# **EFFICIENT FAN-OUT RF VECTOR CONTROL ALGORITHM \***

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## Abstract

A new RF vector control algorithm for fan-out power distribution using reactive transmission line circuit parameters for maximum power efficiency is presented. This control with fan-out power distribution system is considered valuable for large scale SRF accelerator systems to reduce construction costs and save on operating costs. In a fan-out RF power distribution system, feeding multiple accelerating cavities with a single RF power generator can be accomplished by adjusting phase delays between the load cavities and reactive loads at the cavity inputs for independent control of cavity RF voltage vectors. In this approach, the RF control parameters for a set of specified cavity RF voltage vectors is determined for an entire fan-out system. The reactive loads and phase shifts can be realized using high power RF phase shifters.

# **INTRODUCTION**

Fan-out RF distribution with one higher power amplifier feeding multiple cavities may save construction and operations cost of RF systems significantly, especially in non-e- high power SRF accelerator projects. The cost savings can come from the simpler power generation and waveguide installation (minus the additional RF control system.) Figure 1 shows tree-representative power distribution systems for a particle accelerator with N cavities: for example a high power proton accelerator system. Especially with high cavity power, a cavity needs to be driven by a power source to enable precise RF vector control as shown in Figure 1(a). If a fixed power splitter is used in the fan-out system with a high power vector modulator for each cavity as in Figure 1(b), power overhead is required [1]; each cavity load needs one vector modulator that consists of two phase shifters and two (or one) hybrids: the vector modulator must dissipate a certain amount of power to allow the vector control. One other possible approach is shown in Figure 1(c): the cavities are fed through a transmission-line network from a generator. The required power distribution that maintains a specific voltage distribution in the cavities is established with transmission-line phases between the terminals and reactive loads at the terminals. This approach can maximize the RF power to beam efficiency of a fan-out system that can deliver cost savings in operation as well as in construction. In this paper an algorithm for fan-out RF distribution and control as a whole system is presented. In the following paragraphs, a



Figure 1: Comparison of high power RF distribution in a multi-cavity accelerators (a) one generator to one cavity, (b) fan-out with fixed power splitter and vector modulators, (c) fan-out with a transmission line network

procedure for determining the RF control parameters of the system is presented with a simple case example.

# DEVELOPMENT

#### Formulation

Figure 2 shows the fan-out distribution system that employs a network of transmission line sections connecting cavity terminals and reactive loads at the cavity input terminals. N cavities are fed by a single generator at one end terminal through a transmission-line network. Let  $[V^P]$  be a set of cavity voltage vectors for a specific cavity excitation at t = t'. The phase delays in the transmission-line sections between the cavities and the reactive loads that can deliver  $[V^P]$  can be found.



Figure 2: A fan-out RF distribution with reactive loading for a specified set of voltages in the accelerator cavities

<sup>\*</sup> SNS is managed by UT-Battelle, LLC, under contract DE-AC05-00OR22725 for the U.S. Department of Energy.

In Figure 2,  $D_i$  is physical spacing between cavities,  $d_i$  is the electrical length of transmission section between two neighboring cavities,  $V_i$  is the voltage delivered to the cavity input, and  $Z_i$  is the transmission-line characteristic impedance. In this discussion, the physical distances between cavities and the transmission-line characteristic impedances are fixed for the most realistic construction. The cavity inputs are assumed to be matched to the waveguide ports at the terminals for now. Considerations for more general mismatched cases will be discussed later.

A multi-port network can be described using one of the network parameters [2]: a short-circuit admittance matrix

The port admittance matrix for the cavity inputs is simply a diagonal matrix since no mutual couplings can be assumed between the cavities:

$$\begin{bmatrix} Y^{p} \end{bmatrix} = \begin{pmatrix} Y_{in,1} & 0 & \cdots & 0 & 0 \\ 0 & Y_{in,2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & Y_{in,N} & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$
(5)

The transmission line admittance matrix

$$\begin{bmatrix} Y^{T} \end{bmatrix} = \begin{pmatrix} -jY_{1} \cot \beta d_{1} & jY_{1} \csc \beta d_{1} & 0 & \cdots & 0 \\ jY_{1} \csc \beta d_{1} & -j(Y_{1} \cot \beta d_{1} - Y_{2} \cot \beta d_{2}) & jY_{2} \csc \beta d_{2} & \cdots & 0 \\ 0 & jY_{2} \csc \beta d_{2} & -j(Y_{2} \cot \beta d_{2} - Y_{3} \cot \beta d_{3}) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -jY_{N} \cot \beta d_{N} \end{pmatrix}$$
(6)

will be used in this discussion. Each element of the matrix can be obtained by using a two-port network as a building block for the network synthesis. Various configurations are realizable in constructing the transmission-line network [3]. The configuration can be more realistic and simple as a representative design in this work.

