious effects from other elements in the ring, we have developed a computer code called AIOIA which uses measured data (mainly CODs produced by correctors or cavity frequency shifts) to improve on existing linear models of the ring, including modeling thin SQ elements. Since off-diagonal COD matrix elements due to residual coupling are very small, careful calibration of a non-coupled model is important before attempting to model coupling fields. AIOIA was designed to accomplish this task. It resembles LOCO[3] in its purpose. The decision to develop our own software instead of using LOCO was motivated mainly by two reasons: first, LOCO runs on MATLAB, an expensive graphical platform. Second, and more importantly, it is not clear how much of the discrepancies between measurements and model predictions is really due to unknown gains and model parameters values, and how much is due to plain inadequate element modelling. In order to turn things simpler we decided to directly fit Twiss parameters, phase advances and dispersion functions from CODs and tune measurements rather than going all the way to fitting parameters of a predetermined set of element models. For a large number of experimental points, which is typically the case for CODs measurements, stopping at this intermediate step should suffice to provide a reasonable model which can be used to try to compensate for residual coupling as is manifest in the offdiagonal part of the CODs.

AIOIA implements fittings of Twiss parameters, phase advances, dispersion functions and gains for correctors and beam position monitors (BPMs) (in the case of the LNLS ring, 18 horizontal, 24 vertical correctors and 24 monitors). It starts reading a MAD output file with an initial guess for what the linear optics should be. The code then reads three different sets of input files: one which contains COD responses to both horizontal and vertical correctors, a second set with CODs from varying the cavity's radio-frequency (which, apart from yet unknown monitor gains, are measured dispersion functions) and a third one with measured tune values. AIOIA then proceeds by fitting the linear uncoupled optics parameters, and optionally SQ inclutions, to the experimental data set. This is accomplished by minimizing a $\chi^{2}$ that is the r.m.s. deviations between model predictions and measured values:

$$
\begin{equation*}
\chi^{2}=\chi_{c . o_{x}}^{2}+\chi_{\text {c.o.z }}^{2}+\chi_{\eta_{x}}^{2}+\chi_{\eta_{z}}^{2}+\chi_{\mu_{x}}^{2}+\chi_{\mu_{z}}^{2} \tag{1}
\end{equation*}
$$

where each contribution to $\chi^{2}$ in eq.(1) is normalized by
D01 Beam Optics - Lattices, Correction Schemes, Transport
the number of data points and by the estimated data-point errors. AIOIA uses a zero-temperature simulated annealing algorithm for the search. This algorithm proved to be adequate since our MAD8 initial guesses for the optics has been always close enough to converging solutions.

## CALCULATING CODS WITH AIOIA

The calculation of CODs due to correctors kicks is pretty straightforward using the transfer matrix formalism. If $\overrightarrow{\mathbf{r}}_{n}\left(s_{m}\right)=\left(x, x^{\prime}, z, z^{\prime}\right)_{s_{m}}$ is the coordinate vector of the particle at the position $s_{m}$ of a BPM, it can be transported to the location of the corrector kick at $s_{c}$ through the application of the transfer matrix $\mathbf{M}\left(s_{c} \mid s_{m}\right)$. At this longitudinal position the electron is kicked and a vector $\overrightarrow{\mathbf{p}}\left(s_{c}\right)$ corresponding to the dipolar perturbation should be added to the particle coordinate vector. The resulting vector should then be transported back to the original location $s_{m}$ of the BPM, one turn around the ring, by the application of the transfer matrix $\mathbf{M}\left(s_{c} \mid s_{m}\right)$. For the closed orbit solution, the final coordinate vector $\overrightarrow{\mathbf{r}}_{n+1}\left(s_{m}\right)$ should be equal to the original vector, $\overrightarrow{\mathbf{r}}_{n+1}\left(s_{m}\right)=\overrightarrow{\mathbf{r}}_{n}\left(s_{m}\right)=\overrightarrow{\mathbf{r}}_{\text {c.o. }}\left(s_{m}\right)$. The matrix algebra described above gives