In this case, short-circuit admittance matrix [Y] can be used more easily to describe the network system. Network with elements in series connections may be synthesized and analyzed by using open-circuit impedance matrix [Z].

#### Solution Procedure

Using admittance parameters, the whole network admittance matrix  $[Y^S]$  can be constructed and the terminal currents  $[I^S]$  and the terminal voltages  $[V^P]$  are related as

$$\begin{bmatrix} I^{S} \end{bmatrix} = \begin{bmatrix} Y^{S} \end{bmatrix} \begin{bmatrix} V^{P} \end{bmatrix}$$
(1)

where [Y<sup>S</sup>] is the short-circuit terminal admittance matrix of the system that can be expressed as

$$\begin{bmatrix} Y^{S} \end{bmatrix} = \begin{bmatrix} Y^{P} \end{bmatrix} + \begin{bmatrix} Y^{T} \end{bmatrix} + \begin{bmatrix} Y^{L} \end{bmatrix}$$
(2)

where  $[Y^P]$  is port admittance matrix for the cavities,  $[Y^T]$  is admittance matrix of the transmission line network, and  $[Y^L]$  is reactive load admittance matrix.

The voltage vectors that are required to maintain a specific voltage distribution over the cavities are

$$\begin{bmatrix} V^{P} \end{bmatrix}^{t} = \begin{bmatrix} V_{1} & V_{2} & V_{3} & \cdots & V_{N} \end{bmatrix}$$
(3)

 $[I^S]$  contains all zero elements except for the *n*-th terminal that is connected to the generator. The current for the feed terminal is found as

$$Z_f = V_f / I_f \tag{4}$$

The input impedance  $Z_f$  can be found by selecting the element  $Z_{ii}$  in the impedance matrix  $[Z^S]=[Y^S]^{-1}$ . Consider a network consists of N load cavities and the line sections between them in an end-fed case shown in Figure 2.

The reactive load admittance matrix

$$\begin{bmatrix} Y^{L} \end{bmatrix} = \begin{pmatrix} jB_{1} & 0 & \cdots & 0 \\ 0 & jB_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & jB_{N+1} \end{pmatrix}$$
(7)

The total power delivered to the loads is the sum of real power flow to the loads and must be equal to the output power of the generator. The generator voltage and the current are found from

$$P = \left[ V^{P} \left[ V^{P} \right] \right] Y^{P} \left[ V^{P} \right]^{*} = \sum_{i=1}^{N} \left| V_{i} \right|^{2} Y_{in,i} = \left| V_{f} \right|^{2} \left| Z_{f} \right|^{2} = \left| I_{f}^{2} \right| Z_{f}$$
(8)

for a real input impedance. The voltage and the current vectors can be reconstructed to include the generator as

$$\begin{bmatrix} V^{P} \end{bmatrix}^{t} = \begin{bmatrix} V_{1} & V_{2} & V_{3} & \cdots & V_{N} & V_{f} \end{bmatrix}$$
(9)  
$$\begin{bmatrix} I^{s} \end{bmatrix}^{t} = \begin{bmatrix} 0 & 0 & 0 & \dots & I_{f} \end{bmatrix}$$
(10)

A system of equations from Eq. (1) can be established as in Eq. (11) and solved for a set of given cavity voltages  $[V^P]$ : for the *m*-th port the transmission-line characteristic impedances,  $Y_{m-1}$ ,  $(Y_m)$  and cavity spacings  $d_{m-1}$  ( $d_m$ ) are given so that the phase delays  $d_m$  ( $d_{m-1}$ ), and reactive loads  $B_m$  are found:

$$I_{f}\delta_{nm} = Y_{m,m}V_{m} - j\{Y_{m-1}V_{m-1}\csc(\beta d_{m-1}) + Y_{m-1}V_{m}\cot(\beta d_{m-1}) + Y_{m}V_{m}\cot(\beta d_{m}) + Y_{m}V_{m+1}\csc(\beta d_{m})\} + jV_{m}B_{m}$$
(11)

(for m = 1, ..., N+1) where *n* is the feed port index.  $Y_m$  and  $B_m$  must be real if their losses can be ignored. The generator can now be matched with  $Y_{in,N+1} = 0$ ,  $B_{N+1}$ , and  $Z_N$  to the network.