$$
\begin{equation*}
\overrightarrow{\mathbf{r}}_{\text {c.o. }}\left(s_{m}\right)=\left\{\left(\mathbf{1}-\mathbf{M}_{m}\right)^{-1} \mathbf{M}\left(s_{m} \mid s_{c}\right)+\mathbf{D}\right\} \overrightarrow{\mathbf{p}}\left(s_{c}\right) \tag{2}
\end{equation*}
$$

where the matrix $\mathrm{M}_{m}$ is the one-turn matrix at $s_{m}$ and the term $\mathbf{D}$ was added $a d$ hoc in order to take into account the conservation of path length due to the principle of stability of the longitudinal dynamics. The matrix $\mathbf{D}$ has only two non-vanishing elements, $\mathbf{D}_{(12,34)}=$ $-\eta_{(x, z)}\left(s_{m}\right) \eta_{(x, z)}\left(s_{c}\right) / \alpha_{c} C$, and it represents a leading term in a rigorous expression yet to be derived which accounts for an arbitrarily coupled dynamics. Nonetheless this expression should be useful for our purpose.

When there are no coupling elements in the ring eq.(2) is block-diagonal and it simplifies to the more familiar expression[3]:

$$
\begin{align*}
u\left(s_{m}\right) & =\frac{\sqrt{\beta_{u}\left(s_{m}\right) \beta_{u}\left(s_{c}\right)} \cos \left|\psi_{u}-2 \pi \mu_{u}\right|}{2 \sin \pi \mu_{u}} \theta_{u}  \tag{3}\\
& -\frac{\eta_{u}\left(s_{m}\right) \eta_{u}\left(s_{c}\right)}{\alpha_{c} C} \theta_{u}
\end{align*}
$$

with $u=x, z$.

## FITTING THE UNCOUPLED LINEAR OPTICS

The problem of finding a model which describes the residual coupling in the ring in broken into two parts with AIOIA: first all gains, phase advances and Twiss parameters are fitted to match experimental data (including diagonal CODs) for the ring. Then additional thin SQ correctors are inserted within the model and their focusing lengths are adjusted to explain the off-diagonal response functions. With the wigler opened we performed the first part of AIOIA calculations. Fig.(1) shows the improvement 05 Beam Dynamics and Electromagnetic Fields


Figure 1: Difference between measured and modeled diagonal CODs for horizontal (blue) and vertical (red) correctors. (a,d) initial non-adjusted MAD8 linear horizontal and vertical optics model. (b,e) closed orbit difference between measured and model width adjusted monitor and corrector gains. (c,f) model with gains and linear optics adjusted.

(a)
(b)

Figure 2: Fitted gains for BPM horizontal readings (blue), horizontal correctors (dark blue), BPM vertical readings (red) and vertical correctors (dark red).
of the model: left-most plots show residual differences between measurements and calculated CODs from the original MAD optics, with no fitting performed. It shows large deviations from measurements with horizontal and vertical r.m.s. of the order of 80 and $185 \mu \mathrm{~m}$, respectively. (the repetibility of the COD measurement for the 0.1 mrad kick used throughout our study is $\approx 1 \mu \mathrm{~m}$ ).

Fitting the gains for monitors and correctors improved the model significantly, specially for the horizontal dynamics. The r.m.s. dropped down to 17 and $122 \mu \mathrm{~m}$. The gains thus obtained are typical from such fittings[3]. They are displayed in Fig.(2). Monitors AMP01B and AMP09B (at $s \approx 0$ and $s \approx 65 \mathrm{~m}$ ), in particular, presented large deviations from the nominal unit gain. This was not a surprise since spurious systematic readings were recorded before for these two monitors in dispersion function measurements. At the next step, Twiss parameters and phase advances were fitted yielding r.m.s. discrepancies of only $4 \mu \mathrm{~m}$ and $3 \mu \mathrm{~m}$ between model and experiment. This is at the level of measurement imprecisions. Fig.(3) shows fitted beta functions compared to the initial MAD8 model. Sys-

D01 Beam Optics - Lattices, Correction Schemes, Transport


Figure 3: Horizontal (blue) and vertical (red) beta functions before (solid curves) and after (circles) fitting of the linear optics, including gains.
tematic discrepancies between the MAD8 model and the adjusted model can be observed. In particular, the fitted data suggests that a strong vertical beta-beat in the ring is yet to be corrected.

## COMPENSATION OF RESIDUAL COUPLING

Once the BPMs and correctors gains had been calculated we proceeded with testing the modeling of coupling elements in the ring. We closed the wiggler gap, measured both dispersion functions and CODs from the correctors, and fitted again the diagonal linear optics (see Fig.4), but now used the fitted gains from the previous calculation. $\beta_{z}$ changed very little with closing the wiggler gap. On the other hand, the model and measured CODs show a significant change in $\beta_{x}$. This difference was expected because of the low $\beta_{z}$ at the wiggler.

To be sure that the inclusion in the model of thin SQs had been done correctly we checked the simulations against measured CODs with the two SQs in the ring turned on. Measured off-diagonal horizontal and vertical orbit distortions r.m.s. were 85 and $106 \mu \mathrm{~m}$ respectively for the skew focal length value of 100 m set and 0.1 mrad kicks. In the simulations we used this nominal skew focal lengths and the calculated CODs agreed remarkably well with the experimental data: the discrepancy was approximately $5 \mu \mathrm{~m}$, much smaller than absolute measured values.

Next we tried to model the additional coupling from the wiggler by a single localized thin SQ. We were not able to fit this model to the experiment. Only after we distributed a large number of these elements along the wiggler were we able to explain the measured data within a few $\mu \mathrm{m}$. Another simple test that we ran which gave us confidence that all calculations and assumptions me made were correct was to vary the skew focal length in the simulation trying to further minimize the off-diagonal difference CODs r.m.s. The calculation shows that minimum r.m.s. is reached with the rings SQs powered off. This result is consistent with experimental data from comissioning the wiggler in our ring[4], in which we observed no reduction in beam size with varying SQs strengths.
05 Beam Dynamics and Electromagnetic Fields

(a)
(b)

Figure 4: Horizontal (blue) and vertical (red) beta functions fitted with opened wiggler gap (solid curves) and fitted with closed gap (circles).

| straight sections | CODs r.m.s. $[\mu \mathrm{m}]$ | focal length [m] |
| :--- | :---: | :---: |
| $(+0) 05 \mathrm{~A} \& 05 \mathrm{~B}$ | 46 | $\approx \infty$ |
| $(+1) 02 \mathrm{~A}$ | 24 | 37 |
| $(+1) 12 \mathrm{~A}$ | 24 | -35 |
| $(+2) 02 \mathrm{~A} \& 12 \mathrm{~A}$ | 18 | $63 \&-58$ |

Table 1: suggestions of additional skew quadrupoles to compensate for residual coupling fields as measured in the off-diagonal CODs of the LNLS ring.

At last, in possession of a calibrated model for the linear optics of the ring we tried to compensate for the residual coupling by adding SQs in the simulation at places corresponding to available locations in the ring. In the simulations we looked for SQs that generated off-diagonal CODs that when summed with measured values canceled out. This approach is correct to first order in coupling strengths. Table (1) brings a list of possible suggestions for reducing the initial r.m.s. value of $\approx 46 \mu \mathrm{~m}$ further down.

## CONCLUSIONS

We have developed and performed preliminary tests of a computer code that can be used to guide us in suggesting additional coupling-compensating elements for the LNLS ring. Results calculated with it are in accordance with expectations. Nonetheless, additional tests are needed. In the near future we intend to do a number of simple but revealing experiments (such as measurements of beta-function and variation of vertical beam size with SQs strengths) in order to test the code and the assumptions it is based on.

## REFERENCES

[1] K.Ohmi, Phys. Rev. E. 49, 751.
[2] H.Wiedeman, "Particle Accelerator Physics II", Springer Verlag (1999).
[3] J. Safranek, "Linear Optics from Closed Orbits (LOCO), An Introduction", ICFA Newsletter, \#44. Dec. 2007.
[4] R.H.A.Farias et al., "Comissioning of the LNLS 2T hybrid Wiggler", PAC 2005.
[5] X.R.Resende, Internal Communication.

D01 Beam Optics - Lattices, Correction Schemes, Transport