#### Load Matching Consideration

Note that  $V_k = 0$  if no power is applied at the *k*-th cavity input. The cavity input voltage can also be defined with the standing wave if the cavity input is not matched to the

waveguide with a reflection coefficient  $\Gamma(z)$ . If a cavity load is mismatched, the port admittance matrix at the input of a cavity is found as:

$$Y_{in} = Y_o \frac{Y_L \cos\beta d^c + jY_o \sin\beta d^c}{Y_o \cos\beta d^c + jY_I \sin\beta d^c}$$
(12)

where  $Y_o$  and  $d^c$  are the characteristic impedance and the length of the transmission line section connects the cavity to the network, respectively;  $Y_L$  is the cavity terminal input impedance, and  $\beta$  is the phase constant. The voltage standing wave in the transmission line section between the cavity and the input port is

$$V(z) = V_o^+ e^{-j\beta_z} \{1 + \Gamma(z)\}$$
(13)

where voltage reflection coefficient is related to load reflection coefficient  $\Gamma(0)$  as  $\Gamma(z') = \Gamma(0)e^{-2j\beta z'}$ . The transmission-line lengths and reactive loads can be realized by using high power phase shifters. The lengths of the transmission line sections and the reactive loads are related to the phase shifts as

$$\phi_n^T = \beta d_n \qquad \qquad \phi_n^L = \cot^{-1}(-B_n/Y_o) \qquad (14)$$

#### Example

If N cavities are fed through a transmission-line network by a single amplifier connected at the *i-th* terminal somewhere in the middle of the network, the above solution process can still be applied by rearranging the elements in the matrices of the admittance, the voltage, and the current.



Figure 3. An example of proposed fan-out power distribution system. Uniformly spaced 12-loads are fed at the center.

A simple fan-out system as an example is shown in Figure 3. 12 uniformly spaced cavities are assumed to have the uniform 50-ohm input impedances and the transmission line characteristic impedances. A set of arbitrary voltages are specified for the example. The frequency of operation is 805 MHz. The result is shown in Table 1.

Table 1 – An example with 12 loads

Cav	$D_i(m)$	$V_{i}(V)$	Zo (Ω)	$d_i(m)$	$B_i(\Omega)$
1	1.50	1.00 <u>/0 °</u>	50.0	1.8742	-0.0056
2	1.50	1.05 <u>/10 °</u>	50.0	1.8690	-0.0026
3	1.50	1.10 <u>/20 °</u>	50.0	1.8673	+0.0020
4	1.50	1.15 <u>/30</u> °	50.0	1.8664	+0.0056

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5	1.50	1.20 <u>/40 °</u>	50.0	1.8659	+0.0088
6	1.50	1.25 <u>/50 °</u>	50.0	1.8330	+0.0715
7		(3.91 <u>/0 °)</u>	50.0		-0.0544
8	1.50	1.25 <u>/50 °</u>	50.0	1.8330	+0.0715
9	1.50	1.20 <u>/40 °</u>	50.0	1.8659	+0.0088
10	1.50	1.15 <u>/30 °</u>	50.0	1.8664	+0.0056
11	1.50	1.10 <u>/20 °</u>	50.0	1.8673	+0.0020
12	1.50	1.05 <u>/10 °</u>	50.0	1.8690	-0.0026
13	1.50	1.00 <u>/0 °</u>	50.0	1.8742	-0.0056

The phase delays can be found in Eq. (14) from the transmission-line lengths,  $d_i$  and the reactances,  $B_i$ .

## CONCLUSION

The proposed fan-out power distribution system may be able to eliminate power overhead that can help achieve efficient operation. One control system needs to govern all cavities in a system as a whole with the output control of the generator. Using an isolator or a circulator at a cavity input is an option to make the input look critically matched. However, matching the individual cavity load is also possible if desired, since the input mismatch at the cavity loads can be determined by measuring the cavity input impedance and the reflection coefficient. Any cavities missing or disabled in the system can have a 0 voltage vector for solving the equation.

The phase delays and reactive loadings can be realized by using high power fast phase shifters. For practical transmission lines, modification of the system admittance matrices will be needed. To achieve fast system control, fast high power phase shifters are required. Matching lowlevel RF control systems need to be developed and implemented. Manual phase shifters may still be useful in certain applications. The method presented here can also be used with more than one single generator, making the system more redundant in certain applications. The approach can also be considered as a multi-port impedance matching method and can be especially useful for a narrow-band RF system.

#### ACKNOWLEGEMENTS

The author thanks the SNS project for the encouragement of the work and allowing contribution to this conference.

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